Hybrid particle-fluid model of a Hall thruster plume

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I. Introduction

The modelling of electric thruster plumes is a very important issue in view of the increasing importance of such propulsion in all space applications when specific impulse, and not just power, is important, i. e. for satellite guidance, orbit transfer and deep space exploration projects.

In particular the SPT-100[1] can be schematically described as an anode-cathode system, with a dielectric annular chamber where the propellant ionization and acceleration process occur. A magnetic circuit generates an axisymmetric and generally radial magnetic field between the inner and outer poles. This magnetic circuit employs one inner magnet winding and several outer magnet windings. In operation, an electrical discharge of 300 V is established between an anode, which is also a gas distributor, and an external cathode. In this configuration, cathode electrons are drawn to the positively charged anode, but the radial magnetic field creates a strong impedance, trapping the electrons in cyclotron motion which follows a closed drift path inside the annular chamber. The electrons collide with the dielectric walls of the acceleration region, creating lower energy secondary electrons which cannot follow the initial path but gradually diffuse to the anode, thus closing the electric circuit. When the working fluid gets in the annular channel through the anode, it undergoes an intense bombardment by the closed electron drift that ionizes the propellant. The electron-neutral atom collisions also create lower energy secondary electrons which gradually diffuse to the anode. The quasi-radial magnetic field is chosen so that the Larmor radius of the particles and the length of the channel L complies with the conditions:

$$\rho_e << L << \rho_i$$

where ρ_e is the electron Larmor radius and ρ_i is the ion Larmor radius. These conditions allow the electrons to drift along the channel's azimuthal direction and ions to be axially accelerated by the electric field creating the reactive thrust. Models of the inner flows of Hall thrusters have been developed using hybrid fluid-particle approaches[2-4] to aid in the optimization of the performance of the thruster.

Therefore a major concern in the use of these devices is the possible damage their plumes may cause to the host spacecraft and to communication interference of satellites. Indeed an electric thruster, such SPT-100, in operation produces, besides high energy ions (Xenon for his inert and low ionization potential property) responsible of the thrust and electrons emitted to neutralize the positive space charge, also neutral propellant atoms and low energy ions created by charge exchange (CEX) collisions between ions and un-ionized propellant (in which electrons are transferred). These CEX ions are strongly influenced by the self-consistent electric fields. These fields cause CEX ions to propagate radially and to flow upstream, and in doing so they also gain energy. CEX ions, therefore can be deflected back towards the spacecraft, possibly causing contamination and degradation by sputtering on components even when located beyond the line of sight of the exhaust beam.

Thus, the structure of the plasma plume exhaust from the thruster is of great interest and several models[5-9] have been developped simulating the plasmadynamic of the exhaust plume. In this work, a two-dimensional hybrid axisymmetric Particle-In-Cell (PIC) model using Test-Particle-Monte-Carlo (TPMC) collisions to predict the charge-exchange ion environment is presented.

II. Model

Ions and neutrals are treated as test particles and modeled directly, respectively by the PIC and Direct-Simulation-Monte-Carlo (DSMC) methods. Charged and neutrals particles are loaded into the simulation at each time step to simulate the exit flow. The ion and neutral distribution on the exit plane is deduced by linear fits of experimental data that give the magnitude and direction of the ion current and the neutral flux as a function of radial position r:

$$
\begin{aligned}
j_{fit}(r) = & -1.9787241\times10^{3} + 1.2974556\times10^{7}\cdot r^{2} - \\
& 6.3929319\times10^{8}\cdot r^{3} + 1.1640206\times10^{10}\cdot r^{4} - \\
& 7.4550791\times10^{10}\cdot r^{5} \\
\alpha_{fit}(r) = & -1.6672237\times10^{6}\cdot r^{2} + 1.3436936\times10^{8}\cdot r^{3} - \\
& -3.6299391\times10^{9} r^{4} + 3.2502724\times10^{10} r^{5} \\
n_n = & \, 8.0164474\times10^{20} + 5.100755\times10^{23}\cdot r - \\
& 4.8171475\times10^{25}\cdot r^{2} + 1.39654\times10^{27}\cdot r^{3} - \\
& 1.2727662\cdot r^{4}
\end{aligned}
$$

Within a cell, the charge is weighted to the four neighboring grid points and the volumetric charge density is computed by summing over all macroparticles that are in the cell itself. We have factorized the weighting function into two parts: the Ruyten charge density conserving shape factor[10] for the radial coordinate that represents a two-dimensional cylindrical system and the PIC shape factor for the axial coordinate that represents a one-dimensional cartesian system.

The time and space scale of electron dynamic is much smaller (490 times) than that of the ions. For this reason, we model the electrons using a fluid rather than a particle model. Electrons are assumed collisionless and unmagnetized, i.e. to follow a Boltzmann-like distribution modified in order to allow for the effect of non isothermal electron temperature:

$$
n_e(r,z) = n_{ref}\exp[e(\phi(r,z)-\phi_p)/k_B T_e]
$$

where the n_{ref} are calculated equating the electron density to ion density at thruster exit plane and ϕ_p is the electric potential at the thruster exit plane.

The electron temperature is calculated on the basis of the adiabatic approximation (the electrons are assumed to act as an expanding fluid at isentropic conditions):

$$T_e(j,k) = T_{ref}\left(\frac{n_e(j,k)}{n_{ref}}\right)^{(\gamma-1)}$$

where $\gamma = c_p/c_v$ is the ratio of specific heats (equal to 5/3 for monoatomic gases) and the constants T_{ref} and n_{ref} are chosen to closely match the electron measurements[11].

This model improves over the ones already in the literature (see Tab. I) because the electric potential distribution is obtained from the Poisson equation, without assuming the quasi-neutral plasma hypothesis:

$$\nabla^2\phi(r,z) + \frac{\rho_i(r,z)}{\varepsilon_0} - \frac{\rho_{ref}}{\varepsilon_0}\exp[e(\phi(r,z)-\phi_{tu})/k_B T_e(r,z)] = 0$$

Figure 1 shows the grid used for the simulations with the necessary boundary conditions. Along S_1 (the thruster exit plane, which also is the inflow boundary) the electric potential is given, equal to ϕ_p, the sum of the accelerator grid's potential and the contribution of the nearby ions (Dirichlet condition). The wall of the thruster (S_2) is assumed to be biased to the spacecraft potential, which is estimated to be $k_B T_{e\infty}/e$ according to electron and ion currents balance, where $T_{e\infty}$ is the electron background temperature. Along L_1 the symmetry boundary condition is given, i. e. the radial electric field is set to zero while axial variations in potential are permitted (Neumann condition). Along L_2, L_3 and L_4 the open boundary condition is assumed. This assumption is valid when the domain is sufficiently large.

$$\begin{cases} S_1 \rightarrow \phi = const. \\ S_2 \rightarrow \phi = const \\ L_1 \rightarrow \dfrac{\partial\phi}{\partial r} = 0 \\ L_2, L_4 \rightarrow \dfrac{\partial^2\phi}{\partial z^2} = 0 \\ L_3 \rightarrow \dfrac{\partial^2\phi}{\partial r^2} = 0 \end{cases}$$

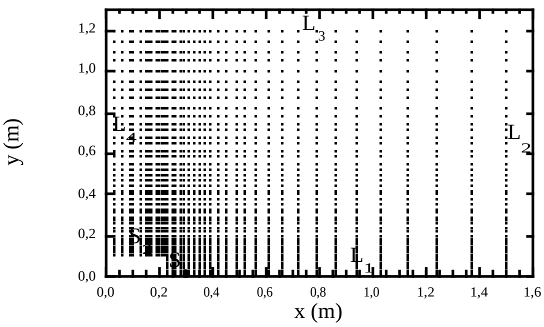

Fig. 1 - Scheme of the simulation domain.

With the Boltzmann distribution for the electron density, the Poisson equation becomes highly nonlinear. The solution is obtained by the iterative Newton-Raphson method, than is discretized allowing for the 5-points expression of the Laplacian in cylindrical coordinates. For large and non-uniform meshes, grid relaxation techniques are the methods of choice and in particular we have used a successive-over-relaxation (SOR) scheme with "chequerboard" ordering and Chebyschev acceleration. Given the potential, the electric field **E** is computed from **E**=-—$\nabla\phi$ that is discretized with the 6-points Boris formula for reducing numerical noise and anisotropy.

Models	Rhee-Lewis [5] 1995	Oh et al. [6] 1999	Boyd et al. [7] 1999	Boeuf et al. [8] 1999	Tajmar [9] 2000	Present model 2003
Domain dimension	2D 0.2x0.3 m	3D 1x1x2.6 m	2D 1x1.3 m	2D 0.5x1 m	3D 2x2x2 m	2D 1.2x1.5 m
Species simulated	Xe^{+}	Xe, Xe^{+}, e^{-}	Xe, Xe^{+}, Xe^{++}, e^{-}	Xe, Xe^{+}, Xe^{++}, e^{-}	Xe, Xe^{+}, Xe^{++}, e^{-}	Xe, Xe^{+}, Xe^{++}, e^{-}
Field solver	Poisson equation	quasi neutr. hypothesis	quasi neutr. hypothesis	Poisson equation	quasi neutr. hypothesis	Poisson equation
Electrons model	Isothermal, unmagnetized, collisionless	Isothermal, unmagnetized , collisionless	Not isoth., magnetized, collisionless	Isothermal, unmagnetized, collisionless	Isothermal, unmagnetized, collisionless	Not isoth., unmagnetized, collisionless
Ion-neutral collis. model	Point source approximat.	DSMC	DSMC	TPMC	DSMC	TPMC
Sputtering	No	Yes	No	No	No	No

Tab. I - SPT-100 plume models characteristics.

In order to move the ions for a time step we have used the leapfrog method. In order to minimize the effect of the energy-conservation violation, typical of a PIC-momentum conserving scheme, we have selected the timestep accordingly to Hockney's optimum-path condition

$$(\omega_{pi}\Delta t)_{opt} = \min\left[\frac{1}{2}\frac{\Delta x}{\lambda_D},1\right]$$

Therefore $\Delta t=1.6\text{x}10^{-7}$ s. and in such a way the Courant-Friedricks-Levy condition that requires particles to move a small distance relative to the width of local potential gradients in each time step and the stability leap-frog condition are satisfied.

Two kinds of ion-neutral scatterings are considered, the momentum transfer and the charge exchange one and similarly for double-ionized particles. Our collision technique is an implementation of the Test Particle Monte Carlo method of Nanbu and Kitatani[12] where the target distribution function is sampled for a virtual neutral particle and the collision probability is evaluated under the assumption of a polarization potential.

III. Results

Figure 2 illustrates the electric potential at the steady state. We can note a fall of 50 V between the exit plane and the stagnation region, where a potential well develops spontaneously. The lobe structures seen directly on the side of the thruster exit are produced by the charge exchange plasma.

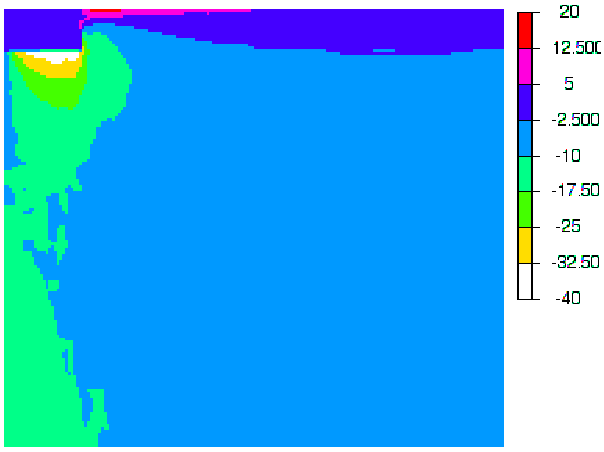

Fig. 2 - Equipotential lines at the steady state.

The code has been tested against several ground measurements showing good agreement. In Figs. 3 the common logarithm of ion charge density is reported. As can be seen, the ions propagate under the effect of self-consistent electric field and collisions with neutrals and ions until a stationary state is reached when the outflow balances the ion emission. Note that a significant component of backflow appears, as well as significant expansion in the radial direction.

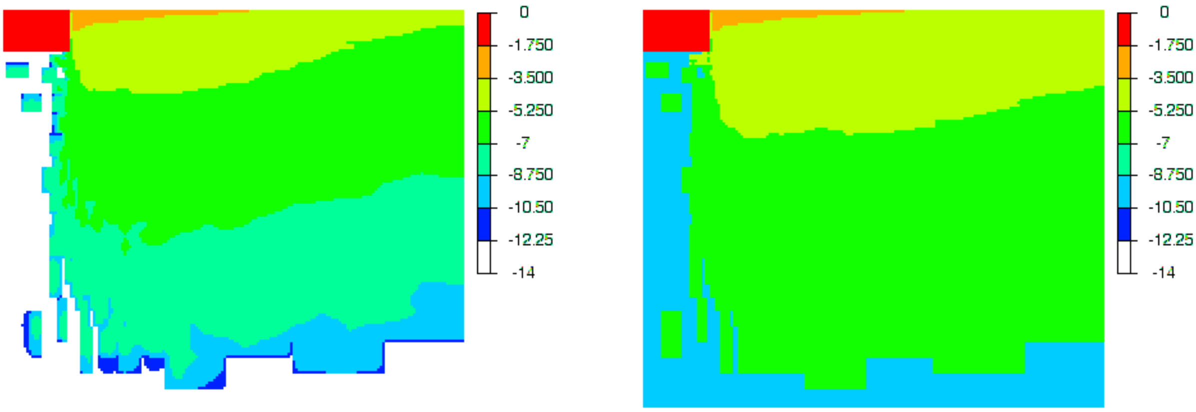

Fig. 3 - Common logarithm of ionic charge density at t=1.6x10^{-5} s. and t=1.6x10^{-4} s.

The electron temperature, reported in Fig. 4, follows, on the basis of the adiabatic equation, the electron density and is indeed strongly dependent on position. The usual assumption of a quasi uniform electron gas with temperature about 1-2 eV holds only quite far from the thruster.

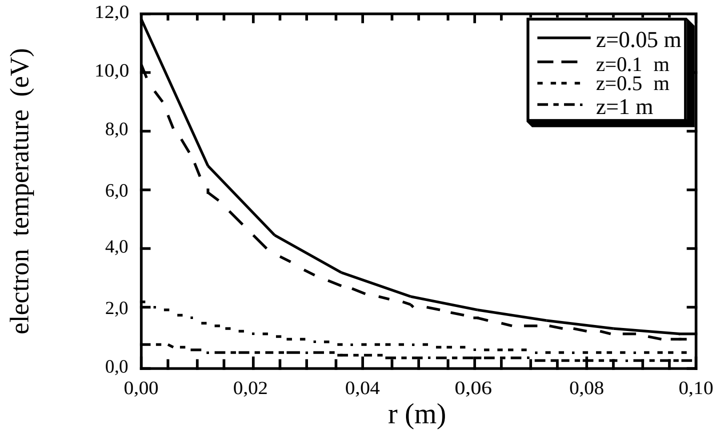

Fig. 4 - Radial profiles of electron temperature at different axial locations.

The present plasma model was used to calculated the energy distribution of stagnating CEX ions hitting the surface at z=0. Results are shown in Fig. 5. It is important to observe that CEX ions hitting the surface have an average kinetic energy about 20 eV, which is close to the sputtering threshold for most metals.

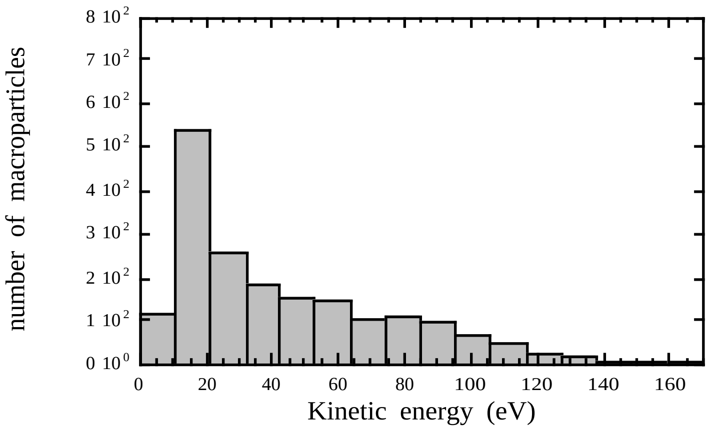

Fig. 5 - Energy distribution of secondary ions impacting the surface at z=0.

Conclusions

A hybrid 2D-3V axisymmetric PIC/TPMC numerical code has been developped for the simulation of plasma plume expansions. The model takes into account the effects of non isothermal electron temperature and of violations of the quasi-neutral plasma hypothesis. Collision processes are included by using the ion-neutral Test-Particle Monte Carlo collision model.

Acknowledgments

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