IEPC-2003-0206
COMPARISON OF MATHEMATICAL MODEL OF A PLASMA PLUME WITH OUTCOMES OF SPACE EXPERIMENTS.

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Abstract
In the report the comparison of outcomes of different space experiments using onboard plasma sources with outcomes of calculations of plasma plume characteristics and an interaction between plasma plumes and space craft elements is presented. In calculations the physical features of both model of plume expansion and an interaction of a plasma plume with conductive and non-conductive surfaces, and also the propagation of radio waves through a large-scale anisotropic plasma non-uniformity were taken into account. The good quantitative consent of theoretical and experimental results is obtained.

Introduction
During many-sided study of plasma plumes performances (see e.g. Ref. [1,2]) the peculiarities of an influence of both the own electric field and an external magnetic field on the plasma dynamic was investigated. During these researches the processes which allow to control plasma dynamic were studied . It is possible to accelerate and slow down the flow [3] to focus or to expand [4] and even to divert it from the initial trajectory by means controlling own fields and currents [5,4].

In conditions of space flight plasma plume of electric propulsion (EP) expires in an environment characterized with both a low density of ionosphere plasma and neutrals of S/C own external atmosphere (SCOA) (n_n~ 10^9÷$10^8$$cm^{-3}$). In this case expansion of plasma isn't quite free since the plume and its secondary plasma have electrical contact with S/C body and conductive background. Unlike of space conditions, during EP operation in vacuum tank the electrical contact of plasma with conductive walls of the bench is very tight. It results in distortion of distribution of both an electric potential and currents in the plume, therefore in distortion of dynamic characteristics of the jet as well. Especially it relates to the periphery of the jet where the flows of a secondary plasma creating by an interaction between plasma plume and background gas (three or four orders of magnitude denser than in space conditions) effect directly on both an expansion of primary jet and a distribution of electric potentials into it and, finally, dynamic characteristics of a jet.

Relying on described above electro physical properties of plasma plumes our main goal at the development of EP plasma plume model was to describe maximum correctly the distribution of fields and currents into the jet according to the onboard spacecraft conditions.

To describe the dynamic of a jet we have used a set of Braginskiy' equations for two-component plasma [6]. These equations take into account the effect of electrical and magnetic fields and also electrical currents on the dynamic of a jet. Particularly, it takes into account thermoelectronic and thermomagnetic effects.

In general case the motion of hypersonic EP plume in an external magnetic field can be described by the 3D system equations, containing elliptical equations also. For the purpose of searching of the rational and correct enough solution of such complicated task we have used the hypothesis of self-similarity of plasma flow (2D and 3D configuration). Basing on it we have developed the self-similar model (SSM), which takes into account, particularly, the processes of electric current generation into plume volume. The theoretical outcomes of SSM modeling of plasma parameters distribution in vacuum tank we compared with experimental data of different kind thrusters: Hall-type, ion engine, arcjet. In Ref. [7] it was shown that the SSM-model more precisely than a lot of other models and approximations describes the plasma parameters in flow periphery. Moreover, the performed comparison leads to important conclusion: since plasma plumes of various mass flow rates, specific impulses and power levels can be described well by SSM-model, it is actual proof of the correctness of hypothesis about a self-similarity of EP plasma flow. It proves visually in experimental measurements: the plumes having especial geometry at initial section (e.g. hexahedral beam exhausted from ion engine [8] or hollow jet exhausted from SPT or TAL [9]) becomes self-similar already at distances 2÷3 gauges from an exit nozzle.

So, the self-similarity feature, prevailing in nature (from micro capillary flow up to nuclear explosion [10]), inheres also in a particular case of EP plasma plume.

In presented work we have compared the calculated parameters of plumes of different propulsions and the effects of their impact on the both a SC structure and onboard equipment with flight data.

Plasma plume expansion in vacuum.

Unlike gas jets, the expansion of plasma ones into vacuum is defined not only by collision processes and initial conditions but also by the internal self-consistent electrical field. The magnitude of the internal electrical field is defined by the gradient of both density and temperature of plasma, external magnetic field and currents in the jet. In the rarefied plasma electrical heat conductivity, thermoelectrical and thermomagnetic effects impact essentially on the distribution of these fields . In the result the value and distribution of electrical fields in the jet differs substantially from Boltzmann`s distributions that are often used in systems of plasma dynamic equations (see e.g. Ref.[11,12]). Ions are accelerated by this field mainly in the directions which accord to maximum values of gradients. In the result the density of the propellant in the peripheral area of plasma plume is relatively larger than in ideal gas jets with the same initial characteristics.

At the initial stage of jet expansion magnetic field effects weakly on plasma dynamic and its expansion is inertial, so that initial stage of exhaust has axially symmetric form. As jet expands, the velocity of plasma motion across magnetic field slows down to the velocity which is equal to transverse diffusion. The motion along field ***B*** accelerates a little under effect of self-consistent electrical field. Difference of velocity along and transverse ***B*** causes a difference of plasma plumes shapes. They get specific elongated forms following the direction of terrestrial magnetic field.

Configuration of the distant area of the plume injected on the LEO orbit will be defined by pitch-angle α – angle between velocity vector of the jet and geomagnetic field vector. If the jet flows along force line ($\alpha \cong 0°$), the plasma plume looks like long narrow needle with much smaller transverse size a than typical longitudinal size l ($a \ll l$). If the jet flows across force lines ($\alpha \cong 90°$), the distant area of the plume looks like petal flattened in the direction ***[VxB]***.

Comparison of the SSM-model plasma plume with space experimental data

Comparison with GEO flight data.

Experiment "Express".

On board the geostationary SC "Express" the probes for measurements of both a density of ion current and an ion energy distribution were mounted at distances of $x_1 = 1m$, $x_2 = 3.8m$, $x_3 = 8.8m$ from exit nozzle. According to calculations executed in Ref. [13] for SPT-100 operated in geostationary conditions the influence of Earth magnetic field upon plasma plume expansion becomes sufficient at distance of $x_b \approx 40m$ from exit nozzle. That is why at calculation of plume parameters on *Express* the influence of geomagnetic field can consider negligible.

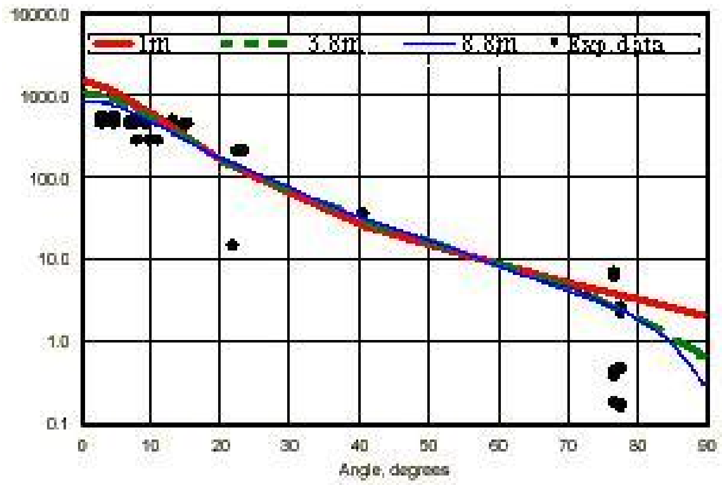

Figure 1 Comparison of *Express* experimental data and SSM-distribution of axial current density, j_x, for different distances .

On Figure 1 in polar coordinates the distributions of axial component of ion currents densities for different distances from SPT are plotted. All of experimental data depicted on Figure 1 and theoretical distribution of current density at different distances also are normalized for distance *1m* from exit nozzle. Normalized flight data are taken from Ref. [14,15].

Theoretical curves *j(θ)* basing on self-similar distributions for axis-symmetrical plume expanding on GEO coincide good enough with flight data. Besides, all of these curves are obtained as an outcome of uniform calculation for all three distances, as for near zone and for far field zone of expansion as well.

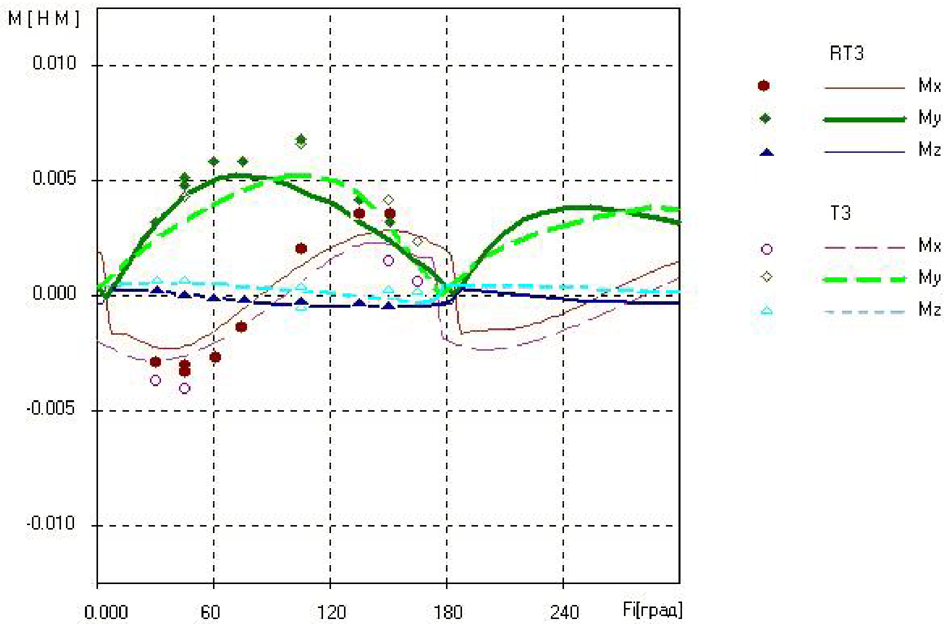

Figure 2 Comparison calculated torque with Express data for T3, RT3 [14]

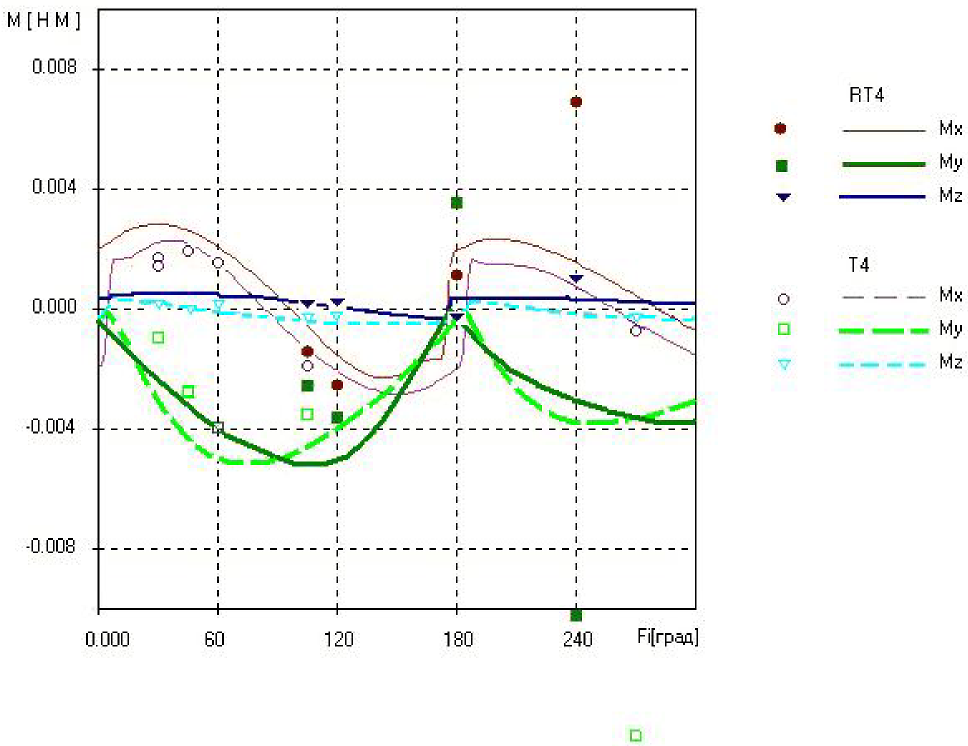

Figure 3 Comparison calculated torque with Express data for T4, RT4 [14]

Figure 2 and Figure 3 show the calculated results of all three torque components *Mx, My, Mz,* which appear on SC *Express* when the SPT operates. In the figures hollow dots relate to experimental values of torque of working thrusters (T3 and T4). Filled dots relate to reserve thrusters (RT3 and RT4).

The angular distributions of three torque components are enough for definition of several plume parameters. In particular, on the assumption of smallness of the relations $\frac{M_z}{M_x} < 0.2$ and $\frac{M_z}{M_y} < 0.1$ for effective magnitude of accomodation coefficients the relation of $\frac{\alpha_n + \alpha_\tau}{2} > 0.9$ holds true. It agrees with the mechanism of effective interaction between attacked surfaces and flow of ions accelerated by the electric field of plasma and wall boundary layer as well. Quantitative estimation on a base of experimental dependencies of *Mx, My, Mz* allows to define identically the values of accomodation coefficients α_n=0.9 and α_τ=0.95 for front surface of the solar battery and $\alpha_n = \alpha_\tau$=1.0 for back surface of the solar battery. Results of torque calculations coincide with experimental data keeping within the range of errors (ΔM~1mNm).

According to SSM-model the density of propellant in a flow periphery is more, than it is guessed by models based on simple conical expansion. It is caused because of an additional expansion of the jet in the far distances under effect of the electrical field. Besides, the SSM- model takes into account the fact that a boundary layer with the potential drop arises close to the surface flown by the plasma. Ions are accelerated additionally there [16]. Both effects increase significantly the density of normal force, which influences on the far area of the solar battery. Due to big arm this increased force makes a significant contribution to *My* and *Mx* values. An use in calculations of the torque of more correct mathematical model of plume allows to get a good agreement with experimental data even without artificial decrease of accomodation coefficients α_n and α_τ (that was required in earlier used models, see e.g. [17])

Comparison with LEO flight data.

Experiment "EPICURE".

Under bench conditions it is not available to register the influence of Earth magnetic field upon EP plasma plume dynamic. However, the fact that a plasma exhaust has no conical shape, but petal- or needle-like one, depending on the direction of the magnetic induction vector, was confirmed in a full-scale experiment within the EPICURE program conducted on SCs "Cosmos-1818" and "Cosmos-1867".

The geometry of compound object "SpaseCraft + PlasmaPlume" at realization of experiments in middle latitudes (above Russia) is shown in proportions on Figure 4. Plasma petal shown on Figure is limited by the level of density of $n_e=2 \cdot 10^{14} m^{-3}$.

The EPICURE plasma generator (the type of arcjet) operated in-flight and produced a flow of Cs^+ ions with the ion flow rate of $\dot{N} = 2 \cdot 10^{20}\ s^{-1}$ and velocity $u = 2 \cdot 10^3$ *mps.* The three-dimensional configuration of a plasma exhaust was verified by several independent methods.

<u>The first one</u> relates to radio measurements. A radio transmitters operated in ranges of wave length λ*=1.8m* and λ*=0.12m* onboard a space vehicle (at the 800-km altitude). A broad network of ground receivers was involved in the experiment. During the transmitter operation a generator of plasma was actuated under a specific cyclorama. During the active period of generation the plasma shielded radio signal and on the Earth the shape of radio shadow was registered (see Figure 5). An analysis has shown that:

1. The radio shadow is nonsymmetrical about a flight course.
2. The radio shadow shape is different for ascending and descending trajectory, i.e. for different directions of the magnetic induction vector about the velocity of a plasma jet.
3. Theoretical curves of radio shadows estimated under SSM models for the cases when the velocity of a jet is directed at diverse angles to the magnetic field have practically coincided with experimental data (dots on Figure 5).

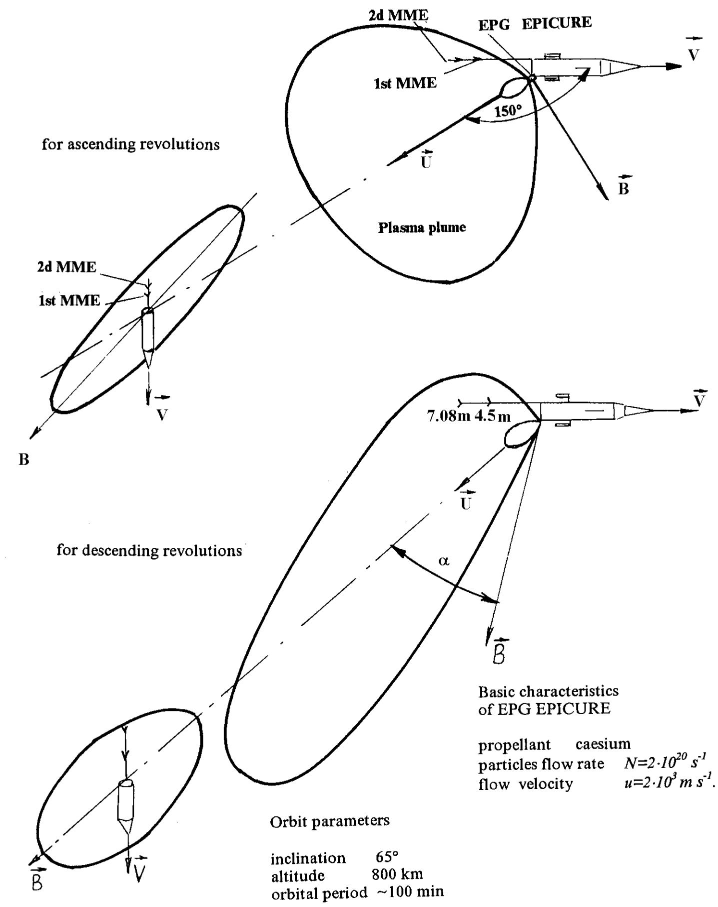

Figure 4 Shape of plasma plume in EPICURE experiment

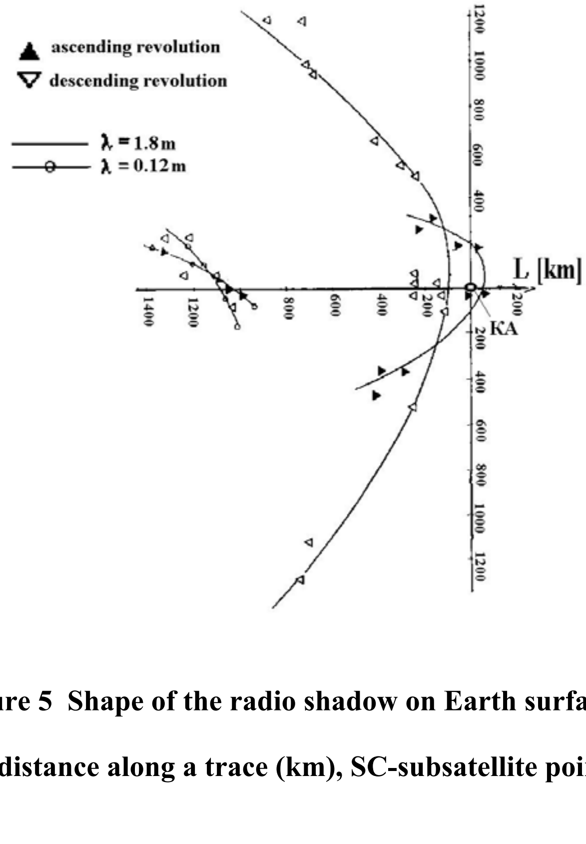

Figure 5 Shape of the radio shadow on Earth surface.

L- distance along a trace (km), SC-subsatellite point

Second case relates to radio probing of plasma petal. Plasma petal has the same radiophysical characteristics like flat-laminar substance. In particular, it tells upon sharply angular anisotropy of reflection performances. The angular diagram of effective area of backward scattering (EABS) of radio waves by plasma petal is shown on Figure 6. The diagram is depicted in coordinates which are the directional cosines of line-of-sight in the “plume’s” coordinate system for the case when pitch-angle α=80°. In the “plume’s” coordinate system the axis OZ coincides with a vector of geomagnetic field induction $\boldsymbol{B}$ (OZ $\parallel$ $\boldsymbol{B}$) in every point of orbit, the axis OX locates in a plan of vectors $\boldsymbol{U}$ and $\boldsymbol{B}$ (when the plasma jet is directed across the magnetic field, i.e. α=90°, axis OX coincides with a vector of jet velocity $\boldsymbol{U}$ (OX $\parallel$ $\boldsymbol{U}$).

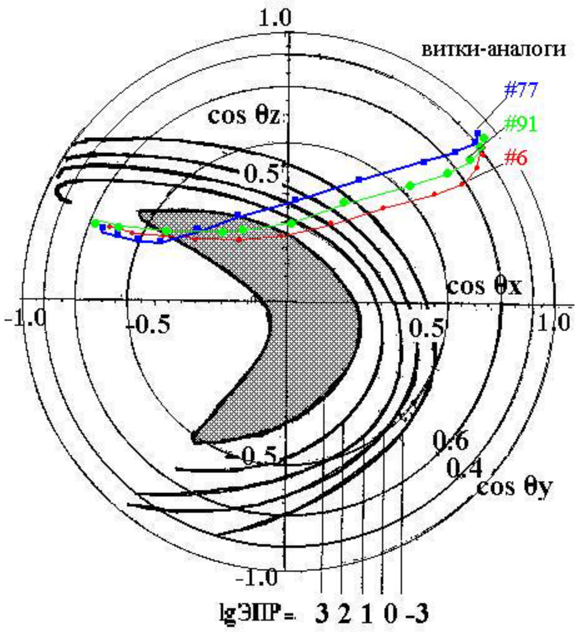

Figure 6 Angular diagram of EABS value of plasma petal

.The values of EABS are presented in non-dimension form; they are normalized relative to square of $(\lambda/2\pi)^2$ (λ- length of radio wave). The computation is executed in the ray approximation for a case of a propagation of flat metric wave.

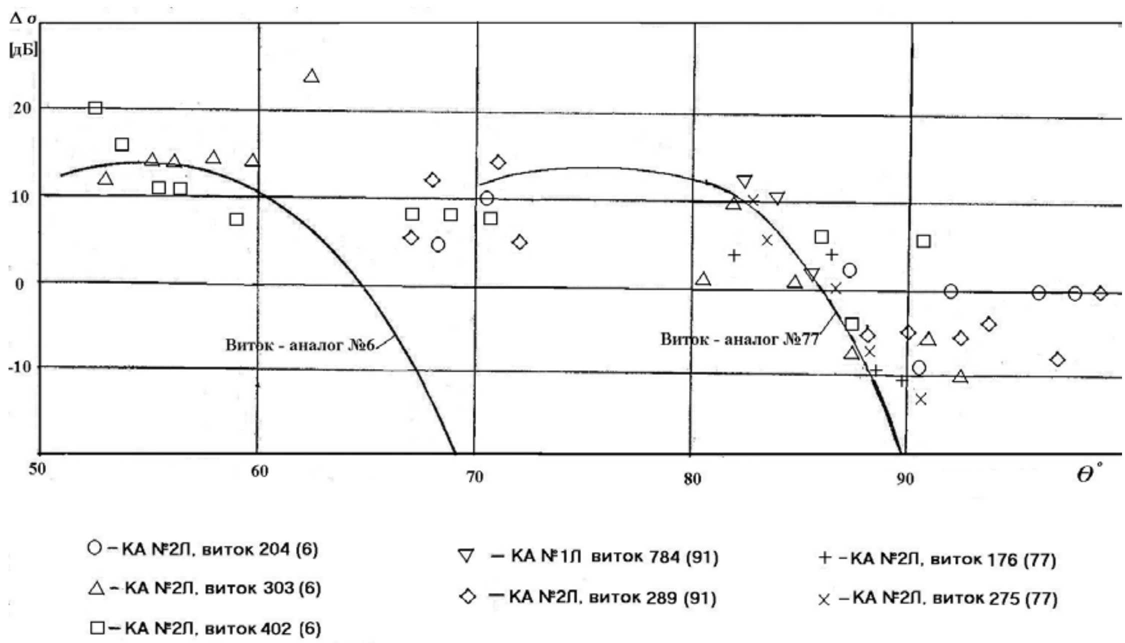

Figure 7 Experimental and theoretical data of effective area-back-scattering of plasma petal .

From the diagram it is visible that in a direction close to normal to the petal, and in a direction “side - behind” also, the magnitude of EABS is rather great. On the diagram these directions fall in shaded area. In remaining directions the value of EABS rather sharply drops.

On Figure 7 the outcomes of calculations of EABS value for two circuit-analogs at two directions of sight (“front - side” and “side”) are submitted. On these circuits the jet effused practically across a geomagnetic field. The points show experimental values for several circuit- analogs. The change of sight-angle for the same circuits is shown on the angular diagram (Figure 6) as dashed lines. The comparison of computational and experimental data allows making a conclusion about accuracy of theoretical representation about three-dimensional, petal-shape configuration of plasma plume on LEO conditions.

The third example is based on magnetic measurements. The geomagnetic field perturbations were registered with two 3-component detectors (Magnet Measurements Equipment – MME sensors, developed in IZMIRAN, Russia), mounted in the boom at distances 4.5 m and 7m from the plasma generator [18]. The measured perturbations of the geomagnetic field ***B*** correspond to the diamagnetic displacement of the field out of a volume of conductive jet. A 3D picture of the geomagnetic field ***B*** perturbations coincides with the 3D model of a plasma petal, in which the expansion across the field ***B*** causes a generation of electric currents. A comparison of calculated results with disturbances of the geomagnetic field ***B*** measured in the EPICURE experiment is shown on Figure 8. The abscissa-axis is the distance *L* from exit nozzle plan along a boom, on which the MME sensors were mounted.

The obtained experimental data allow to draw a conclusion about correctness both of the idea of a system of currents inside a plasma plume and the model of interaction of a dense, low-temperature plasma jet with an external magnetic field.

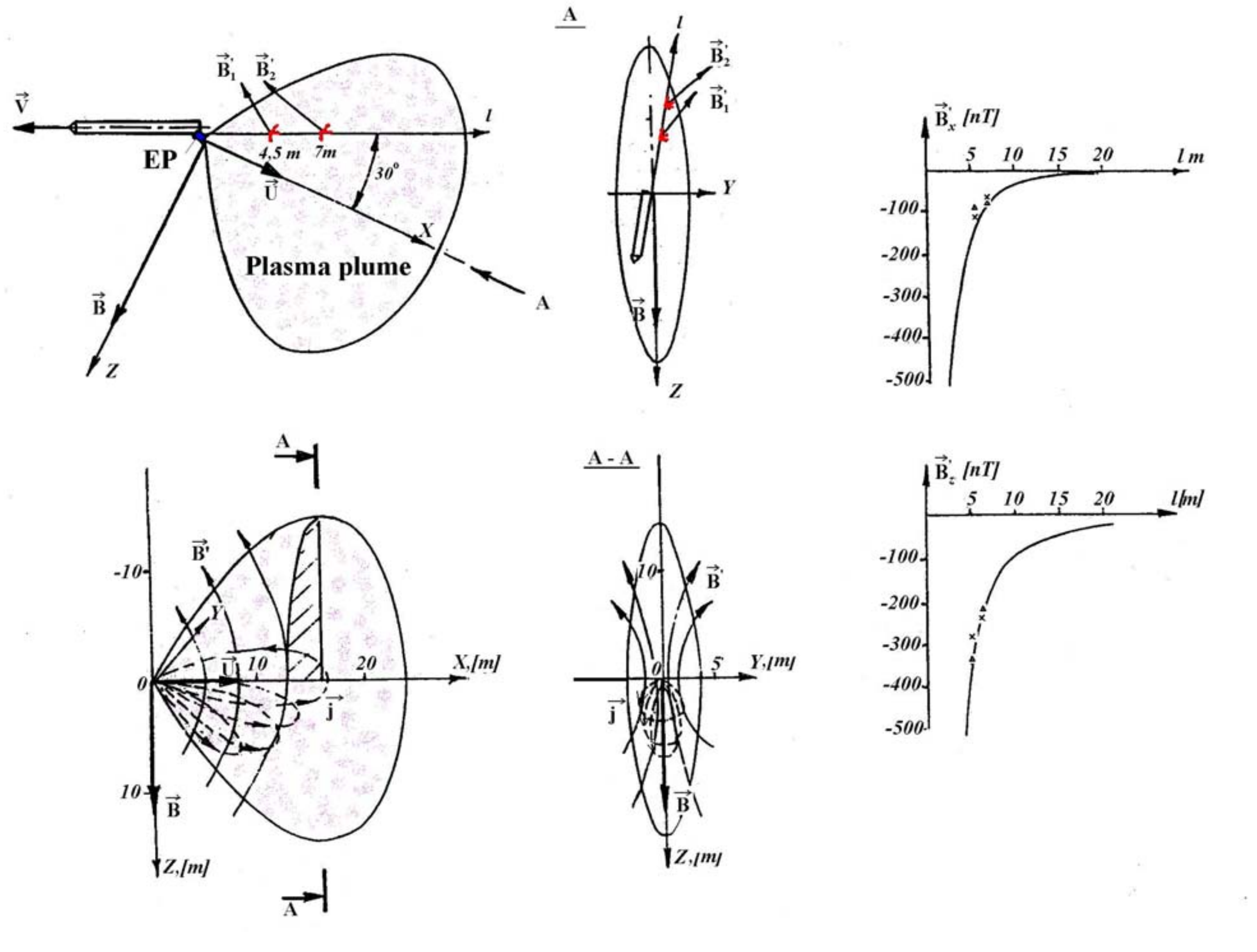

Figure 8 Disturbance of geomagnetic field in EPICURE experiment.

Experiment “Meteor”.

In flight experiment on space vehicle "Meteor" [19] the change of all three components of a torque created due to an impact of the SPT-60 plasma plume on the solar array (SA) panel is fixed during one circuit. The satellite moved on the circular, almost polar orbit with an inclination 82° and with an altitude of ~900 km. Orientation of SC and its solar panels, and direction of the plasma exhaust also were invariable on this circuit. Therefore, the fact of existence of change of torque value testifies directly a change of a plume configuration along the SC motion from equator to a pole. A reason of this fact is the change of value and direction of a geomagnetic field concerning an axis of plasma exhaust. The measured and computed components of a torque are submitted on Figure. All features of time functions *M (t)* agree with changes of a plasma plume configuration :

- Near to equator, where $\boldsymbol{U}//\boldsymbol{B}$, the plume has the shape like a thin spoke, which does not touch almost the SA panels, therefore it does not create the torque - on an Figure value *M~0*;
- Near to a pole, where $\boldsymbol{U}\perp\boldsymbol{B}$, the plume represents rather thin petal oriented in a vertical plane. Center of force application over the SA plane moves during a rotation of a petal plane. Following to this the torque components change.

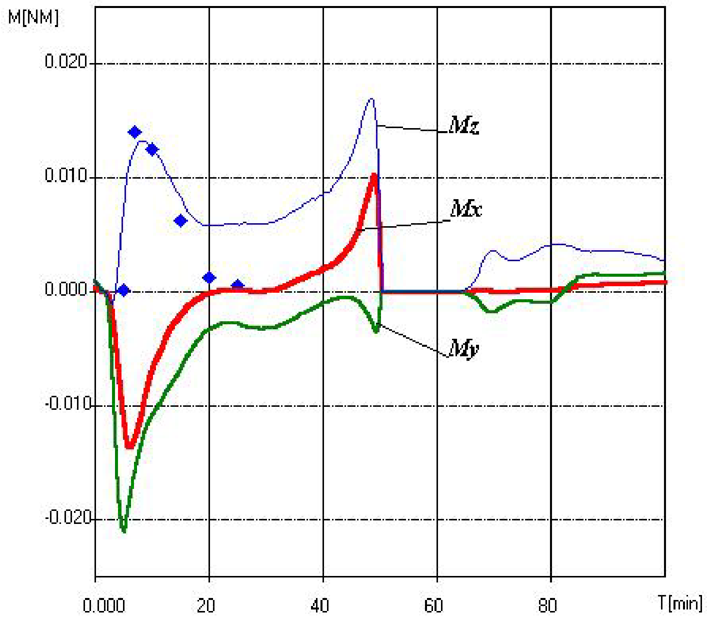

Figure 9 Flight data and theoretical dependencies of change of torque components during one orbit period of *Meteor* SC.

Conclusion

Carried out comparison between results of calculation of parameters distribution in Hall-thrusters exhaust plumes showed the following:

1. Results of space experiments confirm the dominating influence of both internal electrical self-consistent field and external magnetic field on the dynamic of the exhaust plume.
2. Under effect of an internal electrical field the distribution of parameters in exhaust plasma plumes considerably differ from the analogous distributions in neutral gas jets It results in noticeable differences in effects of interaction between plasma plume and space craft.
3. The existence of electrical fields and currents inside a plasma plume stipulates for an additional electrostatic interaction of a plasma with SC' structure elements and onboard systems, which causes, in particular, the increase of effective values of accomodation coefficients.
4. The hypothesis about self-similarity of plasma flows in EP plumes, tested earlier in bench experiments, is now convincingly confirmed by the flight data obtained as on LEO spacecrafts and on GEO satellites as well.
5. Presented model has all advantages of analytic solutions. Among them comparative simplicity of practical using, lack of computational constraints of calculated area, possibility to create on its basis high-speed methods including analysis of experimental estimations in real-time mode and at the same time it doesn't concedes to the most precise numerical solutions.

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