

# END TO END OPTIMIZATION OF THREE DIMENSIONAL CHEMICAL-ELECTRIC ORBIT RAISING MISSIONS

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## Abstract

A low thrust trajectory optimization model is combined with a launch vehicle performance model to derive mass optimized three dimensional end-to-end chemical-electric orbit raising (C-EOR) profiles to geostationary orbit (GEO). Optimized profiles are derived for the Sea Launch, Ariane 4, Atlas V, Delta IV, and Proton launch vehicles. Optimum EOR starting orbits are shown and payload mass benefits are calculated for each vehicle. The payload mass benefit is shown to be between 5.9 and 7.6 kg/day with two SPT-140 thrusters, or up to 680 kg. for 90 days of electric orbit raising. A previously presented analytic staging model is used as a basis for a simple parametric performance model covering multiple launch vehicles. The model is an ideal tool for system level analysis of electric orbit raising missions and matches calculated performance to within 10%.

This work demonstrates Space Systems/Loral's ability to optimize electric orbit raising missions to GEO with major commercial launch vehicles, an important and enabling step to the use of optimized C-EOR trajectories on commercial satellites.

## Nomenclature

$I_{sp}$ = specific impulse (sec)	$m_o$ = spacecraft mass, beginning of orbit raising (kg)
$P$ = Thruster input power (W)	$m_1$ = spacecraft mass, end of chemical orbit raising, before EOR (kg)
$T_2$ = electric thruster thrust (N)	$m_2$ = spacecraft mass, end of orbit raising (payload mass) (kg)
$a$ = semi-major axis (m)	$\dot{m}_2$ = electric propulsion mass flow rate (kg/sec)
$a_{EOR}$ = semi-major axis of EOR starting orbit (m)	$t$ = time (sec)
$a_f$ = target semi-major axis (m)	$\Delta v_{chem}$ = $\Delta v$ for all-chemical orbit raising (m/s)
$a_{GEO}$ = semi-major axis of GEO (m)	$\Delta v_1$ = $\Delta v$ for chemical part of C-EOR mission (m/s)
$c_1$ = effective chemical thruster exhaust velocity (m/s)	$\Delta v_2$ = $\Delta v$ for electric part of C-EOR mission (m/s)
$c_2$ = effective electric thruster exhaust velocity (m/s)	$\Delta v_{2eff} = \Delta v_{chem} - \Delta v_1$
$c^*_2$ = optimum effective electric thruster exhaust velocity (m/s)	$\eta_p$ = thruster efficiency
$e$ = eccentricity	$\eta_v$ = mission planning efficiency

## Introduction

Western commercial communications satellites are generally launched into a transfer orbit and then boosted into geostationary orbit (GEO) using an on board chemical propulsion system (OBS). This process is referred to as orbit raising. Commercial manufacturers are now using electric propulsion systems for orbit raising.<sup>1,2</sup> Electric orbit raising (EOR) occurs over a period of days or months and can save hundreds of kilograms of mass. Although it is possible to accomplish all of orbit raising using EOR, the time required (many months) is prohibitive for commercial applications. Combined chemical-electric missions (C-EOR) provide customers with significantly greater payload capacity and yet maintain an acceptable on-orbit delivery time. A mission with 90 days of EOR can provide a net payload mass benefit of over 630 kg.<sup>3</sup> From the point of view of the satellite owner, optimization requires maximizing the dry mass benefit per day of EOR, or transportation rate. When the power available for EOR is fixed, there is an inverse relationship between thrust and specific impulse ( $I_{sp}$ ) and there is an optimum  $I_{sp}$  that maximizes transportation rate.

In previous work, a series of simple models were developed for combined chemical-electric orbit raising missions.<sup>3</sup> These models treat orbit raising as a series of chemical/electric stages and were used to derive a modified form of the rocket equation for C-EOR missions. A combination numerical/graphical optimization method was used to generate a Sea Launch orbit raising profile that optimizes the overall performance of the launch vehicle, OBS, and electric propulsion systems. It was shown that use of the optimum EOR starting orbit, a subsynchronous circular orbit, results in considerably higher transportation rates than the elliptic starting orbit commonly used today. The multi-stage C-EOR model showed that the difference in performance is caused by differences in the staging strategy associated with different EOR starting orbits.

Taken together, previous work has shown the importance of optimizing the end-to-end orbit raising mission rather than optimizing the low thrust mission without considering the other mission stages. This approach emphasizes the need to consider the whole system, including the launch vehicle, chemical OBS, and electric propulsion stages, when optimizing EOR thrusters, trajectories and steering profiles.

In this paper, we extend the numerical/graphical optimization method to 3D and derive end-to-end optimized trajectories for some major commercial launch vehicles. We also address apparent discrepancies between our previous work on Sea Launch C-EOR missions and work by other authors. We then show how previously developed two stage and three stage C-EOR models can be effectively applied to multiple launch vehicles.

The result is simple model that can be used to calculate C-EOR mission performance to GEO with a variety of different thrusters and launch vehicles. The model is an ideal tool for system level analysis of electric orbit raising missions and for trade studies involving different types of electric thrusters, power systems, and thrust levels.

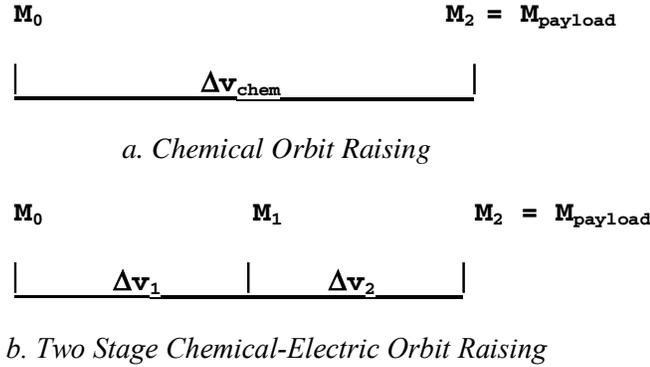
This work demonstrates Space Systems/Loral's ability to optimize electric orbit raising missions to GEO with major commercial launch vehicles. This is an important and enabling step to the use of optimized C-EOR trajectories on commercial satellite missions.

## Previous Work

Considerable work has been done on the optimization of low thrust missions of all types, including orbit raising.<sup>4,5,6,7,8</sup> Much previous work has focused on optimization of the low thrust portion of the mission and has ignored or simplified the chemical portion of the mission. While this approach is appropriate for missions dominated by electric propulsion (such as deep space missions), commercial EOR missions are dominated by the launch vehicle and the chemical OBS. It is important to optimize usage of the chemical stages when optimizing the overall C-EOR mission.

Oleson has calculated optimum EOR starting orbits to GEO for many different launch vehicles using the Solar Electric Propulsion Steering Program for Optimal Trajectories (SEPSPOP).<sup>9</sup> The version of SEPSPOP used contained a simple chemical staging model limited to a single high thrust stage with a circular starting orbit. It was therefore assumed that the launch vehicle placed the satellite and its OBS into a circular parking orbit. This assumption has a significant impact on the overall optimization, which will be discussed in detail later in this paper.

In our previous work, a simple model was presented for a combined chemical-electric orbit raising mission (C-EOR) based on a modified form of the rocket equation.<sup>3,10</sup> In its simplest form, the model considers the overall mission in two stages, one chemical and one electric, as shown in Figure 1. This is referred to as the “two stage” C-EOR model.



**Figure 1: Simple Model of Orbit Raising Missions**

A mission planning efficiency was defined to represent the effectiveness of the thrust trajectory and steering profile relative to an all chemical mission.

$$\eta_v \equiv \frac{\Delta v_{2eff}}{\Delta v_2} \quad (1)$$

The following expression was then derived for payload mass fraction at the end of electric orbit raising.<sup>10</sup>

$$e^{-\Delta v_{chem}/c_1} = \frac{m_2}{m_o} \frac{\eta_p Pt}{\gamma_2 m_2 c_2^2} + 1 \quad \left(1 - \frac{\eta_v c_2}{c_1}\right) \quad (2)$$

This expression serves as the “rocket equation” for a generic C-EOR mission. From it, simple expressions were derived for EOR transportation rate and optimum electric specific impulse at a fixed power level.<sup>3</sup>

$$c_2 = \frac{2c_1}{\eta_v} \quad (3)$$

$$\frac{dm_2}{dt} = \frac{\eta_v T_2}{c_1} - \frac{T_2}{c_2} \quad (4)$$

These expressions are strictly valid for “short duration” EOR missions, but have been shown to accurately model missions of up to 100 days duration when planning efficiency remains constant with respect to time. Additional expressions were derived for EOR transportation rate and optimum electric specific impulse in the following cases.

- Variable efficiency: accounts for changes in efficiency as thrust and specific impulse are throttled at constant power.
- Tank capacity limited: models missions constrained by the size of the propellant tanks for the chemical on board system (OBS)
- Three stage model: includes an electric stage, chemical OBS, and the launch vehicle. Assumes that the chemical OBS  $\Delta V$  remains constant.

These models have been described in detail in previous work and are not further described in this paper. Also in previous work, a trajectory optimization model developed by MIT was combined with a Sea Launch vehicle performance model to derive an optimized 2D end-to-end C-EOR mission profile.<sup>3</sup> The result showed that a subsynchronous circular starting orbit maximizes the mass benefit per unit time, or transportation rate, when launching using a Sea Launch vehicle. This result differs significantly from some previously reported SEPSHOT results for reasons that are discussed below. Because the optimized Sea Launch profile matches the three stage C-EOR model, it performs much better than current methods based on a two-stage inertial fixed steering profile.<sup>4</sup>

This paper begins with a review of the 2D Sea Launch mission optimization. It then discusses differences between the 2D Sea Launch optimization and those of SEPSHOT. The end-to-end optimization method is then extended to 3D to cover multiple launch vehicles. Finally, two stage and three stage mission models are applied to the results to present a simple parametric performance model covering many C-EOR missions to GEO.

### **2D Sea Launch End-to-End Optimization**

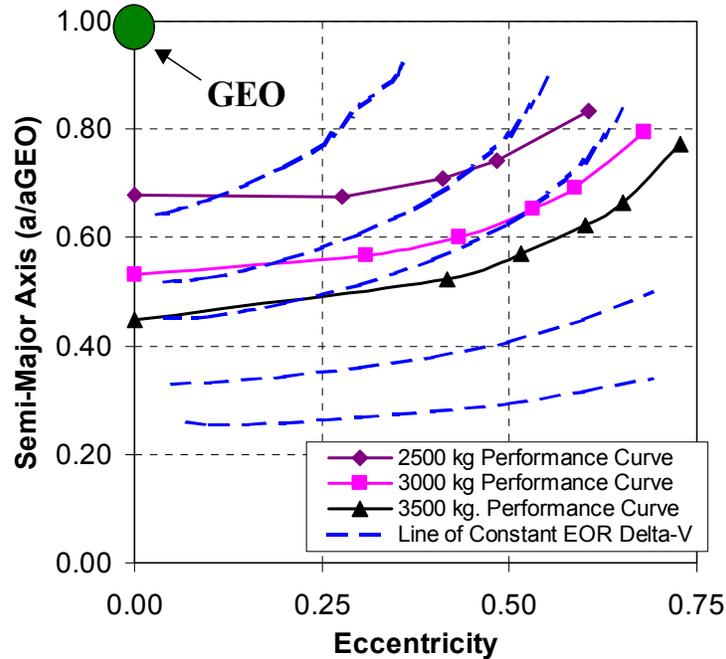
A simulation of a Sea Launch C-EOR mission to GEO was carried out using a low thrust optimization method developed by Martinez. This model was combined with a simple Sea Launch performance model derived from public domain data.<sup>11</sup> Both the MIT model and the overall numerical/graphical optimization method have been previously described in detail.<sup>3,12</sup> A brief overview of the overall method is presented below.

The objective is to identify the family of EOR starting orbits that maximizes EOR transportation rate on a given launch vehicle. To simplify the analysis, the following assumptions are made.

- A fixed amount of power is available for EOR
- The electric propulsion (EP) device is always on (no coast periods)
- The EP device's specific impulse ( $I_{sp}$ ) is constant throughout the mission
- The LV separation orbit is identically zero inclination
- External tidal/multi-body perturbation effects are neglected
- Orbit phasing (aiming for a particular final longitude) is neglected
- There is no limit on fuel tank size onboard the satellite.

Because the EP device is always on during EOR, minimizing electric  $\Delta V$  maximizes both payload mass and transportation rate. Sea Launch has a restartable upper stage and can inject its payload into a range of separation orbits near zero inclination. We begin by considering missions in which the satellite is injected directly into its EOR starting orbit and has no chemical OBS. The overall optimization is done by graphically comparing the launch vehicle's separated mass capability at different injection orbits with the electric  $\Delta V$  required to reach GEO from that orbit.

The electric  $\Delta V$  is calculated using an optimization code developed at MIT. It begins with the standard orbital perturbation equations and then does a two step optimization using LaGrange multipliers. The first step optimizes the steering profile within each orbit to achieve the desired change in orbital elements with minimum  $\Delta V$ . The second step uses orbit averaged equations to optimize the overall path to GEO. The optimization is done numerically and is described in detail in Ref. 12. A contour map is generated of the electric  $\Delta v$  required to reach GEO as a function of starting eccentricity and semi-major axis. Launch vehicle performance curves are superimposed on the contour map and the result is examined to determine the optimum EOR starting orbit. For a given separated mass, the objective is to find the starting orbit that requires the least electric  $\Delta V$  (and therefore the shortest time) to reach GEO. A typical Sea Launch contour map is shown in Figure 2 below. The performance curves are taken from Ref. 11 and assume direct injection into the EOR starting orbit.



**Figure 2: Sea Launch Performance Overlaid on Contours of Constant Electric  $\Delta V$**

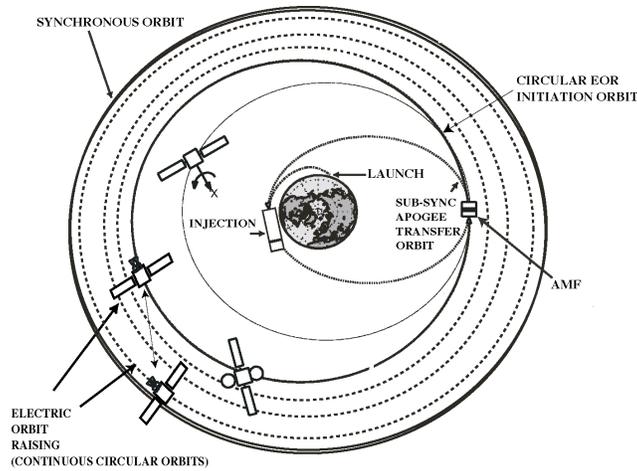
The target orbit in Figure 2 is  $a/a_t = 1$  and  $e = 0$ . Each EOR contour is a group of starting orbits that require the same EOR  $\Delta V$ , and therefore the same amount of time, to reach GEO. Each launch vehicle performance curve is a families of orbits to which Sea Launch can deliver a given separated mass. The optimum separation orbits in Figure 2 are circular orbits located along the left side of the graph. The optimum EOR thrust profile is therefore spiral orbit raising, in which the satellite’s orbit is always circular and the thrust vector is always directed along the velocity vector.

Once the optimum EOR starting orbits are identified, the planning efficiency can be directly calculated and the EOR mass benefit and transportation rate can be derived for a given thrust and power level. Typical performance characteristics for on board chemical and electric thrusters are shown in Table 1.

Thruster	Thrust (N)	$I_{sp}$ (s)	Power (W)
Biprop <sup>13</sup>	445	322	N/A
SPT <sup>14</sup>	0.288	1780	4500
XIPS <sup>15</sup>	0.165	3500	4500

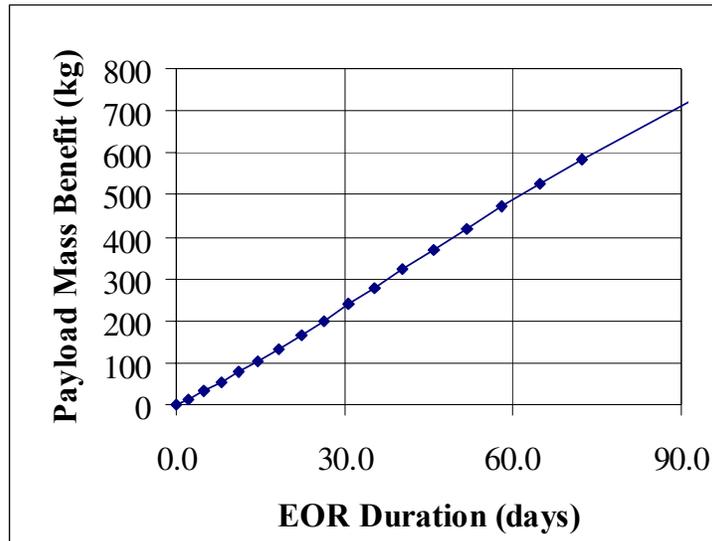
**Table 1: Typical Thruster Performance Parameters**

Although the optimization assumes a direct injection to EOR, conceptually a chemical OBS acts as an extra upper stage for the launch vehicle. As a result, the optimum starting orbit for EOR with the chemical OBS is the same as in the direct injection case, giving the orbit raising profile shown in Fig. 3.

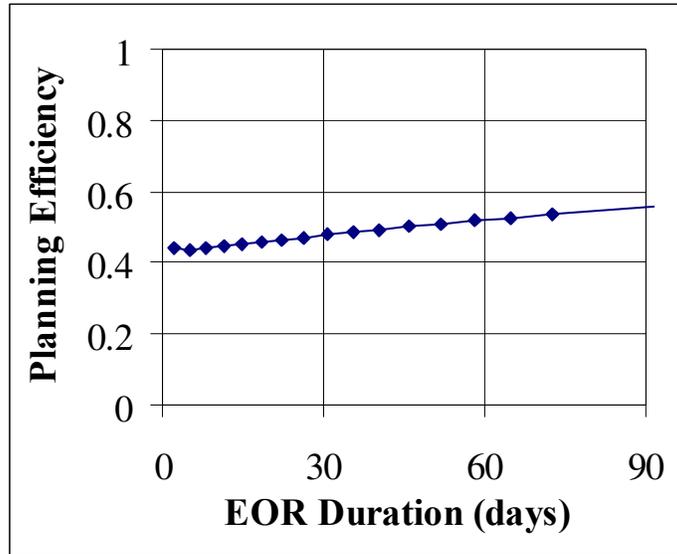


**Fig. 3: Transportation Rate Optimized Sea Launch C-EOR Mission  
(AMF = Apogee motor firing)**

Assuming the use of two SPT-140 thrusters with 9 kW of input power, the following mass benefit and planning efficiency are calculated for a Sea Launch C-EOR mission with an onboard bipropellant orbit raising thruster.



**Figure 4: Payload Mass Benefit vs. EOR Duration on Sea Launch**



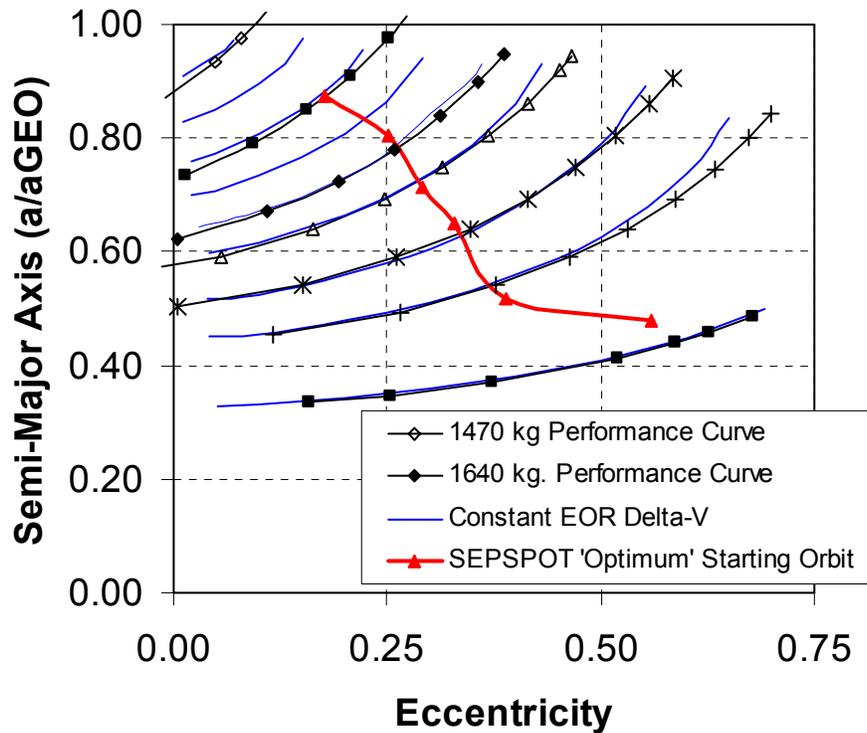
**Figure 5: Sea Launch Planning Efficiency vs. EOR Duration**

The result is a nearly constant transportation rate of 7.5 kg/day, which corresponds to a mass benefit of over 650 kg. from 90 days of EOR. The planning efficiency varies little with EOR duration. This justifies the assumption of constant planning efficiency made when deriving the multi-stage C-EOR rocket equation and performance model.

#### Comparison with SEPSHOT Results

As noted earlier, SEPSHOT is able to calculate optimized EOR starting orbits for some C-EOR missions and has been used to simulate Sea Launch C-EOR missions.<sup>9</sup> This section reviews SEPSHOT results previously reported in the literature and compares these results to those shown in Figure 2 above.

SEPSHOT includes a chemical stage in its optimization, but is limited to a single high thrust stage with a circular starting orbit. As a result, it does not directly simulate the final elliptical separation orbit from the launch vehicle. Instead, the launch vehicle is assumed to place the satellite and its OBS into a low altitude circular parking orbit. To study the effect this assumption has on the overall mission, the graphical optimization method described in the previous section was applied to a satellite with a chemical OBS that starts in a zero inclination 200 km altitude circular orbit. The separated mass is arbitrarily assumed to be 5000 kg and the OBS specific impulse is approximately 320 seconds. A two burn transfer is used to reach an EOR starting orbit between LEO and GEO. Chemical stage performance curves are calculated by varying the burns to generate lines of constant EOR starting mass. The results are summarized in Figure 6 below. Also shown in Figure 6 are “optimum” Sea Launch EOR starting orbits calculated using SEPSHOT. SEPSHOT results are derived from data presented in Ref. 9.



**Figure 6: Circular LEO Starting Orbit Performance Curves Overlaid on Contours of Constant Electric  $\Delta V$**

The results in Figure 6 differ significantly from those in Figure 2. Although somewhat difficult to see, for each mass curve in Figure 6 there is a shallow minimum where the contour is tangent to an EOR  $\Delta V$  line. This is the optimum EOR starting orbit. The optimum starting orbits identified by SEPSPOT roughly overlay these minimum points when  $a/a_{GEO}$  is greater than 0.5. This validates the graphical optimization technique in this regime. When  $a/a_{GEO}$  is less than 0.5, SEPSPOT may be optimizing to avoid the Earth's radiation belts. The graphical method does not account for radiation degradation and therefore gives a different answer in this regime. These results both confirm the accuracy of the graphical method and also demonstrate its limitation at low altitude starting orbits. Commercial C-EOR missions are expected to operate at high altitudes where radiation degradation is not a major factor in the optimization.

The differing results of Figure 2 and Figure 6 are explained by considering the different initial conditions used to generate the two plots. When simulating launch vehicles, SEPSPOT's geometric limitations require that one assume that the launch vehicle uses a circular starting orbit. This matches the assumption used to generate Figure 6. In reality, Sea Launch appears to use an elliptical parking orbit for direct injection missions.<sup>11</sup> Figure 2 is based on actual Sea Launch performance curves and therefore accounts for the true performance of the vehicle. Staging effects further complicate the optimization by changing the relative efficiency of apogee and perigee burns. This effect becomes particularly important when considering use of an OBS. The graphical optimization method includes these effects and is therefore more accurate than SEPSPOT when optimizing the Sea Launch end-to-end C-EOR mission to GEO.

Overall, this comparison demonstrates the necessity of accurately including both the launch vehicle and OBS in the end-to-end optimization.

One advantage of the graphical method is that it provides information about the relative sensitivity of the optimization. When starting from a circular orbit, the chemical stage performance curves nearly overlap the constant EOR  $\Delta V$  curves, resulting in very shallow minima. Because the minima are so shallow, as additional constraints are added to the optimization, a numerical optimizer may have trouble converging on the optimum EOR starting orbit. The contours in Figure 6 suggest that when starting from LEO, overall mass performance is actually relatively insensitive to EOR starting orbit.

### **3D End-to-End Launch Vehicle Optimization**

#### Methodology

This section describes methods used to derive optimized C-EOR mission profiles for the Ariane 4, Atlas V, Delta IV, and Proton launch vehicles. These vehicles do not launch from the Equator, so another dimension, inclination change, is included in the analysis. The basic assumptions are the same as in the 2D case with the following exceptions:

- The launch vehicle separation orbit is a fixed non-zero inclination
- Perigee Velocity Augmentation burns (PVA's) using the OBS are not allowed.
- The geometry of the Low-Thrust portion of the mission is restricted to orbits where the line of apsides and line of nodes are co-linear (i.e. apogee and perigee are at the nodes)

PVA's are low altitude perigee burns conducted using the OBS. They are often operationally inconvenient and are therefore excluded from this analysis.

Launch vehicle performance curves are calculated at a given separation inclination using public domain data from the User's Guides.<sup>16,17,18,19</sup> The separation orbit inclination is chosen to optimize the performance of the launch vehicle and is generally close to the latitude of the launch site. Launch vehicle performance is estimated at different apogees and inclinations by extrapolating data in the User's Guides. An exception is the Proton, which has a restartable upper stage that fires at apogee to raise perigee and lower inclination prior to separation.<sup>19</sup> A separation inclination of 30 degrees was used with this vehicle. This is the highest inclination for which GTO orbit performance is reported in the User's Guide. It should be noted that large satellites may not be able to use high inclination separation orbits due to fuel tank volume limitations. Tank size limits are ignored in both the 2D and 3D optimizations. The significance of this assumption is discussed further below.

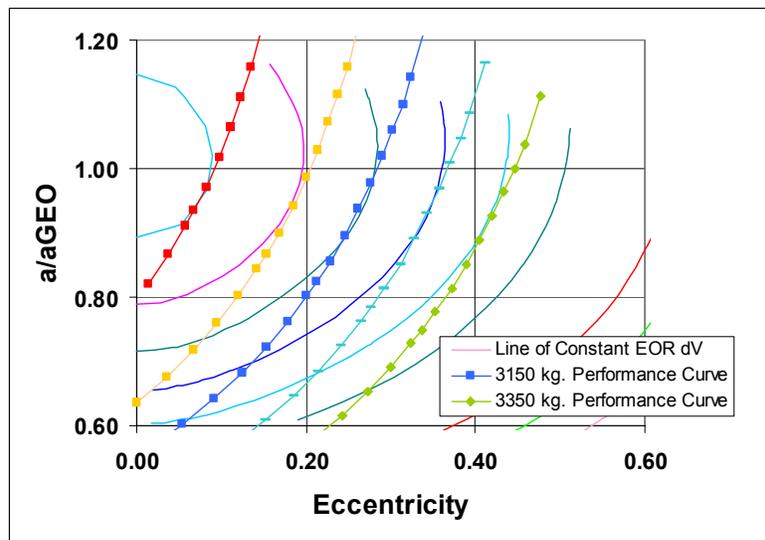
For each vehicle, an all-chemical orbit raising mission is defined as a reference baseline. This mission consists of a GTO injection followed by an apogee burn from the OBS. Because the apogee of the separation orbit is coincident with the line of nodes, a single burn can be used to raise perigee and to change inclination. This is the most efficient configuration for conventional chemical orbit raising and it is expected that it will be the most efficient configuration for C-EOR missions since the majority of the  $\Delta V$  for orbit raising is provided by the launch vehicle and chemical OBS. This geometry is consistent with geometric restrictions required by the 3D low thrust optimizer used for this study.

Because of the restartable upper stage, the separation perigee with Proton is considerably higher than perigee with other launch vehicles. Otherwise, the reference OBS mission profile is similar to that of other launch vehicles.

The electric  $\Delta V$  required to reach GEO was calculated using MITEOR3D, a numerical low thrust optimizer developed at MIT.<sup>12</sup> This program uses a two step optimizer very similar to that used in the 2D case, but optimizes the steering profile in three dimensions. To simplify the analysis, the optimizer assumes that the line of apsides and line of nodes are coincident at the start and end of orbit raising. The steering profiles are self-consistent and maintain the initial orientation of the line of apsides relative to the line of nodes. In practice, perturbations, steering errors, and other factors may disturb this geometry. Although this introduces some errors, the constant EOR  $\Delta V$  contours are generally smooth, so small perturbations should have little effect on overall mass performance.

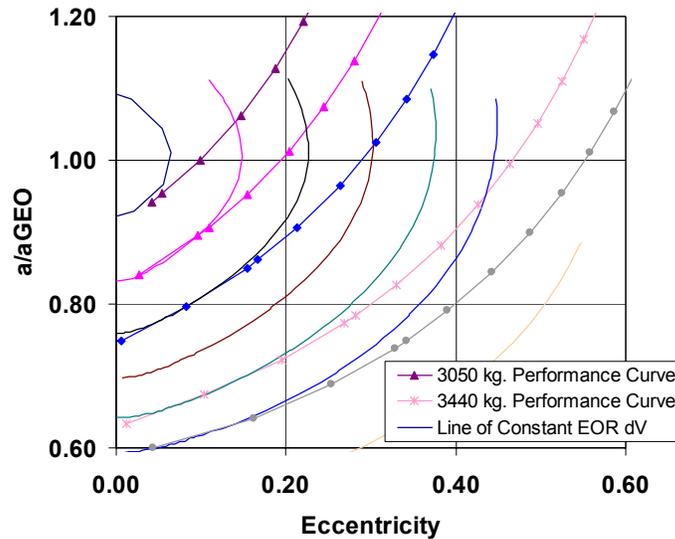
The range of starting orbits considered was limited by the allowable EOR duration to inclinations less than 5 degrees, though the optimizer works across a much wider range of orbits. The optimizer was fast and stable and was used both to identify candidate starting orbits and to calculate final performance values. Contour plots of electric  $\Delta V$  were created and chemical stage performance curves were superimposed on the plots to determine which starting orbits require the least electric  $\Delta V$  (and therefore the shortest time) to reach GEO. The performance curves account for both the performance of the launch vehicle and for the performance of the chemical OBS (see Table 1). EOR starting orbit inclination was varied parametrically to determine what inclination gives the best performance for a given mission duration.

A typical optimization plot for an Atlas V 401 is shown in Figure 7.



**Figure 7: Atlas V 401 Performance Overlaid on Contours of Constant Electric  $\Delta V$  ( $27^\circ$  inclination separation orbit,  $3^\circ$  EOR starting orbit)**

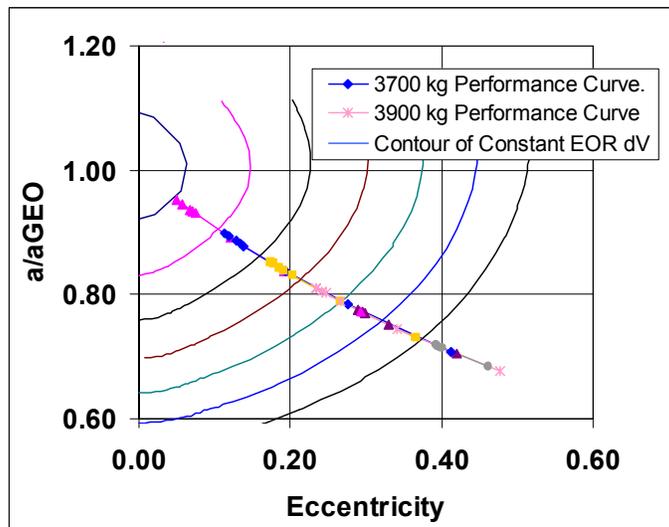
Figure 7 shows EOR  $\Delta V$  contours overlaid on lines of constant EOR starting mass for an Atlas V 401 with an initial separation orbit of  $27^\circ$  inclination. The launch vehicle/OBS performance lines have been spaced so the optimum EOR starting orbit is near the intersection of the performance lines and the EOR contours. The optimum orbits are elliptical and slightly supersynchronous. This enhances the performance of the OBS, which is changing the orbit's inclination by  $24^\circ$ . Figure 8 shows an equivalent plot for an Ariane 44L vehicle.



**Figure 8: Ariane 44L Performance Overlaid on Contours of Constant Electric  $\Delta V$  ( $7^\circ$  separation orbit,  $1^\circ$  EOR starting orbit)**

The optimum starting orbits in Figure 8 are more circular than those in Figure 7 and have subsynchronous apogees. Because the Ariane launches from a lower latitude, the OBS is doing only 6 degrees of inclination change. The system therefore favors optimizing EOR with a spiral orbit over optimizing the OBS with an elliptical one. When launching at zero inclination with a Sea Launch, the optimum orbits are circular.

The optimum orbits for a Delta IV Heavy vehicle are similar to those for an Atlas, but as shown in Figure 9, the optimum orbits for a Proton M/Breeze M variant have a fundamentally different character because of the vehicle's flight profile.



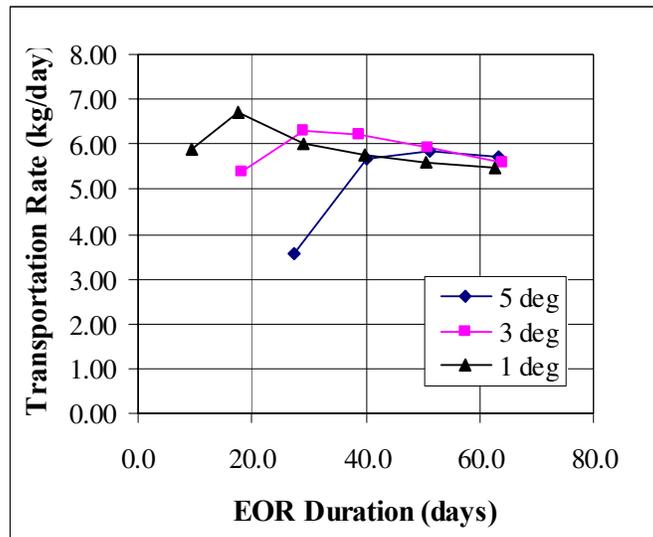
**Figure 9: Proton M/Breeze M Performance Curves Overlaid on Contours of Constant Electric  $\Delta V$  ( $30^\circ$  inclination separation orbit,  $1^\circ$  EOR starting orbit)**

When launching satellites to GEO, the standard Proton M/Breeze M profile uses the restartable upper stage to launch to a fixed apogee (GTO) with a variable perigee and inclination. The OBS further raises perigee and changes inclination, but leaves apogee fixed. As a result, the allowable family of EOR starting orbits lies on a line in Figure 9. Each EOR starting mass has a separate optimum on this line, giving a different travel time to GEO. Supersynchronous and subsynchronous transfer orbits were not considered because only limited data is available for these orbits in the User’s Manual. Different optimum orbits might result if additional separation orbit trajectories are considered, but launch vehicle limitations may prevent the use of many of these trajectories in practice. Interestingly, the optimum launch vehicle separation orbits are very similar for all of the cases shown in Figure 9. The change in EOR starting orbit is primarily due to changes in the use of the OBS.

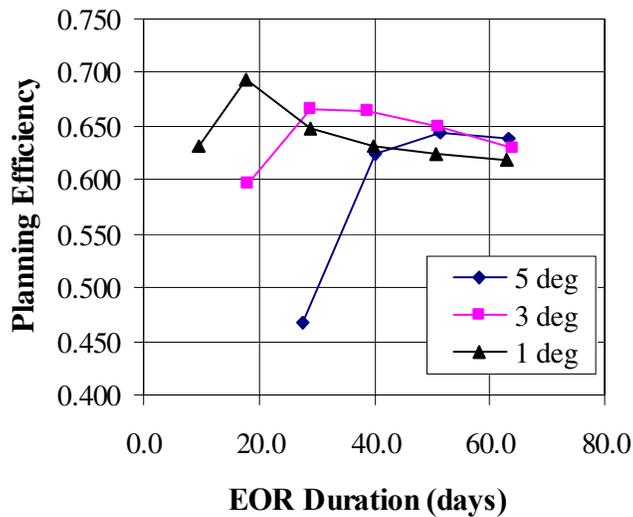
**Results**

For each launch vehicle, optimum EOR starting orbits were identified and planning efficiencies, EOR mass benefit, and transportation rate were calculated as a function of inclination. Unless otherwise specified, the results presented below assume the use of two SPT-140 thrusters for orbit raising the performance characteristics as shown in Table 1.

Figure 10 and Figure 11 show transportation rate and planning efficiency as a function of EOR starting inclination for the Atlas V 401.



**Figure 10: Atlas V 401 EOR Transportation Rate vs. EOR Duration and Starting Inclination**



**Figure 11: Atlas V 401 Planning Efficiency vs. EOR Duration and Starting Inclination**

As one would expect, for short duration missions, it is necessary to start EOR at a low inclination. As EOR duration increases, the optimum starting inclination also increases, creating a family of EOR starting orbits. Looking at the peak of each curve, the optimum transportation rate and planning efficiency are relatively constant. For simplicity, the 3 degree inclination line is omitted in further plots.

The Atlas V comes in different options (401, 402, 501, etc.) which fly similar trajectories but have different separated mass capability. Equation (4) shows that transportation rate is primarily a function of thrust level, which is in turn determined by power available and thrust efficiency. On “short duration” EOR missions (90 days or less), transportation rate is not a function of payload mass. For a given thruster, as long as launch vehicle trajectories are similar and the planning efficiency is constant, the EOR transportation rate of a “light” version of a launch vehicle will be the same as that of a “heavy” version. Changes in separated mass change EOR  $\Delta V$ , which in turn change EOR starting orbit, but the overall mass benefit to the system is constant. Therefore, a transportation rate calculated with one version of a launch vehicle generally applies to other versions of the same launch vehicle.

Figure 12 and Figure 13 show transportation rate and planning efficiency for a Delta IV Heavy (Delta 4H) launch vehicle. Figure 14 and Figure 15 show the same parameters for an Ariane 44L, and Figure 16 and Figure 17 and show them for a Proton M/Breeze M vehicle.

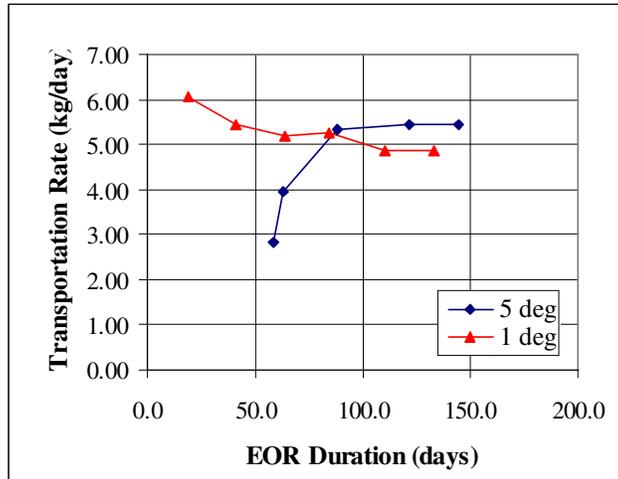


Figure 12: Delta 4H EOR Transportation Rate vs. EOR Duration

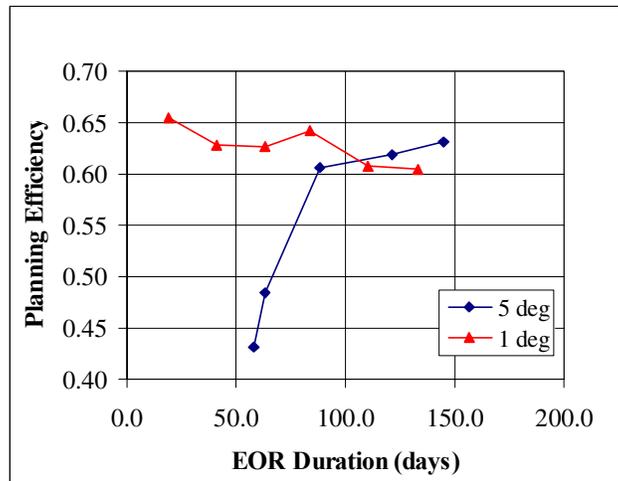


Figure 13: Delta 4H Planning Efficiency vs. EOR Duration and EOR Starting Inclinations

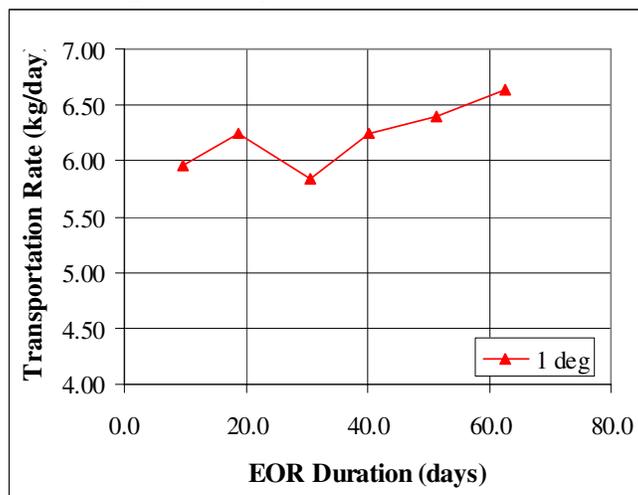
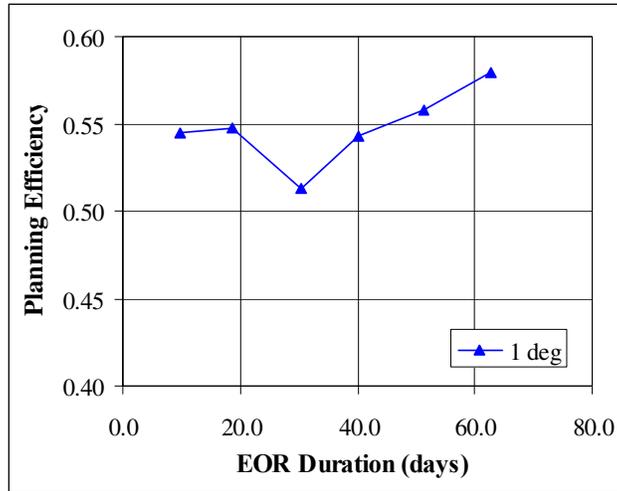
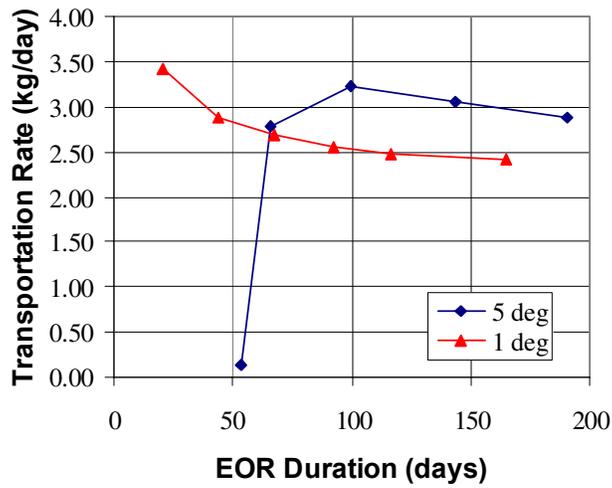


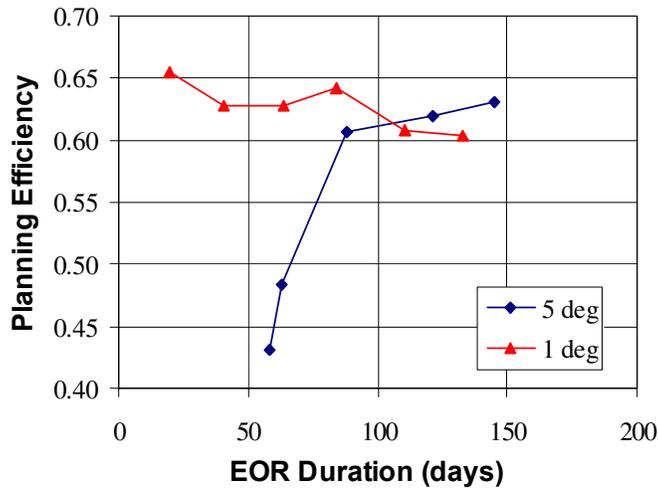
Figure 14: Ariane 44L Transportation Rate vs. EOR Duration



**Figure 15: Ariane 44L Planning Efficiency vs. EOR Duration**

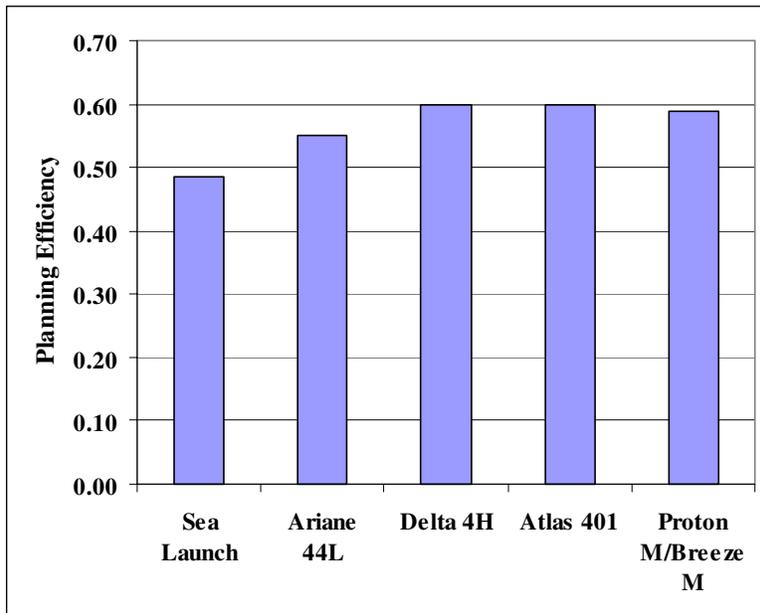


**Figure 16: Proton M/Breeze M EOR Transportation Rate vs. EOR duration**



**Figure 17: Proton M/Breeze M Planning Efficiency vs. EOR Duration**

Overall, the calculated transportation rates fall in a fairly narrow range, between 5.5 and 6.5 kg/day. Figure 18 shows a mean planning efficiency calculated for each launch vehicle based on the optimum starting orbits shown above.



**Figure 18: Planning Efficiency Used for Launch Vehicles**

The planning efficiency lies between approximately 0.45 and 0.60 and remains fairly constant as a function of EOR duration, regardless of the launch vehicle considered. It grows slightly as launch latitude increases. The overall transportation rate varies less with launch latitude, however, because of staging effects.

For missions of 90 days duration, the payload mass benefit is between 500 kg. and 585 kg., a benefit large enough to be economically attractive to a commercial satellite customer.<sup>20</sup>

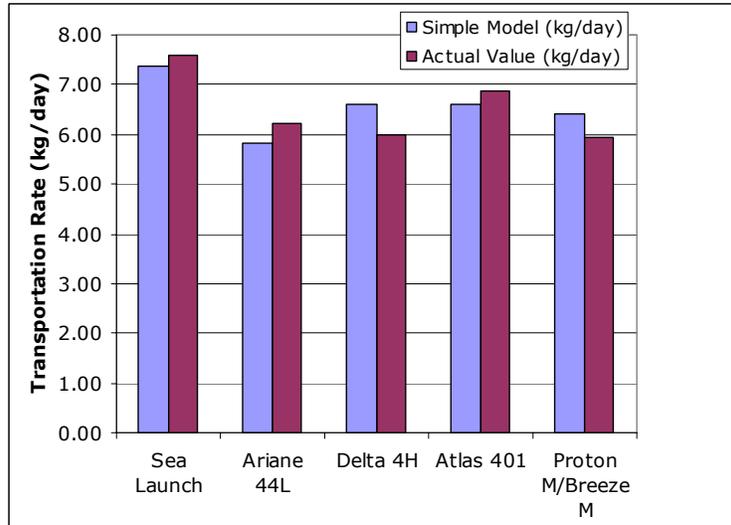
### Application of Two and Three Stage Models

For system level analysis, it is desirable to incorporate the performance of the trajectories shown in the previous section into a single simple model. Because planning efficiency is fairly constant for up to 100 days of EOR, the two and three stage models described as previous work can be applied here. Two and three stage versions of equation (4) have been used to estimate EOR transportation rates for each launch vehicle. The results are summarized in Figure 19 and Table 2. Figure 19 shows calculated performance of each vehicle using the most applicable multi-stage model and the trajectory optimization software.

Table 2 shows each vehicle's input parameters and the type of model applied. The planning efficiencies and actual transportation rates are mean values for each vehicle. The two stage model provides the best estimate for most launch vehicles. Note that the effective  $I_{sp}$  of the launch vehicle may be much lower than the actual  $I_{sp}$  of the chemical engine because of structural mass penalties.

Launch Vehicle	Planning Efficiency	Type of Model	Effective Chemical Isp (sec)	Simple Model (kg/day)	Actual Value (kg/day)
Sea Launch	0.484	Three Stage	240	7.38	7.6
Ariane 44L	0.550	Two Stage	322	5.82	6.2
Delta 4H	0.600	Two Stage	322	6.60	6.0
Atlas 401	0.600	Two Stage	322	6.60	6.9
Proton M/Breeze M	0.588	Two Stage	322	6.41	5.9

**Table 2: Launch Vehicle Modeling Characteristics**

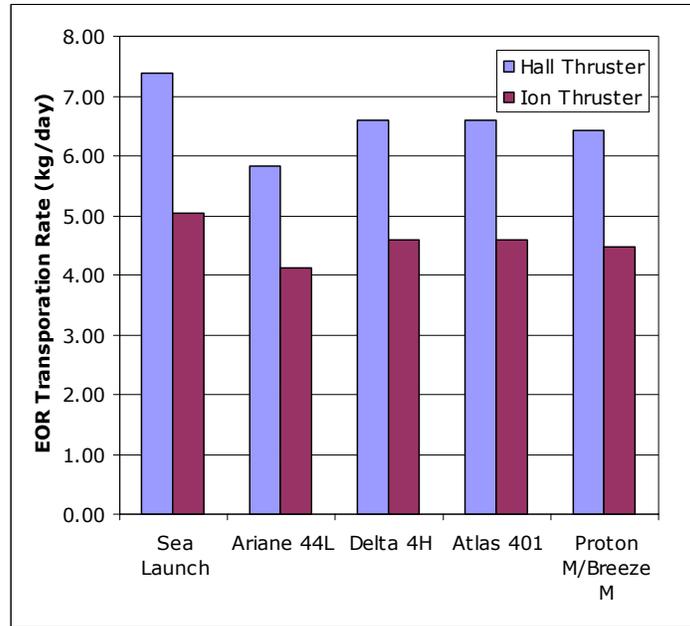


**Figure 19: Comparison of Calculated EOR Transportation Rates vs. Simplified Model (Two SPT-140 Thrusters)**

In general, the match between the multi-stage model and the detailed optimized value is quite good, within 10% of the calculated value. Variations in the accuracy of the model are primarily due to averaging and deviation in the optimized trajectories from ideal two and three stage profiles. The Ariane in particular uses a combined trajectory with elements of the two and three stage models, so its actual performance is better than that calculated using the two stage model.

The values calculated in Figure 19 are trajectory specific and assume use of end-to-end optimized trajectories. They are not representative of the performance currently achieved by satellite manufacturers using non-optimized trajectories.

Using the parameters specified in Table 2, the two and three stage models can be used for system trade studies of different power and electric propulsion systems. For instance, equation (4) can be used to compare the relative effectiveness of ion thrusters and Hall thrusters flying the same optimized trajectory. The relative performance of two XIPS thrusters vs. two SPT-140 thrusters operating at the same power level is shown in Figure 20 below.



**Figure 20: Ion thruster vs. Hall thruster performance for C-EOR missions**

For C-EOR missions, Hall Thrusters consistently outperform ion thrusters by about 30%, or over 150 kg. over 90 days, because of their higher thrust level. This is to be expected, as previous work has shown that Hall Thrusters operate near the ideal specific impulse for commercial C-EOR missions.<sup>3</sup>

One reason that the two stage model applies to most of these launch vehicles is because we have chosen an all chemical reference mission with a high separation inclination. When EOR  $\Delta V$  is added to the mission profile, it can either displace  $\Delta V$  from the launch vehicle or from the OBS. In the cases considered in this paper, the launch vehicle is already using a near-optimum trajectory, so  $\Delta V$  is displaced from the OBS. This makes the mission profile resemble a two stage model.

As was noted earlier, larger spacecraft may be unable to follow the baseline all-chemical mission because of tank size limits on the OBS. In these cases, the launch vehicle flies a less optimized low inclination or supersynchronous trajectory. When EOR  $\Delta V$  is added to these trajectories, it is possible to displace  $\Delta V$  from the launch vehicle instead of the OBS. The result is much closer to the three stage model. In many cases where satellites today fly tank limited or supersynchronous trajectories, substantially higher transportation rates may be achievable than those shown in Figure 20.

## Conclusions

In this paper, we have described a numerical/graphical optimization method that can be used to optimize chemical-electric orbit raising (C-EOR) missions to GEO in 2D and 3D. This method was used to derive end-to-end optimized trajectories for the Sea Launch, Ariane 4, Atlas V, Delta IV, and Proton launch vehicles. Based on our analysis, the following general conclusions have been drawn.

- Using optimized trajectories, EOR transportation rates of 6 kg/day can be consistently achieved on most launch vehicles by a satellite with two SPT-140 thrusters.

These rates assume ideal trajectories with no perturbation or steering losses and use nominal, published thruster performance values. Actual missions must take these losses into account when calculating C-EOR performance.

- Optimum EOR starting orbits tend to increase in eccentricity as launch inclination increases.
- Optimum EOR starting orbits tend to increase in inclination as EOR duration increases.

Because inclination is changed most efficiently when an orbit has a high apogee, launching from a high inclination tends to favor a relatively high EOR starting orbit. The Proton is an exception to this rule because it has a restartable upper stage that is used to raise perigee and change inclination prior to separation. The Proton's optimum EOR starting orbit is also elliptical, but does not follow the same trend as launch vehicles without the restartable upper stage. This result is based on data published in the Proton User's Guide and might not apply if one considers a full range of subsynchronous and supersynchronous starting orbits for this vehicle.

- On C-EOR missions, transportation rate is a function of power available rather than payload mass or power to mass ratio.

Equation (4), which applies to "short duration" EOR missions, shows that EOR transportation rate is primarily a function of thrust level and planning efficiency rather than separated mass or power to mass ratio. A consequence of this observation is that performance numbers calculated with a "light" version of a launch vehicle can also be applied to a "heavy" version of the same vehicle as long as it has a similar specific impulse and flies a similar trajectory. This result should be valid for EOR missions of 90 days duration or less.

An apparent discrepancy between SEPSHOT based optimizations of Sea Launch missions and our previous work on Sea Launch has been addressed. The graphical/numerical optimization method was applied to a mission that assumes the satellite is separated from the launch vehicle in a low altitude circular parking orbit. It was observed that

- The graphical/numerical method reaches the same conclusions as SEPSHOT when optimizing C-EOR missions to GEO where  $a_{\text{EOR}}/a_{\text{GEO}}$  is greater than 0.5.

This serves to validate the results of the graphical/numerical method in this regime. The basic optimizer code, MITEOR3D, does not consider radiation effects. This may cause its answer to diverge from SEPSHOT in lower orbits.

- Because Sea Launch uses an elliptical parking orbit for direct injection missions, SEPSHOT does not accurately optimize the end-to-end Sea Launch mission.

The numerical/graphical methodology uses performance curves specific to each launch vehicle and therefore incorporates the performance of the actual launch trajectory. In general, this comparison demonstrates the necessity of accurately modeling the launch vehicle and OBS when doing an end-to-end optimization.

Simple two stage and three stage C-EOR models have been applied to each launch vehicle. Modeling parameters were defined for each vehicle and comparisons were made to detailed trajectory performance calculations. The results show that

- A simple multi-stage C-EOR model can be used to calculate mission performance on a variety of launch vehicles within an accuracy of about 10%.

The model can be used to calculate C-EOR mission performance to GEO with a variety of different thrusters and power systems. It is an ideal tool for system level analysis of electric orbit raising missions and for trades involving different types of electric thrusters, power levels, and thrust levels. For instance, using the multi-stage model to compare the performance of Hall and Ion thrusters shows that

- Hall Thrusters can provide EOR transportation rates 30% higher than typical ion thrusters operating at the same input power level.

This conclusion is consistent with previous results showing that Hall Thrusters operate near the optimum specific impulse level for electric orbit raising. Taken together, the results show that

- Space Systems/Loral has demonstrated the capability to optimize electric orbit raising missions to GEO with major commercial launch vehicles.

This is an important and enabling step to the use of fully optimized C-EOR trajectories for orbit raising on commercial satellite missions.

SS/L is continuing its work to develop operational orbit raising simulations for C-EOR missions. Areas of interest for future work include development of a fully numerical end to end mission optimization program, the incorporation of variable thrust profiles into the C-EOR model, and optimization of C-EOR missions to non-GEO orbits.

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### **References**

<sup>1</sup> Snyder, J.S. "Electric Propulsion: Year in Review," Aerospace America, Vol. 38, No. 12, 2001, pp. 48-49.

<sup>2</sup> Killinger, R., Kikies, R. et al. "Orbit Raising with Ion Propulsion on ESA's ARTEMIS Satellite," AIAA Paper 2002-3672, 38<sup>th</sup> AIAA/ASME/SAE/ASEE Joint Propulsion Conference, Indianapolis, Indiana, July 2002.

- <sup>3</sup> Oh, D., Randolph, T., Martinez-Sanchez, M. and Kimbrel, S. "End-to-End Optimization of Mixed Chemical-Electric Orbit Raising Missions," AIAA 2002-3966, 38<sup>th</sup> AIAA/ASME/SAE/ASEE Joint Propulsion Conference, Indianapolis, Indiana, July 2002.
- <sup>4</sup> Spitzer, A. "Near Optimal Transfer Orbit Trajectory using Electric Propulsion," AAS Paper 95-215, Feb. 1995.
- <sup>5</sup> Pollard, J.E. "Evaluation of Low-Thrust Orbital Maneuvers," AIAA Paper 98-3486, 34<sup>th</sup> AIAA/ASMA/SAE/ASEE Joint Propulsion Conference, July 1998.
- <sup>6</sup> Kaufman, H.R. and Robinson, R.S. "Electric Thruster Performance for Orbit-Raising and Maneuvering," *Orbit Raising and Maneuvering Propulsion: Research Status and Needs*, Progress in Astronautics and Aeronautics Vol. 89, AIAA, New York, NY, 1984, pp. 303-326.
- <sup>7</sup> Edelbaum, T., H. Malchow, and L. Sackett, "Solar Electric Geocentric Transfer with Attitude Constraints: Analysis", NASA CR-134927, NASA Lewis Center, August 1975.
- <sup>8</sup> Ilgen, M., "A Hybrid Method For Computing Optimal Low Thrust OTV Trajectories", AAS 94-129.
- <sup>9</sup> Oleson, S.R. and Myers, R.M. "Launch Vehicle and Power Level Impacts on Electric GEO Insertion," AIAA Paper 96-2978, 32<sup>nd</sup> AIAA/ASME/SAE/ASEE Joint Propulsion Conference, July 1996.
- <sup>10</sup> Oh, D. and Santiago, G. "Analytic Optimization of Mixed Chemical-Electric Orbit Raising Missions," IEPC Paper 01-173. 27<sup>th</sup> International Electric Propulsion Conference, Pasadena, California, USA, October 2001.
- <sup>11</sup> "Sea Launch User's Guide", Revision B, Boeing Commercial Space Company, Seattle, WA, July 2000.
- <sup>12</sup> Martinez, M. et al. "Optimized Low-Thrust Transfer from Intermediate Orbits to GEO," 2<sup>nd</sup> International Symposium on Low Thrust Trajectories, Toulouse, France, June 2002.
- <sup>13</sup> Stechman, C., Woll, P., Fuller, R., Colette, A., "A High Performance Liquid Rocket Engine for Satellite Main Propulsion", AIAA-2000-3147, 36<sup>th</sup> Joint Propulsion Conference, Huntsville, Alabama, 2000.
- <sup>14</sup> Snyder, J. S., Randolph, T., Oh, D., Sauer, B., Fischer, G., "System-Level Trade Studies of a Dual-Mode SPT for Geosynchronous Communications Satellites", IEPC-01-173, 27<sup>th</sup> International Electric Propulsion Conference, Pasadena, California, 2001.
- <sup>15</sup> Goebel, D., Martinez-Lavin, M., Bond, T., and King, A. "Performance of XIPS Electric Propulsion in On-orbit Station Keeping of the Boeing 702 Spacecraft," AIAA-2002-4348, 38<sup>th</sup> AIAA/ASME/SAE/ASEE Joint Propulsion Conference, Indianapolis, Indiana, July 2002.
- <sup>16</sup> "Ariane 4 User's Manual," Issue No. 2, Arianespace.
- <sup>17</sup> "Atlas Launch System Mission Planner's Guide, Atlas V Addendum," Revision 8, International Launch Services, San Diego, CA, December 1999.
- <sup>18</sup> "Delta IV Payload Planners Guide," Version 2000, The Boeing Company (Delta Launch Services), Huntington Beach, CA, October 2000.
- <sup>19</sup> "Proton Launch System Mission Planner's Guide," International Launch Services, Issue 1 Revision 5, LKEB-9812-1990, McLean, VA, December 2001.
- <sup>20</sup> Randolph, T. and Oh, D. "Economic Benefit Analysis of Chemical-Electric Orbit Raising Missions," AIAA-2002-1980, 20<sup>th</sup> AIAA International Communications Satellite Systems Conference and Exhibit, Montreal, Canada, May 2002.