CRITICAL ASSESSMENT OF A 2D MODEL OF HALL THRUSTERS - COMPARISONS WITH EXPERIMENTS

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ABSTRACT

We present a discussion on the results and predictive capabilities of a 2D hybrid model of Hall thrusters or Stationary Plasma Thrusters (SPTs). It is well known that classical electron transport is not sufficient in the operating conditions of most SPTs to explain the observed thruster characteristics. Our 2D, quasi-neutral, hybrid model uses empirical parameters to describe the anomalous electron transport. We show that assuming that the anomalous transport is due to electron-wall collisions inside the channel, and field fluctuations outside the channel, with properly adjusted anomalous transport coefficients can lead to model predictions in rather good agreement with experimental results over a large range of variations of the operating conditions.

I. INTRODUCTION

Stationary Plasma Thrusters (SPTs), also called Hall Thrusters or Closed Electron Drift Thrusters, are used on-board of satellites as electric propulsion devices. These thrusters are especially suitable for orbit corrections and station-keeping of geostationary satellites and for primary propulsion in interplanetary missions. In a SPT, the thrust is provided by the acceleration of ions with a large axial electric field induced by a radial magnetic field barrier (**E**×**B** configuration) in a plasma generated between two concentric dielectric cylinders. The magnetic field confines the electrons in the channel, which allows intense electron impact ionization in spite of the low gas density. The reduction of the axial electron mobility by the radial B field gives rise to the formation of a large axial electric field to maintain current continuity. The ions, which are not sensitive to the B field are collisionless and are accelerated by this axial electric field to high exhaust velocities, namely on the order of 20 km/s, i.e. much higher than in classical chemical propulsion devices. Consequently less propellant is needed in this kind of thrusters (for a given thrust).

In spite of more than three decades of SPT research and development (mainly in Russia), the physics of SPTs is still not fully understood - although reliable SPT operation has been demonstrated on-board of several Russian satellites. Experimental and numerical modeling efforts have been undertaken to clarify the operation of these thrusters and to develop more powerful thrusters. In this context, we developed a 2D, quasi-neutral, hybrid model. This model combines a particle description of ion and atom species to a fluid description of electrons - like the models of Komurasaki and Arakawa [1] and Fife [2]. A question which cannot be resolved by these models is the anomalously high-electron transport in SPTs : the electron transport across the magnetic field in SPTs is known to exceed by far the values predicted by the classical theory. This anomalous transport is generally attributed to collisions of electrons with channel walls and to field fluctuations or turbulence, but these effects are not well understood and difficult to quantify.

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The 2D hybrid model has been described in Ref. [3]. It includes an empirical treatment (using a set of parameters) of anomalous electron transport. The role of this anomalous transport has been presented in a phenomenological way [4]. More systematic studies have been carried out to reduce the domain of variation of the empirical parameters in comparison with experimental data obtained in the French facility PIVOINE. The aim of this article is to compare the model results (using optimized values of the empirical parameters) with experimental data for various operating conditions. On the basis of this comparison we discuss the reliability and predictive capability of the model. Although anomalous electron transport (and consequently empirical parameters) may depend on operating conditions, we will show that the results of the model are in qualitative agreement with experimental data.
The following sub-sections describe the 2D hybrid model and the experimental set-up. In sections II and III we present respectively the static (time-averaged) and the dynamic (time-varying) behavior of the thruster, and comparisons with experiments. Section IV presents some effects of the empirical parameters. We conclude in section V.

A. Overview of the 2D hybrid model

The model describes the axial and radial dimensions of the thruster geometry (see figure 1). Not only the channel is modeled but also the region beyond the exhaust. The magnetic field is assumed to be unaffected by the discharge and is calculated *a priori*.

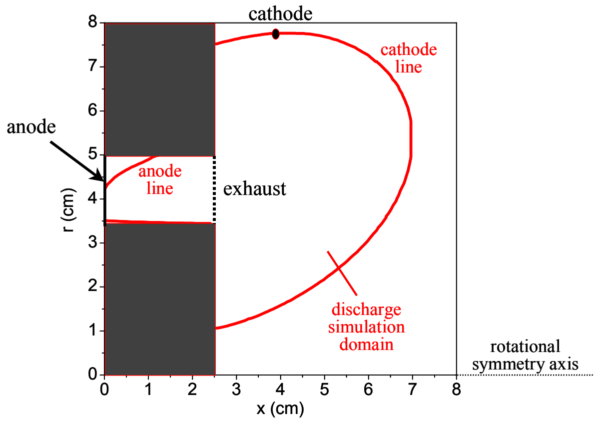

Figure 1 : Simulation domain of the 2D (x,r) model of the SPT-100. The discharge is simulated between the anode and the cathode lines (i.e. two magnetic field lines). The channel is 2.5 cm long and the inner/outer radius of concentric dielectric cylinders measure 3.45/5 cm respectively.

The plasma is modeled using a hybrid approach: while neutral atoms and ions are simulated by a particle in cell (PIC) technique, electrons are described by fluid equations. Only singly charged ions are taken into account. Assuming quasi-neutrality, the plasma density is inferred from the ion PIC simulation and the electric field distribution from the fluid equations for the electron transport.
Along the magnetic field lines the electrons are assumed to be in Boltzmann equilibrium, that is, electron flux due to pressure effects is balanced by electron flux due to the electric field. The electric potential V can be written as the sum of a potential V^* which is constant along the B field lines, and a term proportional to the electron energy ε and to the logarithm of plasma density n [5] :

$$V = V^* + \frac{2}{3e}\varepsilon \ln\frac{n}{n_0} \qquad (1)$$

where e is the elementary charge and n_0 is reference plasma density.
The electron flux $\Gamma_{e,\perp}$ across the magnetic field is described by the drift-diffusion equation

$$\Gamma_{e,\perp} = -\mu_\perp E_\perp n - \frac{2}{3e}\mu_\perp \nabla_\perp(n\varepsilon) \qquad (2)$$

where $\mu_\perp$ is the cross field electron mobility and $\nabla_\perp$ represents the cross field gradient. The electric field component $E_\perp$ perpendicular to the magnetic field is now found by substituting this equation in the current conservation equation

$$e\iint \Gamma_{e,\perp} \mathrm{d}s = e\iint \Gamma_{i,\perp} \mathrm{d}s - I\,, \qquad (3)$$

where $\Gamma_{i,\perp}$ is the cross field ion flux and the integrals are taken along magnetic field lines (surfaces!). The total current I is chosen such, that a specified voltage drop results between the anode and cathode lines (see figure 1). It is assumed that no net current escapes to the insulating channel walls.

The electron mean energy, a parameter that is of crucial importance for both electron diffusion and ionization, is obtained from an electron energy equation [3].

B. Empirical model parameters

The cross field electron mobility is the main parameter determining the electric field distribution and the discharge current, via the equations (2) and (3); the field tends to concentrate itself where the mobility is lowest. The classical theory predicts the cross field mobility to be [6]

$$\mu_{\perp,c} = \frac{e\,\nu_m / m_e}{\nu_m^{\;2} + \left(eB/m_e\right)^2} \approx \frac{m_e \nu_m}{eB^2}\,, \qquad (4)$$

where m_e the electron mass, B the magnetic field strength and ν_m the momentum-transfer frequency of electron-particle collisions. However, the electron mobility in SPTs has been found to be much larger than the values given by the classical expression, probably due to wall collision effects or field fluctuations [7, 8]. A lack of detailed understanding of the responsible mechanisms forces us to treat these anomalous mobility contributions in an empirical way.

Inside the channel we apply the classical expression (4), where we enhance the momentum transfer frequency with a constant contribution that we interpret as being the result of electron-wall collisions, as in Ref. [9]. Our mobility writes (for high enough magnetic fields):

$$\mu_\perp = \mu_{\perp,c} + \alpha \left(\frac{m_e \nu_{\mathrm{ref}}}{eB^2}\right). \quad \text{(inside)} \qquad (5)$$

Here $\nu_{\mathrm{ref}} = 10^7\ \mathrm{s}^{-1}$ is a reference frequency for wall collisions, and α is a constant empirical parameter on the order of 1.

Outside the channel we enhance the mobility by a contribution that follows the Bohm formula for anomalous cross-field electron transport [6]:

$$\mu_\perp = \mu_{\perp,c} + k \left(\frac{1}{16B}\right), \quad \text{(outside)} \qquad (6)$$

where k is a (second) constant parameter.

One should realize that actually the cross field electron mobility is an unknown function of the position.

Realize also, that the cross field electron mobility intervene directly in the cross field electron conductivity :

$$\sigma_\perp = e\,n\,\mu_\perp \qquad (7)$$

Similar to the problem of the electron mobility, the electron energy loss due to collisions with gas atoms is insufficient to explain observed electron energies in SPTs. Additional energy loss is included in our energy equation via an anomalous energy loss coefficient, which we parameterize by an empirical expression [9], introducing a (third) parameter α_ε.

C. Experimental set-up

Static and dynamic results of the model are compared with experiments performed in the French facility PIVOINE described in detail in Ref. [10]. In nominal working conditions : 5 mg/s of Xe flow, 300 V of applied voltage and 4.5 A of coil current, the background pressure was kept at 2.5 mPa. Note that 4.5 A of coil current corresponds to a magnetic field strength at the exhaust at the center of the channel referred as B_0 in the text. Details about performance and dynamic measurement devices can be found in Ref. [11].

II. STATIC BEHAVIOR

This section is dedicated to static (time-averaged) behavior of the thruster. All quantities (except the magnetic field) are averaged over 3 ms, which is sufficient to average out all discharge oscillations. We first show the operation of the thruster for nominal operating conditions. We then present a comparison of some calculated characteristics with experiments for various operating conditions and a discussion about anomalous electron transport. The same set of empirical parameters is used for all the calculations.

A. Nominal operation

We present in this section the operation of the thruster for its nominal operation defined in section I.C. We also point out the role of the empirical parameters on the electric field distribution.

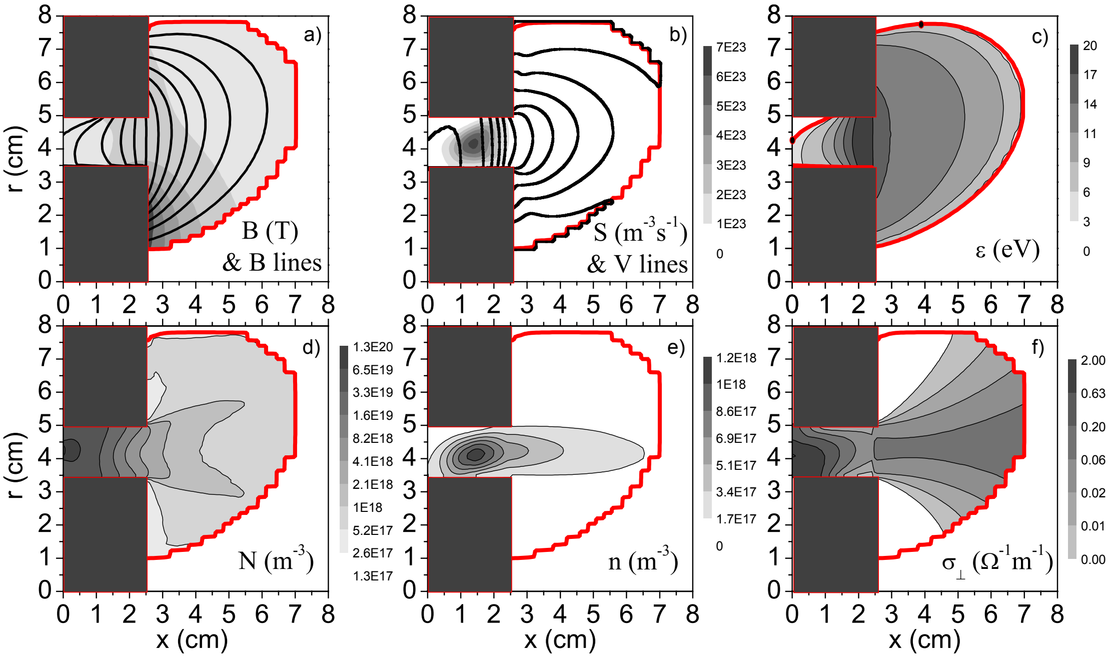

Figure 2 : 2D static results of the model for nominal operation of the SPT-100 with 2.5 mPa of back pressure. The results are averaged over 3 ms. a) magnetic field magnitude B (T) and magnetic field lines ; b) potential lines (spaced by 30 V, 300 V applied voltage) and ionization rate S ($m^{-3}s^{-1}$) ; c) mean electron energy ε (eV) ; d) atom density N (m^{-3}) ; e) plasma density n (m^{-3}) ; f) cross field electron conductivity $\sigma_\perp$ ($\Omega^{-1}m^{-1}$).

The magnetic field generated by the SPT-100 magnetic circuit is radial and maximum at the exhaust (figure 2.a). It confines the electrons along the magnetic field lines (magnetic lens centered at the exhaust) to improve ionization of the 5 mg/s xenon gas flow injected at the anode. A voltage of 300 V is applied between the electrodes. The accelerating potential drop occurs mainly where the magnetic field is strong and 50% of it is located outside the thruster (figure 2.b). The electrons, heated by the electric field, reach mean energies on the order of 20 eV at the exhaust (figure 2.c). They lose their energy by efficiently ionizing the gas (about 90% of the gas flow). The ionization zone is shifted upstream (toward the anode) with respect to the acceleration zone so that the created ions see about 80% of the applied voltage (figure 2.b). The atom density is strongly depleted (from 10^{20} m^{-3} at the anode to 5.10^{18} m^{-3} at the exhaust), especially in the center of the channel (figure 2.d) where ionization rate is maximum (figure 2.b). The plasma density is maximum inside and at the center of the channel and decreases in the acceleration zone (figure 2.e). Comparing figures 2.a and 2.b we can see the influence of the 2^{nd} term of equation (1) on the potential lines: in regions where both plasma density and electron energy are high, the potential lines are more divergent than the magnetic field lines.

The spatial distribution of cross-field electron conductivity (figure 2.f) controls the distribution of the potential (figure 2.b) in the acceleration zone. Indeed, the maximum of electric field, situated close to the exhaust, is associated with a minimum of conductivity in this region (where the magnetic field is high and the atom density is low). Since electron mobility in the acceleration zone is controlled by its anomalous contributions, the empirical parameters play an important role in the distribution of the potential between the inside and the outside the thruster. This point is illustrated in section IV. For the given set of empirical parameters, anomalous electron mobility (and conductivity) is higher outside the thruster so that the electric field is maximum inside ; as observed experimentally [12].

For this set of empirical parameters, the model predictions of thruster performance at nominal operation (a thrust of 80 mN, a total thrust efficiency of 50% and a discharge current of 4.3 A) are also in quantitative agreement with experiments [11].

B. Comparison with experiments – various operating conditions

In this section we present comparisons of the model results with experiments (at PIVOINE) for various operating conditions of the thruster but keeping the same set of empirical parameters. On this basis, we discuss the dependence of anomalous electron transport (and empirical parameters) on operating conditions.

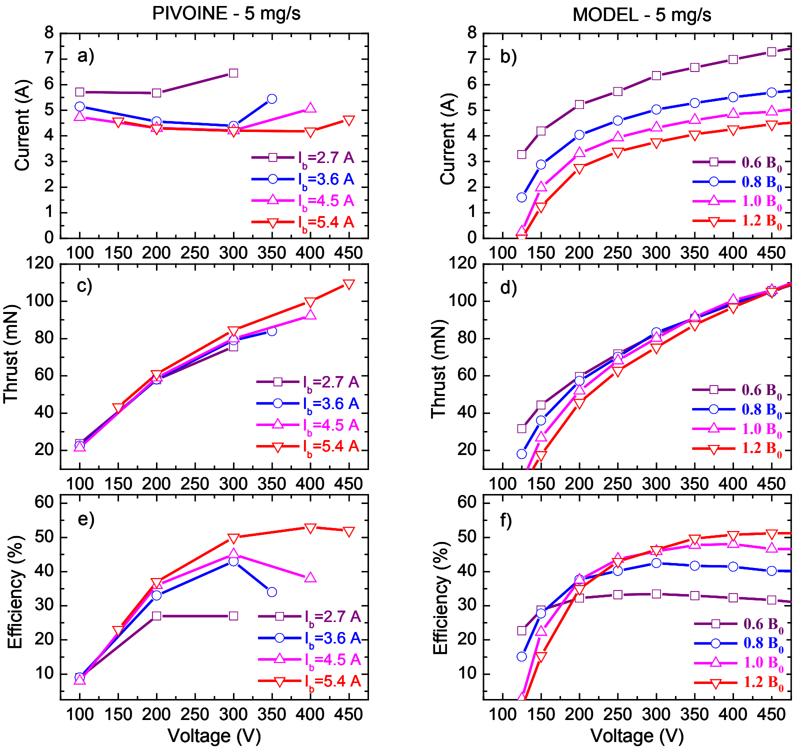

Figure 3 : Comparison between measured [11] / modeled performances - voltage characteristics for various magnetic fields (defined by coil currents in experiments and value of magnetic field expressed in unit of B_0 in the model), a mass flow rate of 5 mg/s and 2.5 mPa of back pressure : a) / b) discharge current (A) ; c) / d) thrust (mN) ; e) / f) efficiency (%).

Experimental and modeled current-voltage characteristics are represented respectively in figures 3.a and 3.b for various magnetic fields and a xenon mass flow rate of 5 mg/s. The model reproduces the shape of the measured current-voltage characteristics but some disagreements appear in the current values especially at low voltages. Anomalous electron transport may be responsible for these discrepancies. It seems that anomalous electron transport is more important in the experiments especially for lower voltages. Moreover the abrupt increase of the current for high voltages is not observed in our model. It could be also related with an increase of anomalous transport. The model of Ref. [13], which uses a detailed treatment of electron-wall interaction, seems to reproduce this. Finally we note that the saturation of the discharge current with increasing magnetic field, observed in experiments (e.g. beyond 3.6 A coil current for 300 V applied voltage), is not reproduced by the model. This could be due to an increase of anomalous electron transport for high values of magnetic field. Apparently, the anomalous electron transport depends on the operating

conditions (voltage and magnetic field), and in order to describe this, our empirical parameters should vary with these conditions.
Experimental and modeled thrust-voltage characteristics are also compared for various magnetic fields. In experiments, the thrust depends essentially on the voltage (variation as the square root of the voltage) but not significantly on the magnetic field (figure 3.c). The model reproduces qualitatively this trend (figure 3.d). It reproduces also qualitatively the experimental efficiency-voltage characteristics, especially the shift of the maximum of efficiency towards high voltages when the magnetic field increased (compare figures 3.e and 3.f).
To conclude, although we kept the empirical parameters constant with voltage and magnetic field, the model predictions of the thruster performance for various operating conditions are in good qualitative agreement with experiments.

III. DYNAMIC BEHAVIOR

In general the simulated discharge is not stable but shows a dynamic (or time-varying) behavior. Two kind of oscillations, which have been reported in the literature, can be distinguished : ionization and transit-time oscillations. In this section, we present these oscillations at nominal operation, as well as a comparison between calculated and measured amplitude of the ionization oscillations for various operating conditions. Note, once again, that the same set of empirical parameters is used for all the calculations.

A. Nominal operation

In nominal operation, the model predicts two kind of oscillations : ionization and transit-time oscillations. Both oscillations coexist and are visible over 500 µs figure 4.

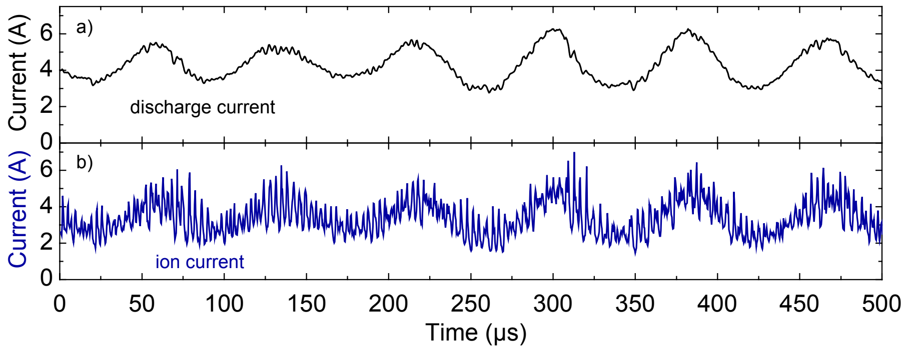

Figure 4 : Modeled dynamic behavior over 500 µs at nominal SPT-100 operation with 2.5 mPa back pressure : a) discharge current (A) ; b) ion current through the cathode line (A).

Low amplitude ionization oscillations can be seen on the discharge current (figure 4.a). They are related to the periodic depletion of the atom flow with a frequency on the order of 10-20 kHz (characteristic of the transit time of the atoms in the ionization zone). These oscillations are observed experimentally and are reproduced by many other models [1-4], [9].
For the given set of empirical parameters, transit-time oscillations are not visible in the discharge current but do appear in the ion current (figure 4.b). They are associated with the periodic extraction of ions from the ionization zone with a frequency on the order of 100-400 kHz (characteristic of the transit-time of ions in the acceleration zone). As these oscillations involve an oscillation of the acceleration zone at the transit time of ions, they affect strongly the ion energy distribution. Some ions can see more than the applied voltage (due to a well known surf-riding effect). These oscillations are also observed in other models [2, 14]. Experimental evidence of transit-time oscillations is reported in Russian literature for early SPTs but not for modern SPTs [15] like the SPT-100 studied at PIVOINE facility. However, ions with energies greater than the applied voltage have been measured at PIVOINE and could be explained by these oscillations. A more detailed description of these oscillations is given in Ref. [4].

B. Comparison with experiments – various operating conditions

We present in this section a comparison between measured and modeled behavior of the amplitude of the current oscillations (ionization type) for various operating conditions. To characterize this, we plot the variations of the standard deviation with voltage and magnetic field for the experimental (figure 5.a) and the modeled (figure 5.b) current values. The mass flow rate is 2 mg/s for experiments (most complete cartography obtained at PIVOINE) and 3.5 mg/s for the model. Note that the experimental trends at 3.5 mg/s and 5 mg/s are quite similar to the results presented here at 2 mg/s.

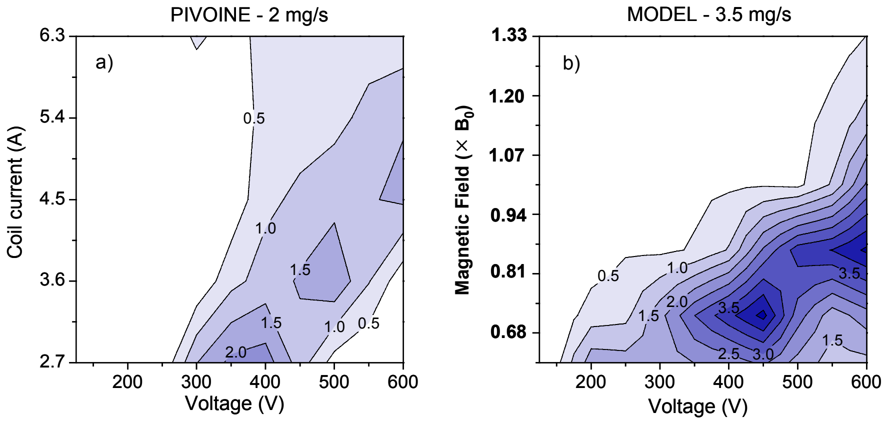

Figure 5 : Comparison between measured / modeled standard deviation of the current oscillations (ionization type) for various voltages and magnetic fields (defined by the coil current in experiments and by its strength expressed in unit of B_o in the model) : a) PIVOINE measurements at 2 mg/s mass flow rate ; b) model results at 3.5 mg/s.

The experimental cartography (figure 5.a) reveals the analogy of the increase of the voltage and the decrease of magnetic field strength (or coil current). An increase of voltage (or a decrease of magnetic field) first induces an increase and subsequently a decrease of the oscillation amplitude. This zone of instability (highest oscillation amplitudes) is clearly shifted towards high voltages when magnetic field is increased. This experimental trend is remarkably reproduced by the model (figure 5.b). However, the orientation of the instability zone in the B-V plane, seems to be slightly different in the model. We find that this orientation is very sensitive to the empirical parameters (that may vary with operating conditions).
To conclude, once again, even though empirical parameters have been kept constant with operating conditions, the model reproduces qualitatively the measured dynamic behavior of the thruster (at least for ionization oscillations).

IV. EMPIRICAL MODEL PARAMETERS

As seen in previous sections, for the same set of the empirical parameters, the model reproduces qualitatively the static and dynamic behavior observed experimentally at PIVOINE. The values of the empirical parameters for this case are : k =0.2, α =1 and α_ε =0.7. This case has been selected after systematic studies and comparisons with experiments at nominal operation of the thruster. We present in this section the major effects of the empirical parameters characterizing the anomalous mobility (α inside and k outside the thruster) on parameters that can be measured experimentally. In this study, the ratio between α_ε and α has been kept constant.
We find that the empirical model parameters (α and k) play a crucial role upon :

(1) The distribution of the acceleration zone between interior/exterior of the thruster
(2) The value of electron current passing the cathode line

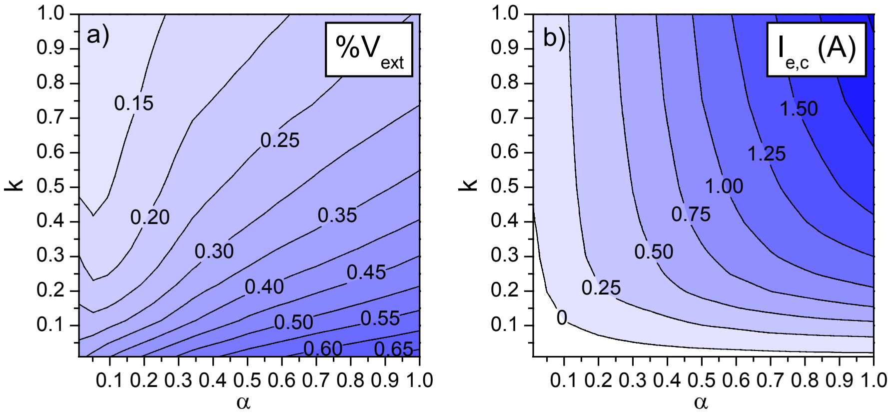

Figure 6 : Variations with empirical parameters α and k of the : a) Part of the applied potential %V_{ext} situated at the exterior of the thruster ; b) Electron current passing the cathode line $I_{e,c}$ (A).

In section II.A, we have shown that the distribution of the potential is essentially controlled by the empirical parameters in the model. That can be seen in figure 6.a where the part of potential at the exterior %V_{ext} is represented as a function of anomalous mobility inside (α parameter) and outside (k parameter) the thruster. This part increases when α/k increases and is constant if we keep α/k constant (this trend breaks down for low values of α because classic electron-atom collisions in the acceleration zone are no longer negligible). Since the part of the potential at the exterior is a parameter that could be experimentally verified (by LIF measurements), it's a way to limit the domain of variations of the empirical model parameters. However we find that this parameter is not sufficient since a same part of potential at the exterior of the thruster can lead to rather different dynamic behaviors [4].

As is to be expected, electron current passing the cathode line $I_{e,c}$ is also very sensitive to the anomalous mobility (figure 6.b). It obviously increases with α and k. This parameter provides another way to check our model against experiments and to limit the domain of variation of the empirical model parameters. Existence of anomalous electron transport is confirmed by the model since without anomalous transport (α=0 and k=0), the model does not find a physically realistic solution.

V. CONCLUSION

The 2D hybrid model uses an empirical approach to treat the cross field anomalous electron transport. Assuming that this transport is greater outside (field fluctuations) than inside (electron-wall collisions) the thruster, the model is in good quantitative agreement with measurements of static and dynamic behavior of the SPT-100 at nominal operation. Although we find that anomalous electron transport may vary with the operating conditions, this agreement is also qualitative for other operating conditions. A better knowledge of the anomalous electron transport in SPTs is necessary to improve the predictability of the model over a wide range of working conditions. Systematic comparisons with experimental data such as performance parameters, discharge, ion and electron currents, distribution of the potential inside and outside the thruster, could help to calibrate the empirical model parameters and reveal their dependence on operating conditions.

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