TWO DIMENSION EFFECTS IN ION FLOW INSIDE INTER-ELECTRODE GAP OF PLASMA-ION THRUSTER ION OPTICAL SYSTEM

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One of the problem in plasma-ion thrusters (PIT) calculations is the absence of mathematical models, which describes ion and field descriptions radial distribution inside the inter-electrode gap (IEG) of ion optical system (IOS).

One dimension models don’t permit to get such descriptions like monochrome coefficient, thrust efficiency, ion lost on acceleration electrode (AE), energy flow density on different parts of AE surface, it’s erosion rate.

The value of electric potential difference via internal, from discharge chamber (DC) side, surface of emission electrode (EE) and external surface of AE has the order of electron temperature and is much less than acceleration voltage. So the boundary conditions for potential may be written as following:

$$\varphi = 0 \ (x = 0 \text{ and on EE surface}), \quad (1)$$

$$\varphi = -\mathbf{U}_0 \ (x = \mathbf{L} \text{ and on AE surface}), \quad (2)$$

where $\mathbf{L}$ – IEG length, including electrodes thickness; $\mathbf{U}_0$ – acceleration voltage; x – axial coordinate.

Ions average thermal motion velosity inside PIT DC is much less than exhaust velocity. Thus, ion flow into IEG can be considered like monochromatic with initial velocity equal to zero:

$$\vec{\mathbf{V}} = 0. \quad (3)$$

Model’s equations system in non-dimensional variables consists of ions continuum eqation, ions motion equation and Poison equation:

$$\nabla \cdot \vec{\Gamma}(x,r) = 0, \quad (4)$$

$$2\left(\vec{V}(x,r)\cdot\nabla\right)\vec{V}(x,r) - \nabla U(x,r) = 0, \quad (5)$$

$$\Delta U(x,r) = \frac{4}{9} n(x,r), \quad (6)$$

where $$\vec{\Gamma}(x,r) = n(x,r)\vec{V}(x,r). \quad (7)$$

The sequence between dimensional and non-dimensional variables and operators is the next:

$$\vec{\mathbf{r}} = \mathbf{L}\vec{r}, \ \mathbf{x} = \mathbf{L}x, \ \mathbf{r} = \mathbf{L}r, \ \vec{\mathbf{\Gamma}} = \mathbf{n}_0\mathbf{V}_0\vec{\Gamma}, \ \mathbf{n} = \mathbf{n}_0 n, \ \vec{\mathbf{V}} = \mathbf{V}_0\vec{V},$$

$$\varphi = -\mathbf{U}_0 U, \ \nabla = \frac{1}{\mathbf{L}}\nabla \text{ и } \mathbf{\Delta} = \frac{1}{\mathbf{L}^2}\Delta,$$

where ($\vec{\mathbf{r}}, \vec{r}$), ($\mathbf{x}, x$), ($\mathbf{r}$, r) – radius-vector, axial and radial coordinares; ($\vec{\mathbf{\Gamma}}$, $\vec{\Gamma}$) – flow dencity; ($\mathbf{n}$, n) – particles density; ($\vec{\mathbf{V}}$, $\vec{V}$) – velocity; (φ, U) – potential. Particles dencity and velocity scales are:

$$\mathbf{n}_0 = \frac{9\varepsilon_0 \mathbf{U}_0}{4e\mathbf{L}^2},\ \mathbf{V}_0 = \sqrt{\frac{2e\mathbf{U}_0}{m_i}}. \tag{8}$$

One dimension solutions of equations system (5) – (7) with boundary conditions (1) – (3) are:

$$U = x^{4/3},\ V_x = x^{2/3},\ V_r = 0,\ n = x^{-2/3}. \tag{9}$$

The complexity in two dimension model building is a result of equation (5) non-linear type for variables, which are in equations (4) and (6). In the first approximation, considering only linear corrections to solutions (9), equations system (5) – (7) can be brought to form:

$$\frac{\partial(nV_x)}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(rnV_r) = 0, \tag{10}$$

$$2V_x\frac{\partial V_x}{\partial x} = \frac{\partial U}{\partial x},\ V_x = U^{1/2}, \tag{11}$$

$$2V_x\frac{\partial V_r}{\partial x} = \frac{\partial U}{\partial r}, \tag{12}$$

$$\frac{\partial^2 U}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right) = \frac{4}{9}n. \tag{13}$$

Inserting of (10) into (12) with consider of (9) gives, in the first approximation, the expression:

$$x^{2/3}\frac{\partial}{\partial x}\left(x^{2/3}\frac{\partial\left(Ux^{-4/3} + 2nx^{2/3}\right)}{\partial x}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right) = 0. \tag{14}$$

Thus, equations (13) and (14) form the closed system, and it's solutions can be written as:

$$U(x,r) = x^{4/3} + \sum_{k=0}^{\infty} U_k(r)x^{5/3+2k}, \tag{15}$$

$$n(x,r) = x^{-2/3} + \sum_{k=0}^{\infty} n_k(r)x^{-1/3+2k}. \tag{16}$$

It is possible to show, inserting (16) into (13), that ignorance of variables n_k, beginning from $k = 1$ gives the relative error:

$$\delta U_1 \approx 0.026 \text{ with } \delta U_{k+1} << \delta U_k. \tag{17}$$

In other words, two dimension shape of n distribution influence on U distribution is much less then divers influence.

The neglecting of initial velocity dispersion gives the solution with strict border between ion beam area and vacuum. Considering (17) this border can be considered like cylindrical surface with the radius, which is equal to EE radius $\mathbf{R_0}$ (non-dimensional – $R_0 = \frac{\mathbf{R_0}}{\mathbf{L}}$), and Poison equation can be written as:

$$\Delta U(x,r)=\begin{cases}\frac{4}{9}\left[x^{-2/3}+n_0(r)x^{-1/3}\right], r\le R_0\\ 0, R_0<r\le R\end{cases}, \quad (18)$$

where $R=\frac{R_0}{\sqrt{\xi}}$ and ξ – transparncy of EE (relation of summary orifices square to entire EE surface square).

The solutions of equation (18) can be found like:

$$n_0(r)=\sum_{k=1}^{K}N_k\Phi_k(r), \quad (19)$$

$$U(x,r)=x^{4/3}+\sum_{k=1}^{K}u_k(x)\Phi_k(r),\ r\le R_0, \quad (20)$$

$$U(x,r)=\sum_{n=0}^{N}v_n(x)\Psi_n(r),\ r>R_0, \quad (21)$$

where:

$$\Phi_k(r)=\frac{I_0(\alpha_k r)}{I_0(\alpha_k R_0)} \text{ and } \Psi_n(r)=\frac{I_1(\beta_n R)K_0(\beta_n r)+K_1(\beta_n R)I_0(\beta_n r)}{I_1(\beta_n R)K_0(\beta_n R_0)+K_1(\beta_n R)I_0(\beta_n R_0)},$$

and I_0 и K_0 – modified Bessel functions of the first and second type.

Considering EE thickness like small one it is possible to write:

$$v_n(x)=\frac{V_n}{\pi n}\sin\beta_n x,\ V_n=Const,\ v_0=1, \quad (22)$$

$$u_k(x)=\frac{2}{5}N_k\frac{\Gamma(8/3)}{\alpha_k^{5/3}}F_{2/3}(\alpha_k x), \quad (23)$$

where Γ is gamma-function and aunctions $F_v(x)$ are determined like:

$$F_\nu(x)=\frac{1}{\Gamma(\nu)}\int_0^x z^{\nu-1}\sin(z-x)dx. \quad (24)$$

Coefficients α_k and β_n must be chosen for execution of boundary conditions (1) and (2):

$$\beta_n=\frac{L}{L_0}\pi n,\ F_{2/3}(\alpha_k)=0, \quad (25)$$

where L_0 – MEG length without electrodes thickness.

Coefficients V_n and N_k must be chosen for execution of boundary conditions (1) and (2) and the continuum of potential φ and electric field tension radial direction E_r take place on the border of ion beam. The distributions of φ and E_r are shown on the Fig. 1 and 2. Number 0 relates to vacuum area and number 1 – to ion beam.

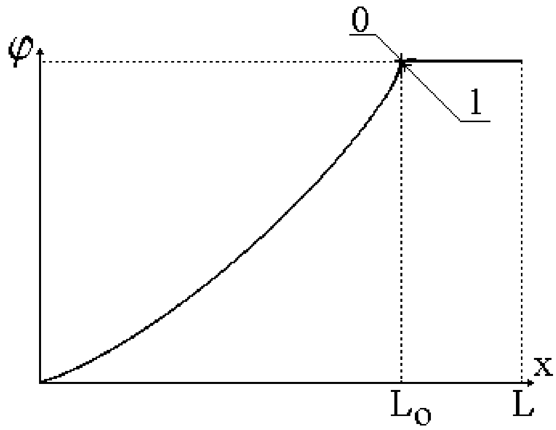

Fig. 1. Potential distribution via ion beam "border".

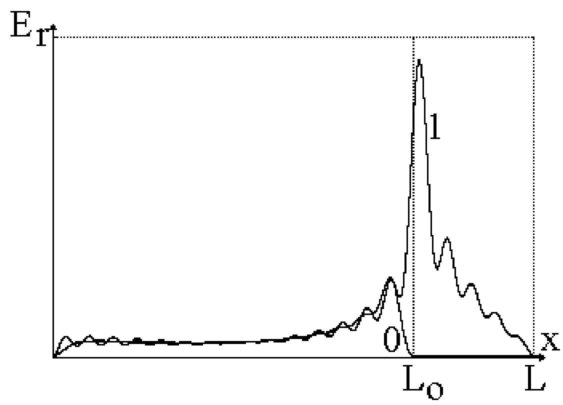

Fig. 2. Electric field tension distribution via ion beam "border".

The equal-potential lines are shown on Fig. 3.

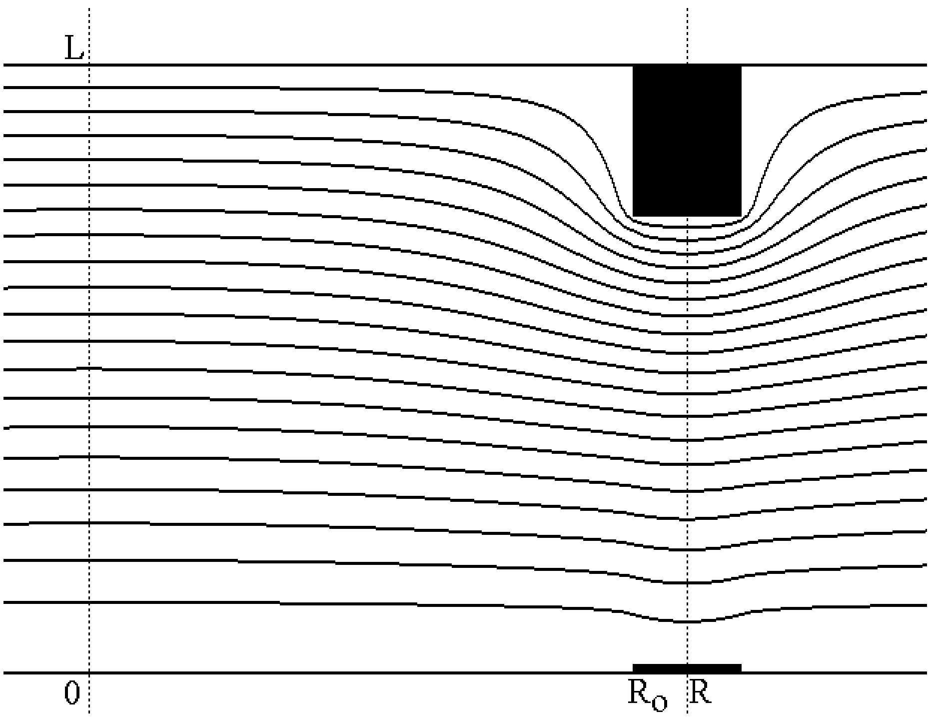

Fig. 3. Potential distribution inside inter-electrode gap.

The point $\mathbf{x} = \mathbf{L}_0$, $\mathbf{r} = \mathbf{R}_0$ – inner angle of AE is the special point for E_r – this variable value depends on the direction of coming to this point. This provides in numerical solution to recognized non-coincidence of curves 0 and 1 on Fig. 2 in this point area with size of order of $\mathbf{L}_0 / N$. The finite number of components in (20), (21) bring also the oscillations in obtained solutions (Fig.2) with the step of the same order.

The solutions, which are obtained, can be used while the calculation of distribution of particles