MAGNETIC NOZZLE PLASMA EXHAUST SIMULATION FOR THE *VASIMR* ADVANCED PROPULSION CONCEPT

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ABSTRACT

An MHD simulation study is being carried on to investigate the efficiency of the thrust generation process and the properties of the plasma detachment from the nozzle in the VASIMR experiment.

The goal of the modeling effort is to provide an analysis tool for the nozzle performance and for optimizing the design of the plasma acceleration section and of the magnetic field profile.

The model is being developed specifically for obtaining comparisons with laboratory measurements in the VASIMR experiment. Preliminary, two-dimensional results (in cylindrical geometry) on the evolution of the plasma profiles in the nozzle magnetic field are here reported.

INTRODUCTION

VASIMR (Variable Specific Impulse Magnetoplasma Rocket, [1]) is an electric propulsion concept currently under experimental development at the Advanced Space Propulsion Laboratory, NASA Johnson Space Center.

An RF (Helicon) discharge generates plasma (typically ionized Hydrogen or Helium) confined by a solenoidal magnetic field. The plasma is then further heated by a RF booster (in the ion cyclotron resonance frequency range) and finally flows through a magnetic nozzle where the thermal plasma energy is converted into kinetic energy along the longitudinal direction, enabling plasma detachment to produce thrust.

The system has no electrodes and the magnetic field ensures magnetic insulation of the plasma from the material surfaces. By powering the plasma source and the heating antenna at different levels it is possible to vary smoothly of the thrust-to-specific impulse ratio while maintaining maximum power utilization.

A MHD simulation modeling effort (*NIMROD* code [2,3]) is being carried on to study the plasma parameters and the detachment process of the plasma plume in the magnetic nozzle.

In the model, a plasma source feeds a continuous flow through a nozzle-shaped magnetic field. Profiles of plasma density, temperature and flow velocity are then recorded at different times, while the plasma plume is formed in the magnetic field. Current simulations are set to model the plasma environment up to few meters from the nozzle throat: at that distance the plasma exhaust parameters reach values comparable with the ionospheric plasma background [4].

A MHD modeling effort with the *NIMROD* code [2,3] has been started for studying the details of the nozzle acceleration process and the properties of the plasma detachment from the nozzle. The simulation results are to be used to investigate the nozzle performances and to optimize the

design parameters of the plasma acceleration section and of the magnetic field profile of the engine.

First *NIMROD* runs have been benchmarked with a two-dimensional particle trajectory code showing satisfactory results [4]. The present simulation effort is geared towards obtaining quantitative answers in a direct comparison with experimental measurements in the laboratory.

MODEL DESCRIPTION

The goals of the simulation are to study the plasma detachment, the plume collimation (allowing a numerical measurement of the effective thrust) and to perform optimization studies of the magnetic nozzle.

The fundamental plasma parameters to be followed are density, temperature and flow velocity. In addition, the perturbation to the magnetic field in the nozzle produced by plasma currents must be computed by solving for the self-consistent magnetic field in the plasma

The geometry is cylindrical, with plasma flowing along the axis, as in fig. 1. While basic studies can be performed in two dimensions (r-z), assuming azimuthal symmetry, the full three-dimensional model will be required for nozzle optimization studies and quantitative predictions capable of affecting the design of the experiments.

All these requirements lead to the need of a 3-D, nonlinear compressible MHD model. The *NIMROD* code has the required features and was here used with few modifications.

The code implements Maxwell's equations in the low-frequency approximation, a resistive MHD generalized Ohm's law along with equations for the particle conservation, center-of-mass velocity evolution, and temperature evolution respectively, as follows:

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times \mathbf{E} = 0 \tag{1}$$

$$\mu_0 \mathbf{J} = \nabla \times \mathbf{B} \tag{2}$$

$$\mathbf{E} = -\mathbf{V} \times \mathbf{B} + \eta \mathbf{J} \tag{3}$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{V}) = 0 \tag{4}$$

$$\rho \left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \nu \rho \nabla \mathbf{V} \tag{5}$$

$$\frac{n}{\gamma - 1} \left(\frac{\partial k_B T}{\partial t} + \mathbf{V} \cdot \nabla k_B T \right) = -p \nabla \cdot \mathbf{V} - \nabla \cdot \mathbf{q} + Q \tag{6}$$

where all units are MKS, $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic induction, $\mathbf{V}$ is the particle flow velocity, $\mathbf{J}$ is the plasma current, n is the number density (assuming quasi-neutrality $n=n_e=n_i$), T is the fluid temperature, k_B is the Boltzmann constant, p is the total fluid pressure, ν is the viscosity, $\mathbf{q}$ is the heat flux vector (related to n and ∇T through a non-isotropic thermal conductivity) and Q is a heat source density that includes Ohmic and viscous heating.

The particle number density n and mass density ρ are related through the mass per ion, pressure and temperature follow the ideal gas relation, $p=2nk_BT$, and $\gamma=5/3$ is the ratio of specific heats. The single temperature of Eq. 6 represents the total fluid temperature in the assumption of rapid equilibration between the ion and electron components. NIMROD has also a provision for running a two-fluid model where a separate equation for electron temperature is evolved and additional terms are included in the generalized Ohm's law. Electron pressure can be also included in the single fluid model as described in [5].

In the continuity equation (4) a particle diffusion term could be required for simulations over long time-scales, like transport time-scale in fusion plasmas, where physics effects other than those captured in the MHD model are influencing the density profile evolution.

The numerical implementation of these equations in is done via finite element formulation as described in [6]. Fourier transform is applied along the third, periodic, dimension (here the azimuthal coordinate). The code requires a modified version of the Faraday's law that includes a numerical diffusion term for reducing the divergence of $\mathbf{B}$ to negligible levels.

MHD SIMULATION OF THE VASIMR PLASMA EXHAUST

The modeling of the magnetic nozzle is a critical issue for the design and development of the VASIMR engine. In the present laboratory experiment most of the activity has been focused on the development of the plasma source and of the RF booster section.

The analysis of the magnetic nozzle and of the related issues like plasma detachment, nozzle efficiency and optimization and stability of the plasma flow would require an upgrade of the current experimental set-up. Computer modeling of the nozzle plasma is therefore of critical importance at this stage of the project development.

The approach that is being followed is based on a MHD model of the plasma in the magnetic nozzle region. The NIMROD code is used with few modifications to fit the VASIMR open-end geometry. The present effort concentrates on basic tests of the physics of the plasma flow in the exhaust magnetic field.

The model considers a cylindrical region that encloses part of the magnetic nozzle diverging field lines (Figure 1). The present simulations have been conducted in two dimensions, therefore, with respect a cylindrical coordinate system of Figure 1 (r-z-θ), an azimuthal symmetry is assumed.

The simulation domain has a radius smaller than the last coil of Figure 1, so that only a limited part of the diverging magnetic field region is included. However, in the present VASIMR experiments the plasma radius is a fraction of the coil radius and this allows enough "room" in the model for analyzing the effect of the plasma expansion across the field lines before any significant interaction with the boundaries would take place.

Since the magnetic nozzle region will vent into the in the open space, no outer physical boundaries should be included in the model. The present simulations have been then conducted within a region large enough to maintain the plasma profiles nearly unchanged near the boundary of the computational domain. This region is discretized by means of a non-uniform mesh to reduce the computational effort. The zero flux "natural" boundary conditions imposed by the finite element formulation are here imposed. Further comparisons with open-end boundary conditions models are in progress.
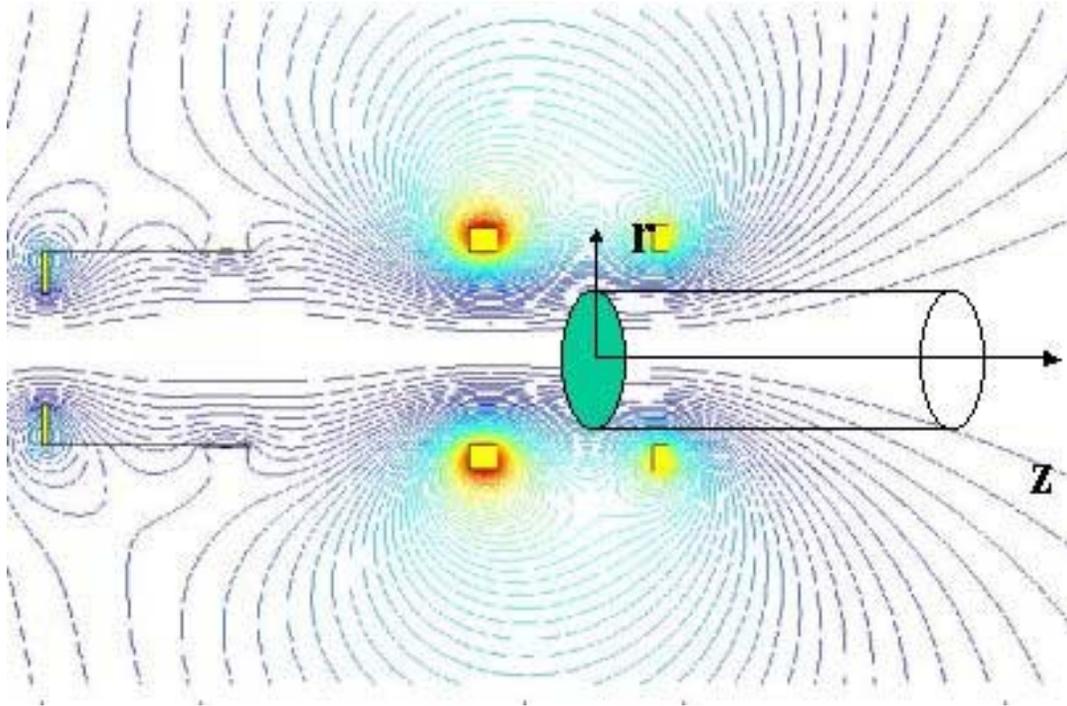

Figure 1 – Simulation domain for the magnetic nozzle region

BASIC SIMULATION SCENARIO

The goal of this simulation effort is to reproduce the behavior of a dynamically evolving plasma in a magnetic nozzle from a quantitative point of view. Basic tests aimed at reproducing the fundamental aspects of the physics are being conducted.

The exhaust plasma is, in general, expanding both in the longitudinal and in the radial direction: the longitudinal motion starts with a drift velocity of the injected plasma near the left side of the domain (referring to Figure 1). This is simulating plasma that is coming from the VASIMR (Helicon) source and that has been already heated in the RF booster section.

The injected plasma is therefore carrying a net drift velocity in the longitudinal direction, in addition to a thermal velocity spread. The longitudinal motion will be then enhanced by the effect of the magnetic nozzle that produces a longitudinal acceleration at the expense of the thermal energy in the perpendicular plane. Eventually, provided that the exhaust has enough energy, this process would lead to a detachment of the plasma from the magnetic field.

The thermal conductivity is included in the model equations but the simulations here reported consider an adiabatic approximation as a baseline. The effect of the (anisotropic, due to the presence of the magnetic field) thermal conduction will be reported later.

EVOLUTION OF THE RADIAL PLASMA PROFILE

While the plasma is traveling along *z* a pressure gradient is causing a force in the radial direction: this force is, in general, only partially balanced by the magnetic pressure of the nozzle field. An advective flow is then produced in the radial direction, causing an expansion of the plasma profile.

In addition, a diffusive flow can also be present because collisional effects. Since the model is based a resistive MHD formulation this contribution is included with the proper choice of plasma resistivity.

Simulations with different parameter setting have been performed to reproduce the different (advective and/or diffusive) character of the radial plasma profile evolution. Table 1 summarizes the four typical situations.

	$\eta=0$ (Ideal MHD)	$\eta\neq0$ (Resistive MHD)
$p=nk_BT \leq B^2/2\mu_0$ (Low pressure)	Confinement	Diffusion
$p=nk_BT > B^2/2\mu_0$ (High pressure)	Advection	Advection+Diffusion

Table 1

effect of the plasma current that modifies the total magnetic field is shown for the same two cases discussed above.
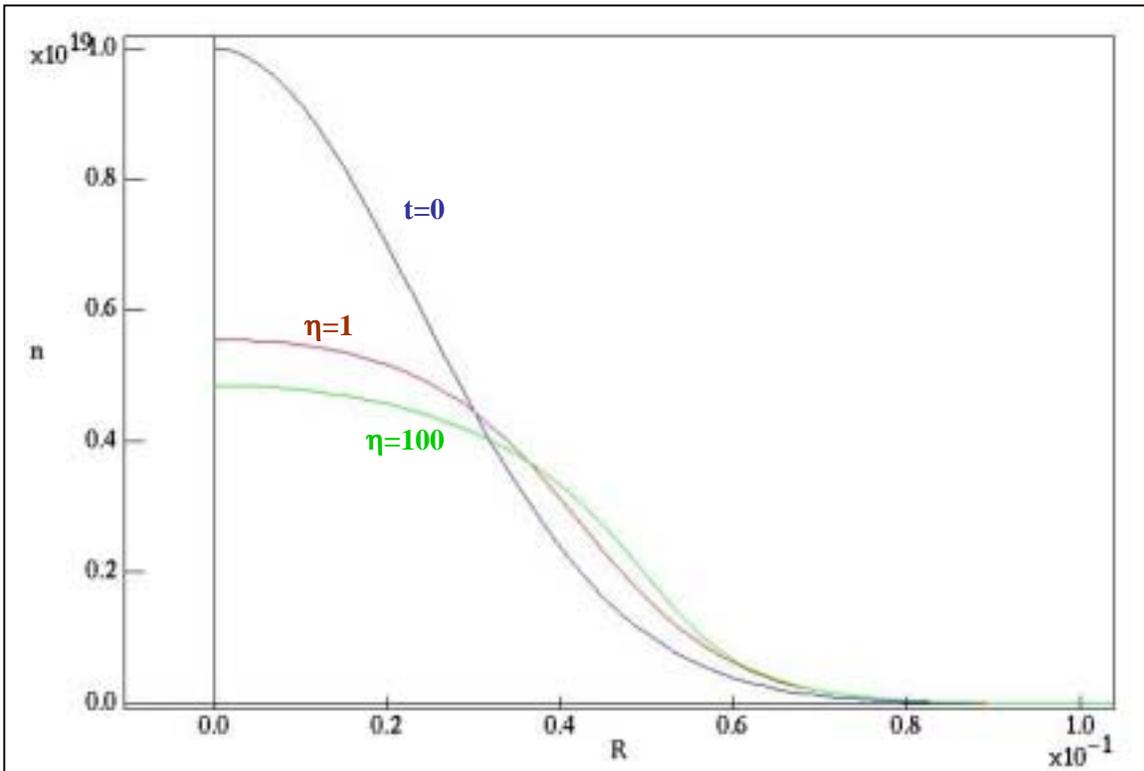

Figure 2 – Ideal vs. resistive MHD radial flow (high-pressure, advection-dominated case)

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In the NIMROD code a numerical viscosity term is included in the temperature equation, to provide a dissipation for onset of numerical instabilities. The value of viscosity used in all these simulations is the same, and while is not the smallest converged value, it produces a small effect that does not change the character of the solution. Additional numerical effects due to the discretization and iterative solutions may also play a role in the observed diffusion process: because the still qualitative character of the results here reported, a complete convergence study of these numerical effects has not been performed.

An example of advective radial expansion is shown in Figure 2. Here the model was set to reproduce a plasma column in a uniform magnetic field of 0.01 T along the z-axis. The plasma is loaded uniform along the z (longitudinal) direction and with initial radial density profile, as shown. Periodic boundary conditions were imposed in the z direction, while the domain along the r coordinate stretches up to 2 m, about 20 times the length reported in Fig. 2.

The curves show the radial density profile evolution at a fixed z-location, chosen in the center of the domain. The initial plasma pressure on axis is set as more than twice the magnetic pressure: a radial advective is established flow is causing the flattening of the profile, as expected. The resistive MHD case exhibits a further flattening due to the diffusion, not present in the ideal MHD case (in the simulation, nearly ideal, with resistivity reduced by a factor of 100).
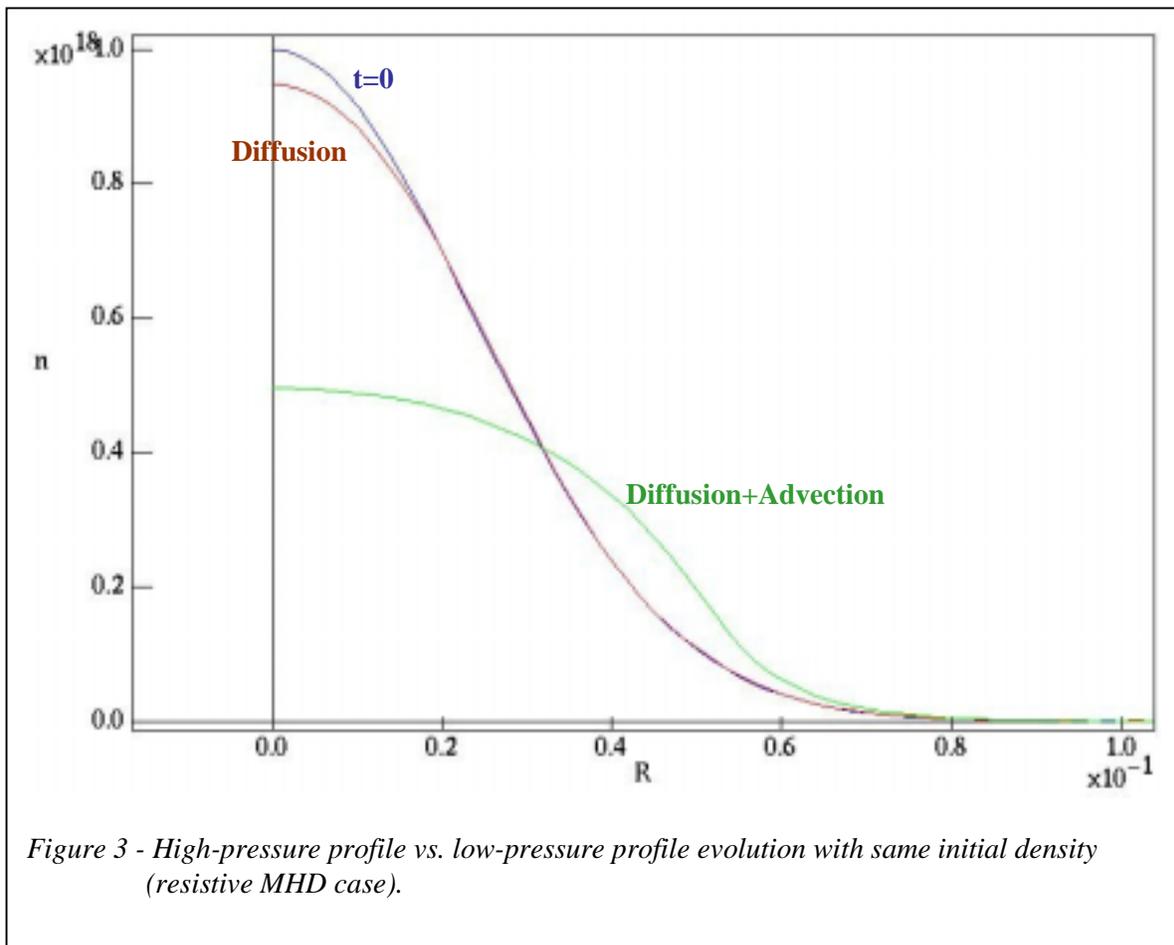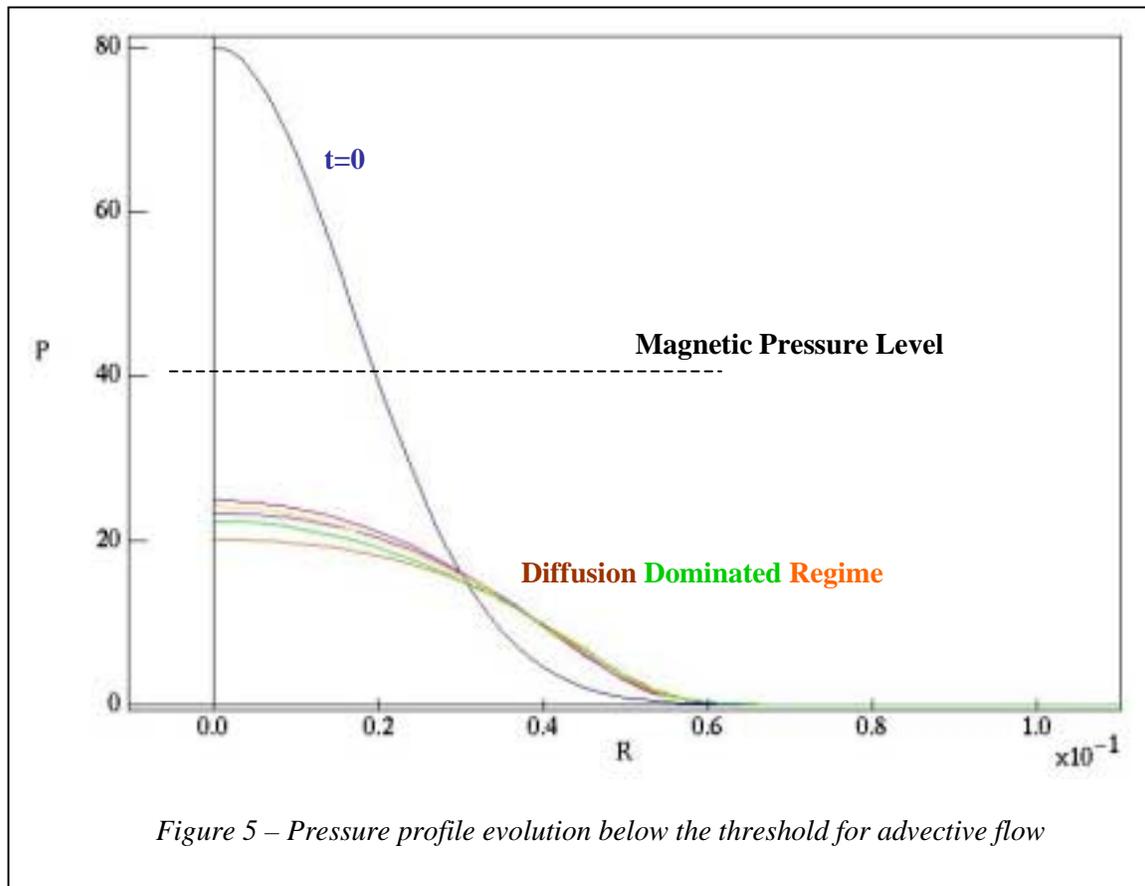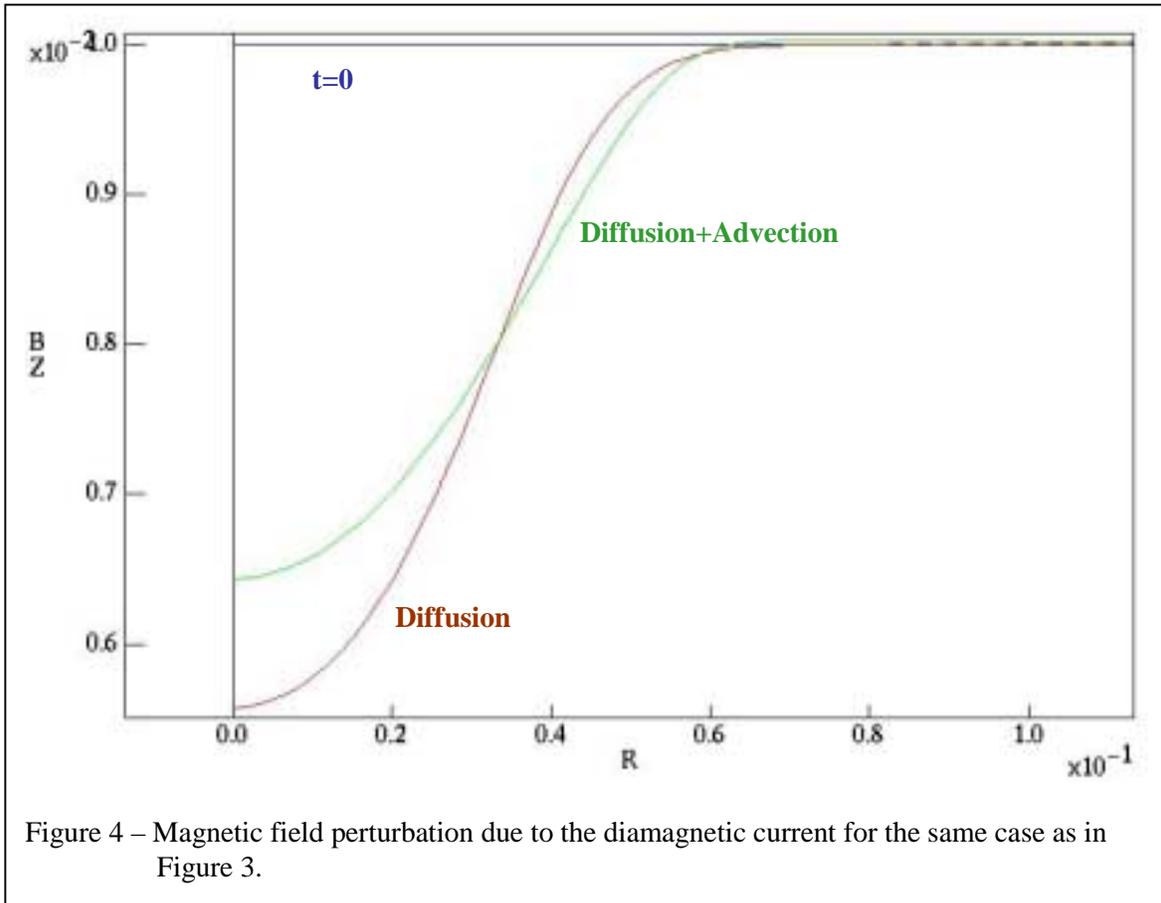

Figure 3 - High-pressure profile vs. low-pressure profile evolution with same initial density (resistive MHD case).

A different situation is depicted in figure 3: here the plasma resistivity and the radial density profile are the same but two different temperature profiles have been loaded to produce two different values of the kinetic pressure, respectively above and below the magnetic pressure (that is constant along the radius). The evolution of the density profiles for both the low-pressure and high-pressure cases are shown after the same time interval (about 2 μs) and compared with the initial condition. A significant difference in the profile amplitude is caused to the advection. While the physics somewhat contains an artifact, since the resistivity should be a function of the temperature (as, for example, in the Spitzer formula), this test allows to compare easily the effect of a case subject to advection and diffusion with a case affected only by diffusion. In Figure 4, the Finally in Figure 5, the evolution of the pressure profile for the high-pressure case is shown. The different curves are plotted at an interval of about 0.5 us and after the initial, advection-dominated decay, the pressure profile falls below the magnetic pressure level (computed for 0.01 T magnetic field). In these conditions the advective flow is halted and the further evolution becomes a damped oscillation of the profile amplitude dominated by the diffusion

While these simulations are intended to be essentially of qualitative character and a more comprehensive and detailed validation is in progress. Further tests will provide a basis for a near future a benchmark with experimental measurements in the laboratory as the model is being "tuned" for the specific requirements for a simulation of the VASIMR plasma exhaust (in particular in relation to the plasma injection source and to the open-end boundary conditions).

Figure 5 – Pressure profile evolution below the threshold for advective flow

Figure 4 – Magnetic field perturbation due to the diamagnetic current for the same case as in Figure 3.

EVOLUTION OF THE LONGITUDINAL PLASMA PROFILE

A critical goal of the present simulation effort is to build a model that is able to reproduce the conditions for plasma detachment in a nozzle-shaped magnetic field. The present simulations are starting to tackle the problem of the proper modeling the plasma source and the open boundary along the longitudinal direction in order to avoid unphysical effects.

The present approach considers a simulation domain discretized with a non-uniform mesh composed of three regions smoothly connected together. In Figure 6 the longitudinal profile of the z-component of the magnetic field is shown along with the discretization mesh (along z). The central region of the mesh starts from z=0 and is constituted of a uniform mesh, in the case shown up to z=0.2 (units are in meters). This is the region where most of the "action" is going to take place during the simulation interval.

On the left side, for z<0, a "source region" is considered. In this region the plasma is loaded with an uniform initial velocity and, because its relative large size provides a sufficient "reserve" of plasma for the injection during the observation interval of the simulation. This region effectively isolates the central part of the mesh from the left boundary and allows a continuous smooth injection. This method was found less noisy, for relatively large time steps, than imposing a perturbation source term in the time dependent equations.
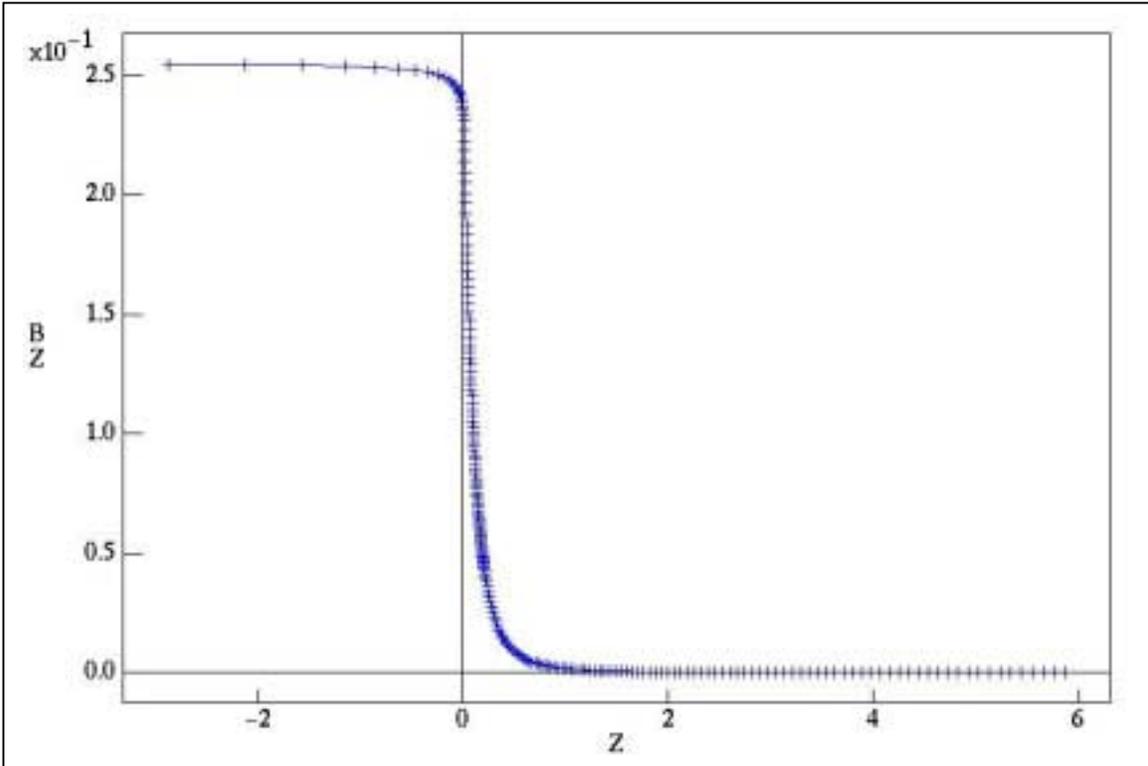
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Figure 6 – Initial B$_z$ profile in the longitudinal direction showing the non-uniform mesh and th source region (z<0)
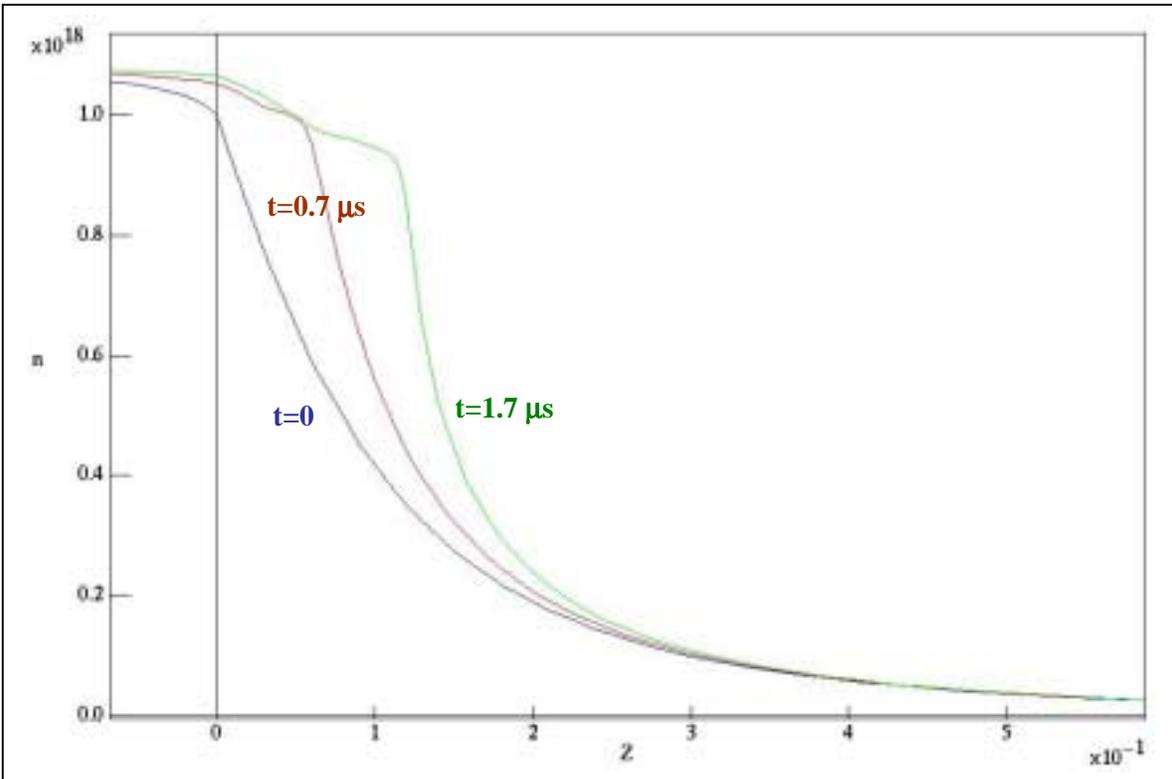

Figure 7 – Plasma injection and evolution profile evolution along the z-axis

The last region, on the right side of figure 6, effectively simulates an "open" domain by inserting a large distance, by means of a non-uniform mesh, between the plasma-filled region and the boundary of the simulation domain.

The time interval for the simulation will be chosen in order to avoid any significant plasma flow near the boundary. Longer intervals will require a larger mesh. In alternative, open-end boundary conditions model can be imposed. One of these models is presently under testing. The mesh along the radial direction is of similar structure, except that the source region is not present.

With the model described first tests of plasma injection and propagation through the nozzle have been initiated. The magnetic field profile of Figure 6, for *z>0,* is effectively the magnetic field produced by the coils in the VASIMR experimental setting [1],[4] and has the same pattern as the one depicted in Figure 1.

The result of a plasma injection test is shown in Figure 7: the plasma is fed from the source region and in *z>0* has an initial density profile proportional to the magnetic field (then has the same shape as B_z in Figure 6). As the injection takes place the plasma profile evolves rapidly: the injection velocity (as well as the velocity of the initial profile) was $4 \cdot 10^4$ m/s and the profiles in figure 7 are taken at 0.7 and 1.7 ms. The plasma has a maximum density on axis of 10^{18} m^{-3} and a temperature of 10 eV. The profiles shown represent only the variation along the *z*-axis, at the same time the profile is evolving, more slowly in the radial direction (initially, almost across the magnetic field, given the shape of the magnetic nozzle as depicted in Figure 1).

DISCUSSION AND CONCLUSIONS

Preliminary runs with the NIMROD code for the simulation of the VASIMR plasma exhaust have been reported. Simple qualitative tests of the code capabilities in modeling flowing plasma in open-end geometry have been presented. These results they constitute a first "sanity check" that will lead to more complete analysis of the fluid model and, eventually, a benchmark against with experimental plasma profiles from the VASIMR experiment.

The modeling with the NIMROD code eventually will constitute a powerful analysis tool for the transient phase of the plasma exhaust transit across the magnetic nozzle. In general, it is not even clear that an actual steady state phase should take place: at high plasma densities in fact the plasma detachment process is expected to exhibit a time-dependent behavior that includes a magnetic reconnection of the field lines in the detachment region. This could lead to a periodic formation of ejected plasma structures that build up, stretch the field lines and eventually detach, similarly, for example, to what occurs on the Sun surface during a coronal mass ejection.

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