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ANALYSIS OF HALL-EFFECT THRUSTERS AND ION ENGINES FOR EARTH-TO-MOON TRANSFER

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ABSTRACT

Analytical methods were combined with actual thruster data to create a model for predicting the performance of systems based on two types of electric propulsion thrusters, Hall-effect thrusters and ion engines, for Earth-to-Moon transfer. This paper presents the analysis of flights from an initial Earth-centered trajectory to Moon neighborhood. Analysis performed on the basis of the restricted three body equations showed that the required velocity increment could be closely approximated by a smooth logarithmic function of the specific impulse and specific power. The possible applications of different electric thrusters were considered and possible flight characteristics were determined for initial spacecraft masses from 100kg up to 1500kg.

1. Introduction

Electric thrusters have long been known to be an efficient means of propulsion for space missions. Thirty-years Stationary Plasma Thrusters (SPT) space operation demonstrated their high reliability. The remarkable NASA Solar Electric Propulsion Technology Application Readiness (NSTAR) ion engine operation as the primary propulsion for the Deep Space 1 (DS-1) probe demonstrated high versatility for varying flight tasks and satellite performances.

Earth-to-Moon transfer with electric propulsion essentially differs from one with chemical transfer. As a rule, a transfer with chemical thruster is a two-impulsive Hohmann maneuver. This maneuver consists of three stages: i) thrust impulse on the initial orbit ii) coasting flight on an elliptical orbit iii) thrust impulse in the apogee of this elliptical orbit. The flight time from initial circular orbit with altitude 200km to Moon orbit, is less than 5 days. The required propellant mass at specific impulse I_{sp}=350s is approximately 68% of initial on-orbit spacecraft mass. The transfer with electric propulsion is motion along a spiral trajectory as a result of continuous thruster operation. As is well known, transfer with electric propulsion is much slower than one with chemical thruster. However, as large loads are transported by slow ships and not by fast planes, the same approach may well apply for space flights. Furthermore, it is possible to consider a mission with a number of small electrically propelled spacecraft consecutively initiating flight from low Earth orbit, "slowly" moving in space in the course of one year, and simultaneously observing condition of space at various points.

This paper presents a general analysis of electrically propelled spacecraft flights from an initial Earth-centered orbit to Moon neighborhood.

2. Problem Statement

For the Earth-to-Moon transfer depicted in Fig.1, the system equations describing the spacecraft trajectories correspond to those of the restricted three-body problem. The Earth and Moon will be assumed to move in circles around their common center of mass (barycenter) and their gravitational fields will be assumed spherical. X_e, X_m , the Earth and Moon centers distances to barycenter are given by,

$$X_e = R_m \frac{m_m}{m_e + m_m} = R_m \frac{\mu_m}{\mu_e + \mu_m}, \; X_m = R_m - X_e,$$

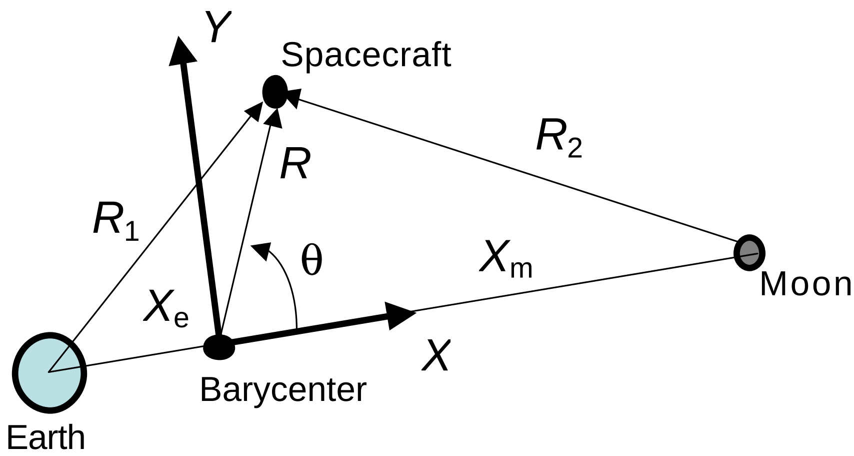

Fig.1: Spacecraft in Earth-Moon Rotating Frame

where R_m is the constant Earth-moon range, m_e, m_m, $\mu_e = Gm_e; \mu_m = Gm_m$ are the Earth and moon masses and gravitational constants respectively and G is the universal constant of gravitation. R, R_1 and R_2 are the spacecraft's distances tobarycenter and to centers of Earth and Moon, respectively.

The equations of motion of the spacecraft in the Earth-Moon rotating coordinate system, centered at their barycenter, are given by (Ref.1)

$$X'' = 2\omega Y' + \omega^2 X - \left(\frac{\mu_e}{R_1^3} + \frac{\mu_m}{R_2^3}\right) X - \frac{\mu_e X_e}{R_1^3} + \frac{\mu_m X_m}{R_2^3} + a_x \quad (1)$$

$$Y'' = -2\omega X' + \omega^2 Y - \left(\frac{\mu_e}{R_1^3} + \frac{\mu_m}{R_2^3}\right) Y + a_y \quad (2)$$

where X, Y are the spacecrafts coordinates, $R^2 = X^2 + Y^2$; w is the orbital angular Moon velocity; a is the control acceleration, $a^2 = a_x^2 + a_y^2$.

The control acceleration is $a = \frac{F}{M_{s/c}}$, where F is thrust and $M_{s/c}$ the actual spacecraft mass, with initial value M_0 and final value M_f. The spacecraft mass $M_{s/c}$ for constant propellant mass flow rate $\dot{m}_s$ is given by $M_{s/c} = M_0 - \dot{m}_s t$ with t the burning time.

In an electric engine, propellant mass flow rate, thrust and control acceleration are respectively given by,

$$\dot{m}_s = 2\frac{P}{I_{sp}^2}\eta_t \quad (3)$$

$$F = \sqrt{2\dot{m}_s P \eta_t} = 2\frac{P}{I_{sp}}\eta_t \qquad (4)$$

$$a = \sqrt{2\frac{M_0}{M_{s/c}}\dot{m}_s P_s \eta_t} = 2\frac{M_0}{M_{s/c}}\frac{P_s}{I_{sp}}\eta_t \qquad (5)$$

with h_t thrust efficiency, a function of specific impulse; P power consumption; $P_s = P/M_0$ specific power and I_{sp} specific impulse.

Let us define spacecraft normalized mass

$$\overline{M} = \frac{M_{s/c}}{M_0} = 1 - \frac{2P_s\eta_t}{I_{sp}^2}t \qquad (6)$$

Control acceleration is thus given by,

$$a = \frac{2P_s\eta_t}{\overline{M}I_{sp}} \qquad (7)$$

Now that all necessary equations are defined we shall determine the spacecraft trajectories from initial Earth-centered orbit to Moon sphere of influence. We shall assume that constant thrust is applied along the spacecraft velocity vector,

$$a_X = a\frac{X'}{\sqrt{(X')^2 + (Y')^2}};a_Y = a\frac{Y'}{\sqrt{(X')^2 + (Y')^2}} \qquad (8)$$

Even though optimal thrust is varying both in amplitude and direction, due to the almost flat characteristics of the cost function this approximation is good enough to study the influence of various thrusters on overall system performance. The calculations goal is determination of the flight time and normalized final mass for difference thruster performances.

3. Input data

The possibilities of use of various thrusters as given in Table 1 will be discussed in this work. Their respective data was obtained from public available sources.

3.1. Specific impulse

The values of specific impulse considered for calculations are: $I_{sp} = 10, 20, 30, 40 \ km/s$. Thrusters' performances are all within this range.

3.2 Relative power

Present achieved relative power levels are 3-5W/kg (Ref.3). Predicted relative power can be more than 6W/kg. For calculations following values were employed: $P_s = 3, 5, 7\ W/kg$.

Table 1: Electric Thrusters Characteristics

Thruster	Operation mode	Specific impulse, km/s	Power consumption, W	Efficiency	Country of origin
KM-37	1	14.050	200	0.403	Russia
HETI	1	13.200	215	0.350	Israel
KM-37	2	16.400	300	0.496	
HETI	2	13.000	520	0.480	
SPT-70	1	14.000	750	0.500	Russia
HETI	3	16.300	790	0.500	
TAL-D38	1	17.900	800	0.510	Russia
NSTAR	3	29.420	1018	0.554	USA
SPT-100	1	15.000	1350	0.500	Russia
PPS-1350	1	17.200	1500	0.510	France
NSTAR	2	31.420	1579	0.618	
NSTAR	1	31.270	2325	0.618	

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3.3.Propellant and efficiency

Xenon is assumed the propellant for all thrusters. Efficiency according to Ref.2 is assumed

$$\eta_t = 0.75 e^{-5/I_{sp}} . \qquad (9)$$

3.4.Initial orbit

We assume an initial low Earth circular orbit with radius $R_0 = 6800\ km$, initial angle θ, shown in Fig.1 is determined for each case in order to reach Moon sphere of influence.

3.5. Constants

The constants employed for all simulations are:

$$R_m = 384710\ km;\ \ \mu_e = 3.986\times 10^5 km^3/s^2;\ \ \mu_m = 4.903\times 10^3 km^3/s^2;\ \ \omega = 2.683\times 10^{-5} rad/s.$$

4. Results of calculations

Results of calculations at constant operation modes are presented in Table 2. The calculations were realized at constant operation modes (specific impulse and power are constant) and at variable operation modes. The way to reach the optimal variable operation mode was obtained in Ref.1: optimal control acceleration is a decreasing function of flight time in order to achieve a maximum final mass. In this work, actual implementation of the optimal control acceleration was approximated the following way: The mass flow rate and/or the specific impulse were piecewise changed with specific impulse increased within the range 10 to 40km/s and specific power decreased within the range 7 to 1W/kg. With this approximation, thrust decreased by a factor of five at end of flight. The maximal thrust decrease was limited by existing thruster performances. Results for variable operation mode are presented in Table 3.

Table 2: Constant Operation Mode Results

Specific power, W/kg	Specific impulse, km/s			
	10	20	30	40
	Flight time, days			
3	161	253	332	402
5	104	169	226	274
7	78	127	172	212
	Normalized final mass			
3	0.621	0.809	0.879	0.914
5	0.590	0.787	0.862	0.902
7	0.572	0.775	0.853	0.894

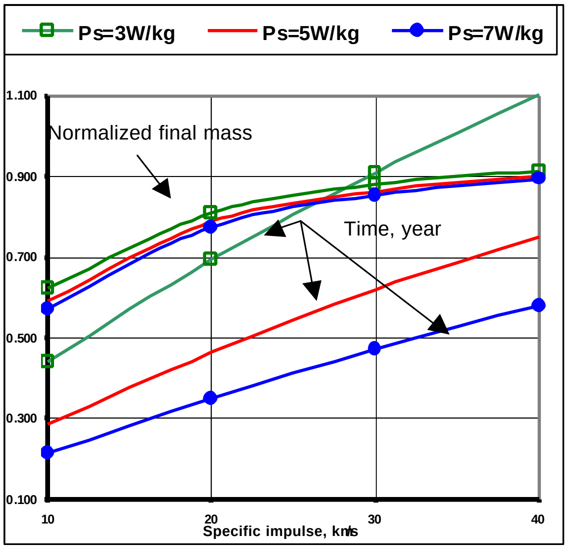
Table 3: Variable Operation Mode Results

Specific power (average), W/kg	Initial specific impulse, km/s	Specific impulse (average), km/s	Ratio of initial to final thrust	Flight time, days	Relative final mass
3.38	30	30	2.4	317	0.868
2.35	30	28	5.7	427	0.867
3.38	20	20	2.4	235	0.794
2.35	20	18.7	5.7	324	0.792
4.48	20	20	1.3	185	0.791
4.64	30	30	1.2	242	0.863

Data obtained at constant operation mode are visualized in Fig.2. Normalized final mass decreases by less than 5% for a specific power increase from 3W/kg to 7W/kg while flight time decreases more than twice for the same specific power increase. Final spacecraft mass decreases with increasing specific power. In order to avoid mass loss specific impulse is increased and, hence, also flight time. Flight time increase is approximately 5-8 days per 1km/s of specific impulse increase. All obtained data are visualized in Fig.3 where normalized final mass is shown as a function of specific impulse. As usual in electric thrusters the final mass function is *fairly flat*. On the other hand, small mass increase is accompanied by sizeable flight time increase. This specific characteristic of electric propulsion systems has an important consequence: even under a propulsion or power system partial failure, the delivered final mass will remain almost the same, at the expense of a later mission completion.

For constant thrust, velocity increment is given by,

$$\Delta V = -I_{sp} \ln \overline{M} \qquad (9)$$



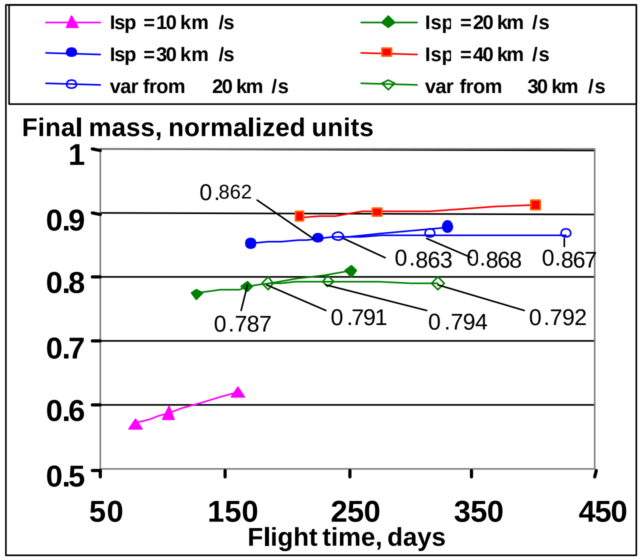


Fig.2: Transfers in constant operation mode

Fig. 3: All operation modes

The value of ΔV for the various thruster cases is shown in Fig. 4 as a function of thruster parameter $I_{sp} / P_s \eta_t$.

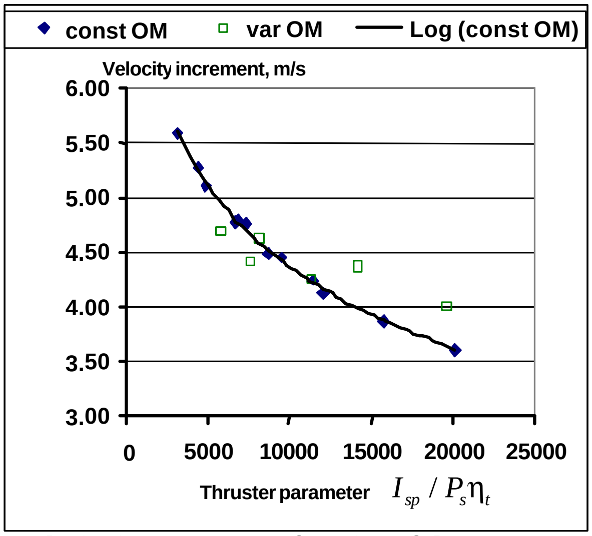


Fig.4: Velocity increment as a function of thruster parameter

Required velocity increment can be closely approximated by a smooth logarithmic function,

$$\Delta V = -1.064 \ln(I_{sp} / P_s \eta_t) + 14.166 \qquad (10)$$

In this approximation the constants appearing in Equation (10) were calculated with specific impulse in km/s, specific power in kW/kg, velocity increment in km/s and flight time in days.

From Equations (9) and (10) follows that normalized final mass is

$$\overline{M} = (I_{sp} / P_s \eta_t)^{1.064/Isp} e^{-14.166/I_{sp}} \qquad (11)$$

In view of Equation (9) and equality: $1 - \overline{M} = \dot{m}_s T / M_0$, flight time is

$$T = (1 - \overline{M}) I_{sp}^2 / 172.8 P_s \eta_t \qquad (12)$$

The following dimensions were employed in Equations (10)-(12): Specific impulse-km/s, Specific power-kW/kg, Velocity increment –km/s; Flight time -days.

The thruster parameters averages were used in calculations for variable operation modes, and the same equations (10)-(12) can be also used for assessment of flight performances with variable operation modes. The deviation from the logarithmic approximation line is below 400m/s. This corresponds to less than a 3% spacecraft mass error at average specific impulses between 20 to 30 km/s.

5. Thrusters Options

We shall now assess possible flight characteristics for different initial spacecraft masses taking into account thrusters' performances from Table 1.

Flight time and normalized final mass can be directly obtained from Equations (11), (12). However, actual final mass still requires further analysis. In general, power capacity of a spacecraft with an electric propulsion subsystem can be divided into three parts. The first part is necessary for spacecraft basic functions (structure, attitude and thermal control, communications, etc.). The second part is intended for the payload. In general, it is this part that is assumed for use completely or partially by the electric propulsion system during transfer. The third part is some additional power required by the electric propulsion system. It is not required when using chemical propulsion. So, the final mass value as obtained from Eq.11 should be decreased to take into account the additional mass requirements of the power system. Based on previous experience we shall correct for the final mass according to the following expression,

$$M_{fr} = \overline{M}_f M_0 - (P_s - 3M_0)/\alpha \qquad (13)$$

where α is power system specific power, assumed to be $\alpha = 30W/kg$.

Results of calculations are presented in Fig. 5-10 for various values of initial spacecraft mass. The number of thrusters for each case assured that normalized final mass $\overline{M}_{fr} \geq 0.5$. The following notation was used for the various thrusters: **AAAA (BB_CCC*D)**, where **AAAA** is thruster name; **BB-** rounded off specific impulse, km/s; **CCC-** power consumption, Watt; **D** –number of thrusters operated simultaneously.

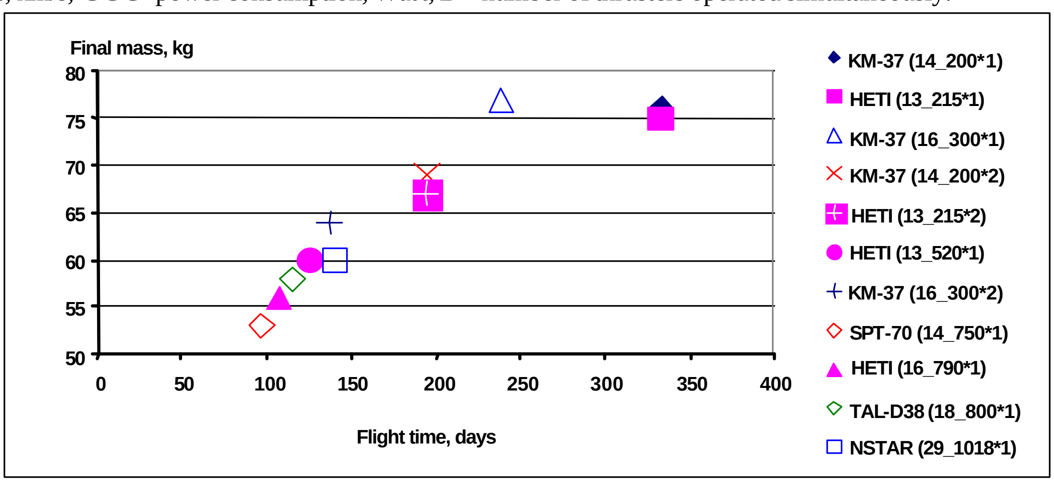


Fig. 5: Initial spacecraft mass 100kg.

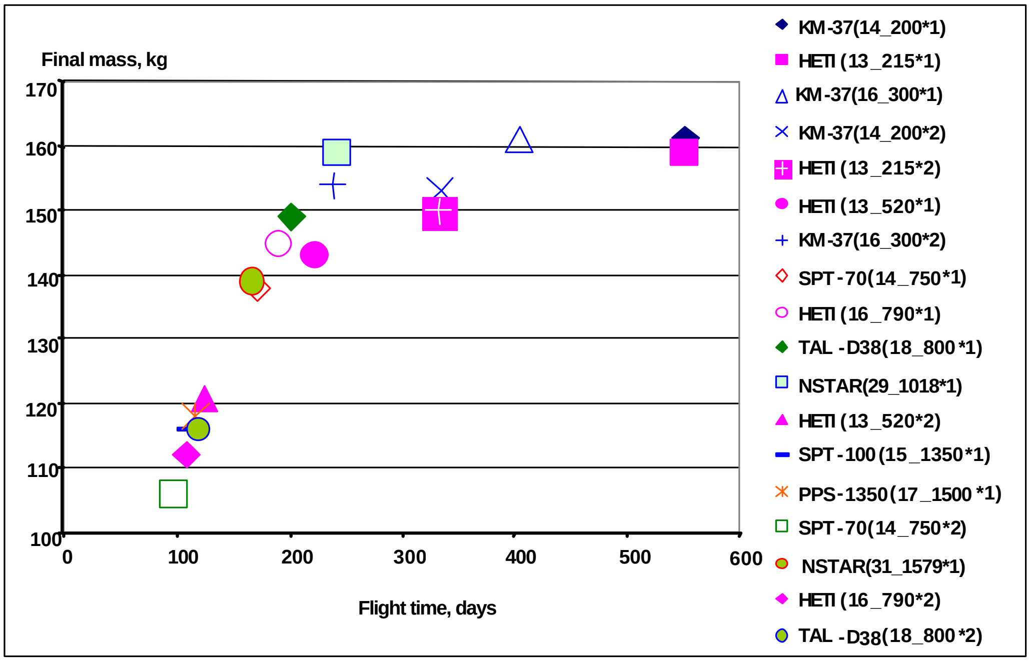


Fig. 6: Initial spacecraft mass 200kg

The thruster lifetime characteristics were not taken into account in the calculation of the diagrams. Evidently, the smallest thrusters cannot operate during the required time. It is necessary to add additional thrusters, thruster selection unit, tubes, valves and cables. As a result final mass will be decreased by 8-12 kg. It would seem that more powerful thrusters, operating in an on-off mode, can replace less powerful thrusters completely. But on-off operation will result in an increase of propellant consumption and reduction of final mass.

All M_{fr}-T diagrams with points representing different types of thrusters have similar characteristics. We see both, groupings of thrusters and empty intervals. Groupings of thrusters are a positive characteristic. In this case, more than option is available to fulfill the mission with different types of electric thrusters.

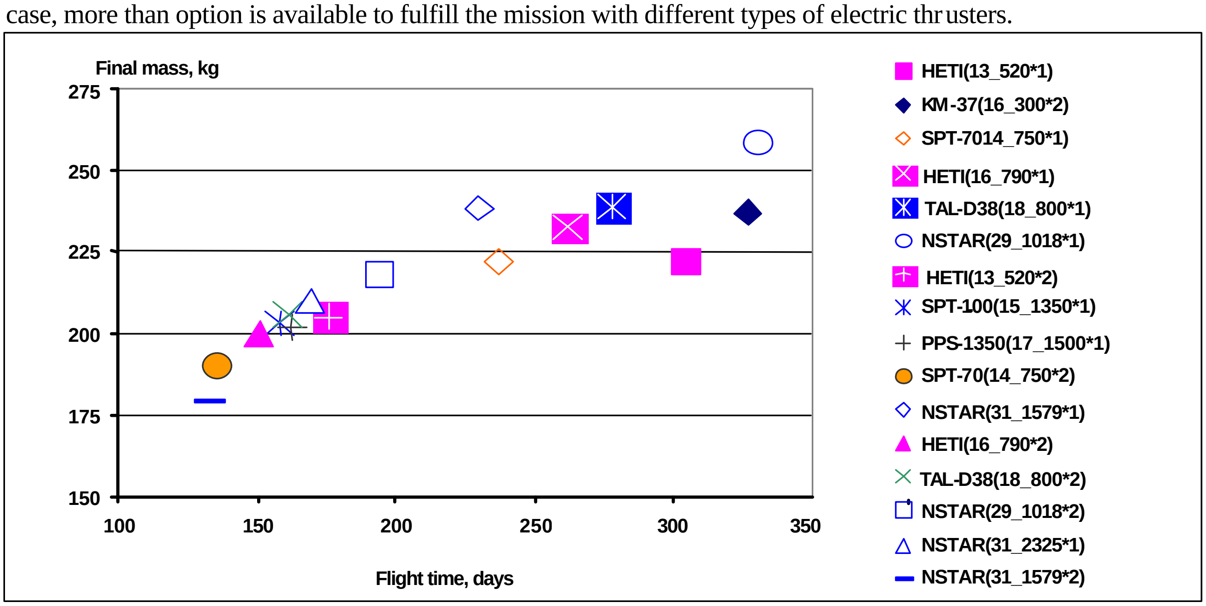


Fig. 7: Initial spacecraft mass 300kg

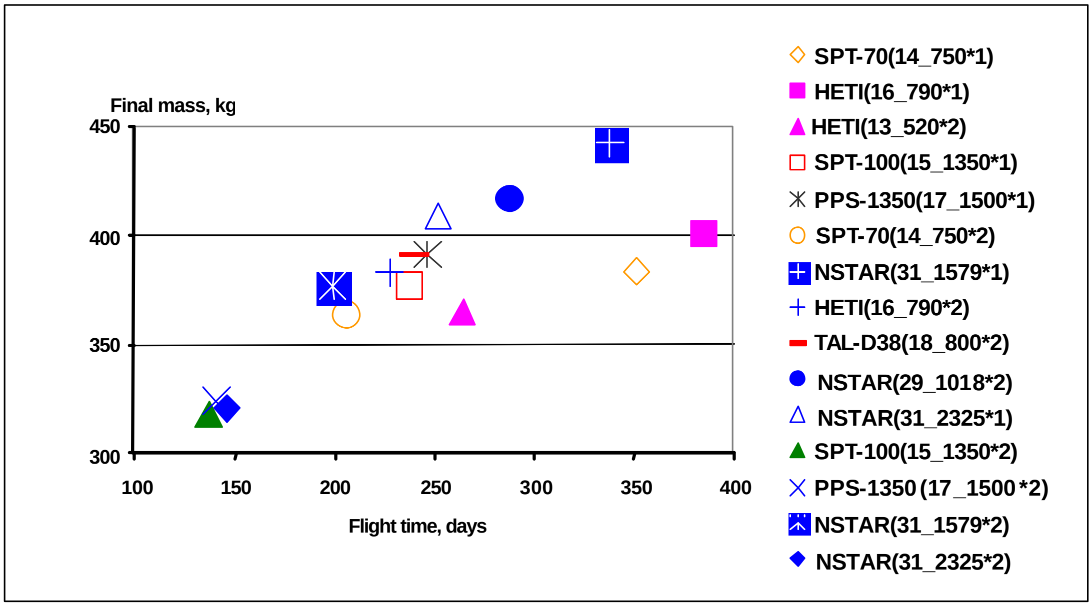


Fig. 8: Initial spacecraft mass 500kg

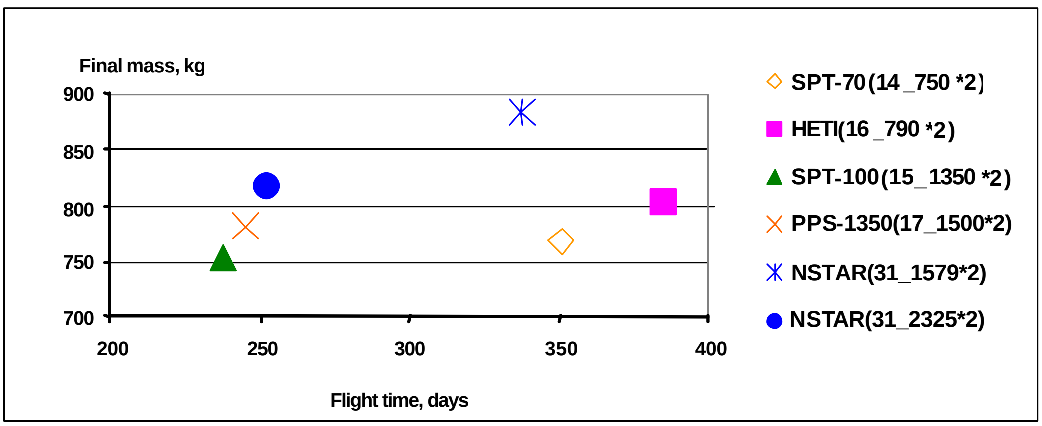


Fig. 9: Initial spacecraft mass 1000kg

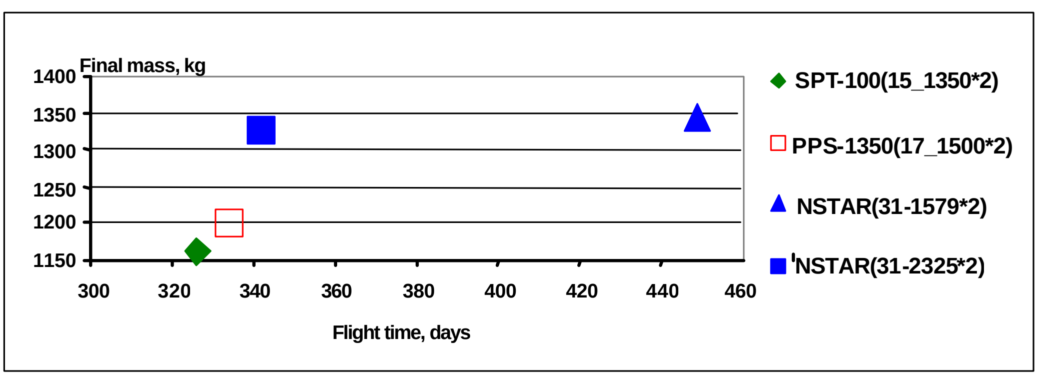


Fig. 10: Initial spacecraft mass 1500kg

Enlarging admissible operation modes of existent thrusters can fill the diagrams empty intervals. NSTAR is presently the best example. This approach is simpler and less expensive than new thruster development for every case. Range of operation modes should be considered as one of the main characteristics of an electric thruster, the same way as specific impulse and lifetime.

6. Summary and Conclusions

Computations performed on the basis of the restricted three-body equations of motion enabled to determine the final spacecraft mass, flight time and required velocity increment. Analysis of the results showed that required velocity increment for Earth to moon transfer could be approximated by a smooth logarithmic function of a basic electric thruster parameter, the ratio of specific impulse to specific power.

The possible applications of different engines with their actual characteristics taken into account are calculated for initial on-orbit spacecraft masses from 100kg up to 1500kg. Number of employed simultaneous thrusters was defined according to thruster parameters. Examining obtained points on final mass-flight time plane it is possible to see a grouping of thrusters (many thrusters with similar performances) and many empty intervals. Enlarging admissible operation modes of existent thrusters can fill the discovered empty intervals. Range of operation modes should be considered as one of the main characteristics of an electric thruster, the same way as specific impulse and lifetime.

The proposed thruster parameter and obtained mass and time equations can be used for definition of the required ranges of specific impulse and power at selected spacecraft masses and estimated flight durations.

7. References

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