The Charged Particle Beam System of an Electrostatic Ion Thruster as an Antenna Beacon aboard a Satellite

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Abstract: The present work investigates the electron wave amplification in the combined electron /ion beam system of an electrostatic ion thruster. It also investigates the rf radiation into free space due to the propagation of an amplified wave along the length of the beam system. Theoretical investigations indicate that the modulating rf signals >600MHz are amplified within the combined electron/ion beam system of an ion thruster and the beam system also acts as rf antenna. This suggests a useful role for the thruster charged particle beams as an antenna beacon aboard a satellite. The power gain of the amplified signal, along the length of a UK 10 ion thruster beams, is estimated as 1.54dB per cm length at 5 GHz. The role of the thruster internal plasmas in the generation of rf noise aboard the satellite has also been investigated.

Nomenclature

A	= vector potential
$\bar{A}$	= atomic No
C	= capacitance
E	= electric field
e	= electronic charge
H	= magnetic field
J	= current density
j	= imaginary quantity
j_e	= electron current density
j_{eo}	= fluctuating electron current density component
j_i	= ion current density
j_{io}	= fluctuating ion current density component
k	= wave vector
L	= inductance
ℓ.	= dipole length
m	= electron mass
n_e	= electron number density
n_i	= ion number density

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R = (ω^2_{ip} / ω^2_{ep}),square of ratio of ion to electron plasma frequencies
T = temperature

T_e = electron temperature
T_{eV} = temperature in electron volts
T_i = ion temperature
t = time
u_1 = velocity of one stream particle beam
u_e = electron beam velocity
u_i = ion beam velocity
u_{iit} =velocity of ith particles
V = voltage

$\overline{V}$ = volume

$\bar{v}$ = DC velocity
$\tilde{v}$ = fluctuating velocity
$\tilde{v}_e$ = fluctuating electron velocity
$\tilde{v}_i$ = fluctuating ion velocity

W = (ω / ω_{ep}), ratio frequency to electron plasma frequency
x = distance
Y = surge admittance
Z =charge state
z = [k ÷(ω_{ep} / u_e)] , rationalised wave vector
ε_o = permitivity
ε_r = relative permitivity
ξ = collision frequency
λ = wavelength
μ = permeability
ρ = charge density
ρ_o = initial charge density
ρ_e = electron charge density
ρ_i = ion charge density
σ = conductivity
$\overline{\sigma}$ = complex conductivity
τ_{ei} = electron ion relaxation time
ω = frequency
ω_{ep} =electron plasma frequency
ω_{ip} = ion plasma frequency
ω_{1p} = plasma frequency of one stream particles
Γ = (=jk), propagation constant

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I. Introduction

Many Deep Space communication facilities, such as the ESA 35 m reflector ground station at New Norcia Australia, utilise reflectors configured to operate at S (2.2-2.3 GHz) and X (8.025- 8.4 GHz) bands. Here, the pointing is achieved on a programme track basis, where the antenna pointing is based on the calculations of the expected position of the spacecraft. Unlike the systems used to track lower earth orbit (LEO) spacecraft, there is no auto-track system, such as mono-pulse or electronic beam squint. In the present system, calibration is necessary to account for pointing error effects such as the thermal distortion and the mechanical pointing errors. For the New Norcia system, the pointing errors amount to about 15 milli-deg (mdeg), and this are within the antenna beamwidth at X-band, of approx. 70 mdeg.

Currently the New Norcia is being used as a ground reception for the Mars Express mission that utilises S and X-bands. The latter in particular, is being used for the long-haul return link to Earth. Venus Express is a future mission that is being designed to operate on the 'programme track basis' at S and X frequency bands.

The operation of the up and down communication links at even higher frequencies in the region of the Ka-band (>30 GHZ) is now being considered. Plans are afoot to implement these changes in the ESA owned facilities. The US Deep Space Network (DSN) is already supporting the proposed higher frequency band. However the use of Ka-band will place increasing demands on the pointing system, where the pointing-errors are expected to approach the beam-width of the antenna.

A low frequency beacon/antenna system installed on the spacecraft would dramatically aid the acquisition of the spacecraft, due to the larger beamwidths involved. Such a system will enable the 'health' of the spacecraft to be monitored continuously and spare the valuable ground resources, to support other multiple interplanetary spacecraft requirements.

This concept has been recently demonstrated on the NASA Deep Space 1 (DS1) mission where a beacon with a series of discrete frequencies or codes selected by on-board intelligence (rather than telemetring all data to ground) was used.

Further, when the spacecraft resources (ie mass, volume and power) are critical and limited, there is a distinct advantage in combining the science/ telemetry return link and the beacon system. The data rate is reduced and this avoids the need for a conventional high gain antenna (HGA) and the associated transmitter. A low gain antenna (LGA) with effective isotropic radiated power (EIRP) could be used for tele-command.

Another benefit of the beacon/antenna configuration becomes apparent when missions such as the Smallsat Intercept Mission Object Near Earth (SIMONE) are examined. This mission concept, developed by QinetiQ and partners as part of an ESA study, is based on 120 kg spacecraft equipped with 1 kW high SI gridded ion thrusters to achieve a rendezvous with a Near Earth Object (NEO), and here the conservation of spacecraft resources is paramount.

It is suggested in this paper that the requirements of a beacon/antenna aboard a satellite may be met utilising the inherent plasmas and the beam system of the electric propulsion (EP) system, thus saving on additional hardware. In principle, this type of beacon could also be modulated with low bit rate or even the science data itself.

All EP systems comprise an ionic (commonly xenon or argon ions) discharge beam and a neutralising electron beam derived from a hollow cathode also called a neutraliser hollow cathode. The hollow cathode in addition to supplying the e-beam also presents an internal plasma zone. The Kaufman type electrostatic ion thruster also incorporate a sizeable internal plasma zone fed by a DC power supply source invariably superimposed with inherent high frequency noise components.

It is shown in this paper that:

i) the conductivity of the internal plasma zones of both the hollow cathode and the electrostatic ion thruster approaches the DC metallic limit at frequencies < 10^7 Hz, and as such these plasmas may be used as conductive electrical elements. Further, at some specific frequencies the internal plasmas exhibit negative conductance. In the former mode, especially the internal plasma of an ion thruster in conjunction with a noisy power supply unit and unscreened interconnecting cables, acts as a magnetic dipole antenna.

ii) the combined two beam system of electron and ion beams may be modulated to achieve wave amplification with rf radiation into free space.

iii) Utilising the plasma reflection properties at specific frequencies, the two beam system may even be used as a regenerative system for the scientific data transmission.

II. Plasma as a Conductive Element within an Electrical Circuit

A. Plasma Conductivity

The plasma conductivity in a simplified form, taking into account the viscous damping (proportional to velocity) to represent the influence of the ions and the molecules, may be obtained from the equation of motion for the electrons

$$m\frac{d^2x}{dt^2} = -eEe^{j\omega t} - \xi m\frac{dx}{dt} \tag{1}$$

Here $Ee^{j\omega t}$ is the net oscillatory field imposed by sources outside the plasma and the internal field associated with electron space charge. ξ is the collision frequency and the second term on the RHS represents the rate of loss of momentum. Using the complex notation where $d/dt \rightarrow j\omega$, the displacement (x) and the velocity (dx/dt) ,

$$x = \frac{eEe^{j\omega t}}{m\omega(\omega - j\xi)} \tag{2}$$

$$\frac{dx}{dt} = -\frac{eEe^{j\omega t}}{m(\xi + j\omega)} \tag{3}$$

But the current density

$$J(=\bar{\sigma}Ee^{j\omega t}) = -en\frac{dx}{dt} \tag{4}$$

and the complex conductivity

$$\bar{\sigma} = \frac{ne^2}{m(\xi + j\omega)} \tag{5}$$

Thus for an applied frequency ω less than the collision frequency ξ ($\omega < \xi$), the conductivity is substantially real and metallic i.e.

$$\sigma = \frac{ne^2}{m\xi} \tag{6}$$

B. Radiation from a Plasma Discharge fed from a Noisy Power Supply Unit

At relatively low frequencies, less than the electron collision frequency ξ (= 1.3x 10^{10} Hz), the conductivity, as shown in Sect.A, of such a plasma is real and approaches the metallic limit. Thus a loop of unscreened conductors (Fig.1) containing a series connected plasma, and fed from a DC voltage source superimposed with an rf signal f_0, acts as a magnetic dipole antenna provided f_0 is less than the electron collision frequency.

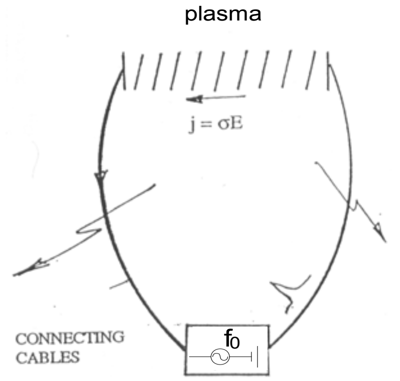


Figure 1. Plasma as a conductive electrical element

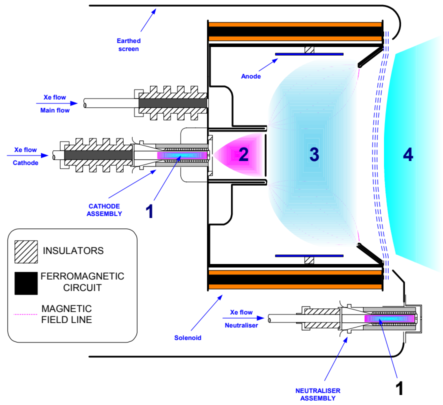


Figure 2. Various plasmas in UK10 Ion thruster

C. Conductive Plasmas within an Electrostatic Ion Thruster

A number of plasmas (designated as 1, 2, 3 & 4 in Fig.2) exist within an electrostatic ion thruster. These give rise to various closed loop circuits as is obvious from the circuit diagram in Fig.3.

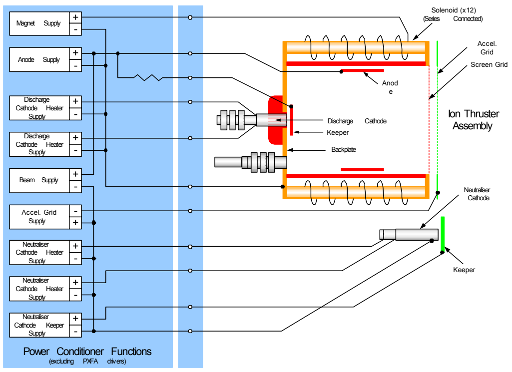


Figure 3. Various closed electrical-loop circuits in an electrostatic ion thruster

III. Plasma as a Two-Terminal Oscillator

A plasma within a pair of negative and positive electrodes, for the purpose of rf performance, may be represented as a series combination of an emission limited and a space charge limited diodes. The former simulates the conditions near the cathode electrode and the latter near the anode electrode. Small signal analysis show that both emission limited and space charge limited diodes exihibit[1] a negative conductance at specific fundamental frequency and at its harmonics. For UK10 ion thruster, Fig.4 illustrates the superimposed conductance curves for both emission limited and space charge limited modes. The negative conductance occurs at 60MHz and at its 2nd harmonic 120MHz for both modes. Thus the plasma as a whole is also expected to

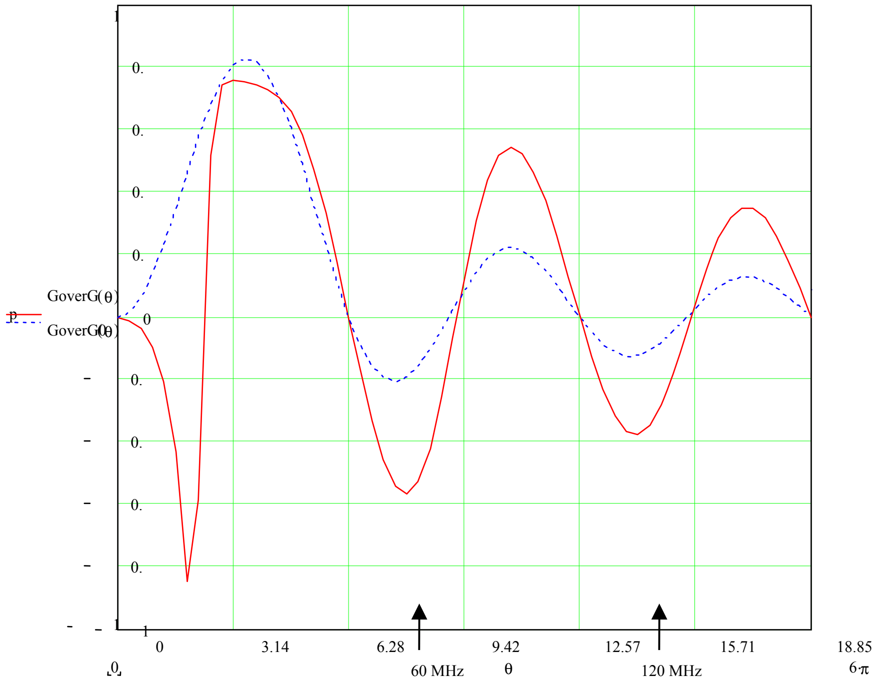


Figure 4. Plasma conductance in a diode mode
Vertical axis: Conductance (G). Red (Gp): conductance due to emission limited diode.
Blue (Go): conductance due to space charge limited diode.
Horizontal axis: Radians ($\theta = \omega T_o$).
Electron transit time $T_o = d\,(2m/eV)^{1/2}$; frequency(ω)

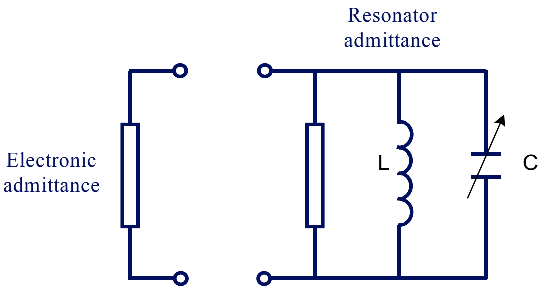
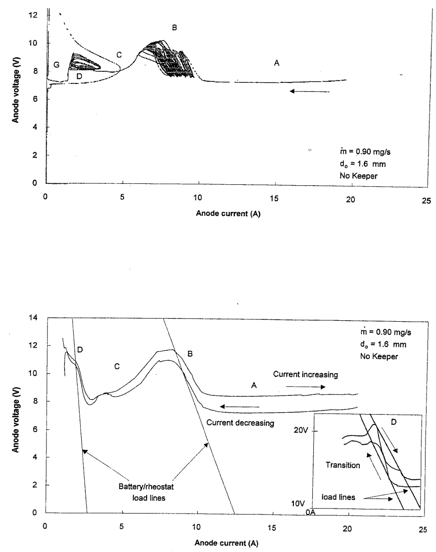


Figure 5. Plasma as a 'two-terminal' oscillator
A negative conductance element connected across an oscillatory circuit, promotes oscillations



negative conductance of a hollow cathode plasma.
Upper trace: due to a noisy DC power supply source. Lower trace: due to a DC batter ysource.

exhibit similar characteristics, and as such it is useful in sustaining oscillations in an oscillatory LC circuit (or a tuned crystal) , as a two- terminal oscillator[2]. This is illustrated in Fig.5.

A hollow cathode is known to exhibit negative volt-amp characteristics at some definite applied DC excitation conditions (Fig.6) and oscillations corresponding to stray LC parameters are generated.

IV. Wave Amplification within the combined Electron/Ion Beam System of an Electrostatic Ion Thruster

A disturbed or modulated[3] single charge particle beam gives rise to two space charge waves, one travelling slightly faster and the other slightly slower than the beam DC velocity. The amplitude of these propagating waves does not change with distance. However, when a combined system of two-beams, electron/electron or electron/ions, are modulated, wave amplification takes place as a result of interaction between the two streams of charged particles.

In general, when a stream of charged particles of charge density ρ_1 and velocity u_1 is injected into the space occupied by another stream of density ρ_2 and velocity u_2, then in addition to energy exchange between particles in the two streams caused by scattering, there also occurs a partial conversion of kinetic energy of the particles into the energy of electromagnetic fields associated with space charge fluctuations.

Most of the fundamental work related to the wave amplification due to interaction with charged particle beams, is attributed to Pierce[4,5] and Haef[6]. Haeff's work relates to wave amplification using two electron/electron beams of differing velocities. Pierce's work relates to wave amplification in both electron/electron and ion/electron beams.

However, for electron/ion beam system, Pierce considered two cases: i) stationary ions, and moving electrons and ii) the creation of ions due to impact ionisation of electrons with neutrals, and the resulting slow movement of the ions in the presence of fast moving electrons.

The present work is largely based on Pierce formulations[1,2]. However, in the present problem unlike Pierce's problems, both the ion and the electron beams move initially at appreciable and independent velocities. Further, Pierce and Haef have considered cases where the profile and the velocities of the beams do not change with distance. In the present case both beams expand with distance and especially the electron beam velocity decreases with distance. All these features have been taken into account in the present analysis.

It is shown in this work that at any given signal frequency ω, the wave amplification is mainly determined by the ratio of the velocities of the two beams, and the square of the ratio of the plasma frequencies of the two beams. The amplification however, decreases with an increase in the ion to electron beam velocity ratio. At any given frequency, the peak amplification occurs at a definite velocity ratio of the two beams, and for a system where the ratio of the electron beam to the ion beam velocities increases with distance, the wave amplification decreases.

A. Interaction of Wave/Particle Beams

1. Basic Interactions

The modulation[7] of a charged particle beam of charge density ρ_o and initial velocity u_o gives rise to superimposed space charge waves of small amplitude, where;

$$\rho = \rho_o + \rho_1 e^{j(\omega t - kx)} \tag{7}$$

and

$$\upsilon = u_o + \upsilon_1 e^{j(\omega t - kx)} \tag{8}$$

Here ρ and υ represent the charge density and velocity respectively. The subscripts 0 and 1 represent the amplitude of the DC and the fluctuating components respectively. The exponential terms $e^{j\omega t}$ and e^{-jkx} represent the sinusoidal time and the spatial variations of the waves respectively where k is the wave vector.

The current density J from Eqs (7) and (8) assuming a linear case (neglecting the product $\rho_1\upsilon_1$ which is small) is

$$J = \rho \upsilon = \rho_o u_o + (\rho_o \upsilon_1 + \rho_1 u_o) e^{j(\omega t - kx)} \tag{9}$$

Here $\rho_o u_o$ is the DC component and the terms within the bracket for the coefficient of the exponential term represents the amplitude of the fluctuating component of the current density. It is also obvious that the fluctuating charge gives rise to fluctuating electric field E or voltage V, and may be represented as a propagating wave:

$$V = V_1 e^{j(\omega t - kx)} = V_1 e^{j\omega t - \Gamma x} \tag{10}$$

Here the wave propagation constant Γ (=jk), in terms of the velocity u_{ith} and the plasma frequency ω_{ithp} of the ith component of the beam particles, (normally derived using Poisson's equation, conservation of charge, and the momentum equation as shown in Appendix.1) may be expressed as

$$\sum_1^i \frac{\omega_{itp}^2}{(\omega + j\Gamma u_{it})^2} = 1 \tag{11}$$

2. One-beam System

For a one-beam system (designated by the subscript 1), the propagation constant Γ from Eq. (11) is

$$\frac{\omega_{1p}^2}{(\omega + j\Gamma u_1)^2} = 1 \tag{12}$$

Or

$$\Gamma = j\left(\frac{\omega \pm \omega_{1p}}{u_1}\right) \tag{13}$$

For an imaginary propagation constant, the spatial amplitude of the wave, determined by $e^{-\Gamma x}$, is unaffected. Computing the wave velocity ($v = \omega/k$) using Eq. (13) yields two waves of velocities

$$\upsilon\ (= \omega/k) = \left(\frac{\omega}{\omega \pm \omega_{1p}}\right) u_1 \tag{14}$$

One wave travels faster and the other slower than the particle beam velocity u_1.

3. Two-beam System (electron /ion beams)

The Poisson's equation for a 2-beam electron/ion system is

$$\nabla^2 V = -\frac{1}{\varepsilon_o}(\rho_i - \rho_e) \tag{15}$$

But from Eq.10

$$\nabla^2 V = \Gamma^2 V \tag{16}$$

And Eq.(15) reduces to

$$\Gamma^2 V = -\frac{1}{\varepsilon_o}(\rho_i - \rho_e) \tag{17}$$

Here the wave propagation constant Γ is determined for the known values of the fluctuating densities of the ions ρ_i and the electrons ρ_e, and these are in turn evaluated using the current density, continuity and the force equations for the particles as shown in Appendix. A.

From Eq.(A18) in appendix .A, the fluctuating density component for the electron species in terms of the wave vector k is

$$\rho_e = +\frac{\Gamma^2 V \omega_{ep}^2 \varepsilon_o}{(\omega - k u_e)^2} \tag{18}$$

where, V and ω are the applied RF voltage and frequency respectively

ω_{ep} is the electron beam plasma frequency,

u_e is the initial electron beam velocity

Similarly, the ion density from Eq.(A33) Appendix A, in terms of the wave vector and other ion beam parameters is:

$$\rho_i = -\frac{\Gamma^2 V \omega_{ip}^2 \varepsilon_o}{(\omega - k u_i)^2} \tag{19}$$

Here u_i and ω_{ip} are the ion beam initial velocity and the ion plasma frequency respectively.

Substituting equations (18) and (19) into Poisson's equation (15a) gives:

$$\Gamma^2 V = \left[\frac{\omega_{ip}^2}{(\omega - k u_i)^2} + \frac{\omega_{ep}^2}{(\omega - k u_e)^2} \right] \Gamma^2 V \tag{20}$$

This equation is satisfied when

i) $\Gamma = 0$

ii) V=0 , or

iii) $$\left[\frac{\omega_{ip}^2}{(\omega - k u_i)^2} + \frac{\omega_{ep}^2}{(\omega - k u_e)^2} \right] = 1 \tag{21}$$

The solution for the wave vector k satisfying the condition (iii) in Eq. 21 is of interest in the present analysis. It determines the amplitude of the wave. Eq.(21) is quartic in k , and both numerical[7] and algebraic[10] solutions are possible.

Taking the ratio of the ion to electron plasma frequencies as

$$\frac{\omega_{ip}^2}{\omega_{ep}^2} = R \tag{22}$$

and the ratio of the signal frequency to electron plasma frequency as

$$\frac{\omega}{\omega_{ep}} = W \tag{23}$$

and normalising the ratio of the wave vector k as

$$\frac{k}{\omega_{ep}/u_e} = z \qquad (24)$$

Eq. (20) reduces to:

$$1 = \frac{R}{\left[W - \left(\frac{u_i}{u_e}\right)z\right]^2} + \frac{1}{(W - z)^2} \qquad (25)$$

This equation, for a two-beam system comprising a fast electron and a fast ion beam, is similar to the expression obtained by Pierce[5] for a two-beam system comprising a fast electron beam and a slow ion beam.

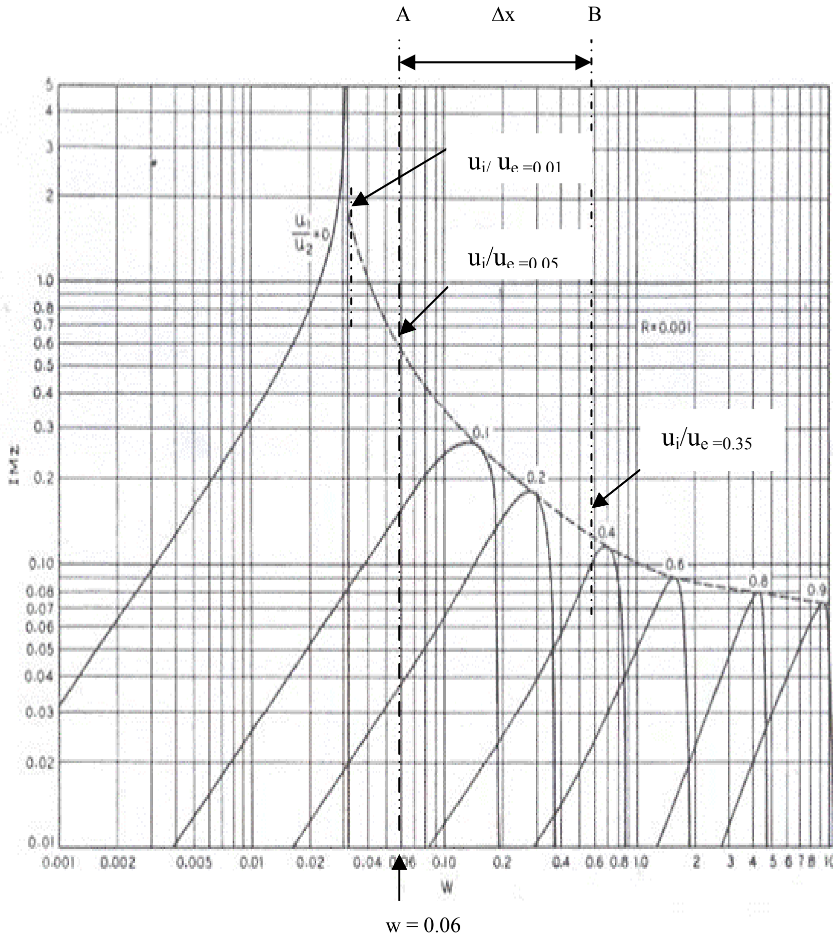


Figure 7. Wave gain parameter Imz against W the ratio of signal frequency to electron plasma frequency.

R(=0.003 for UK 10 ion thruster): square of the ratio of the ion plasma frequency to the electron plasma frequency.

u_1/u_2 (= u_i/u_e): ratio ion beam to electron beam velocities. Δx: range for wave amplification.

W: ratio of applied rf frequency to electron plasma frequency

Equation (25) is quartic in z and may be solved numerically, where the imaginary part of z relates to the wave amplification. Fig.7 illustrates a typical numerical plot[5] of imaginary z (Img z) against the ratio W, with R and the beam velocity ratio u_1/u_2 (=u_i/u_e) as parameters. An algebraic solution[10] for k in the quartic equation (21) is also available.

4. *Wave Amplification within UK 10 Ion Thruster Beams*

The system parameters for the UK10 ion thrusters are as follows:

Electron beam from neutraliser hollow cathode

No. density $n_e=10^{12}cm^{-3}$

Electron plasma frequency f_{ep} (=$10^4 n_e^{1/2}$) = 10GHz

or, ω_{ep}= 60x 10^9 rads/s

Keeper voltage = 20V.

Electron beam velocity u_{oe} (=$10^6 V^{1/2}$) = 4.4x10^6 m/s.

Xenon ion beam from ion thruster

No. density $n_i = 10^{12}cm^{-3}$ at 1mN.

or 10^{15} cm^{-3} at 20mN.

Ion beam velocity u_{oi} = 50km/s.

Table.1 lists the derived parameters for the evaluation of wave amplification in UK 10 ion thruster.

The plot of wave amplification factor Imz against W in Fig.7 and based on Eq.(25) may be approximated for UK10 ion thruster system using Fig.7. The results of Fig.7 are based on R=0.001 whereas that for UK10 thruster R= 0.003 (Table.1). For modulating frequency ω=600MHz, W=0.06, and the optimum amplification is expected to occur at a velocity ratio u_i /u_e= 0.05. However the initial velocity ratio u_i/u_e for the two beams is 0.01, and the corresponding characteristic velocity ratio curve does not intercept with the vertical at W=0.06. Consequently there is no beam amplification. But as the velocity ratio increases (due to a reduction in the electron beam velocity), the wave amplification starts at point A , corresponding to W=0.06 and velocity ratio 0.05. Progressively, decreasing wave amplification then persists over the region marked AB. Point B corresponds to the velocity ratio 0.35, beyond this the characteristic velocity ratio curves do not intercept with the vertical at W=0.06 and there is no wave amplification.

The estimation of the distance Δx over which the wave amplification (for the range of velocity ratios 0.05-0.3) persists, is not straightforward. However, it may be approximated invoking the relaxation process. An assumption is made that the electron beam, slows down with distance, and finally attains the velocity equal to that of the ions. This relaxation process may be expressed as

$$\frac{dT_e}{dt} = \frac{T_i - T_e}{\tau_{ei}} \tag{26}$$

where, T_i is the ion temperature in equilibrium, and time τ_{ei} characterises the relaxation rate of approach to equilibrium of the electron beam.

For a plasma the relaxation time

$$\tau_{ei} = \frac{3.5\times10^8 \bar{A} T_{eV}^{3/2}}{n_i Z^2 \ln\wedge} \sec \tag{27}$$

Here,

$\bar{A}$ is atomic no

n_i the number density of ions, and

Z is the charge state

For the present xenon system:

$\overline{A}$ =131
T_e=1.1eV
$n_i=10^{12}-10^{15}cm^{-3}$
Z=1
ln =10

and from Eq.27, the relaxation time constant $\tau_{ei} = 10^{-6}$s.

The distance Δx for the electron beam velocity to decrease from $4.4x10^6$ m/s to $5 x10^4$m/s is strictly the integral $\tau_{ei} \int u_e$ du or approximately, it is the product of τ_{ei} and the average velocity change yielding approximately Δx = 2.2m.

From Fig..7, the Imz varies from a peak value 0.6 at the velocity ratio u_i/u_e = 0.05 to negligible value at a velocity ratio 0.4. Taking an average value for Imz = 0.3 over this range, and using x (= Δx) = 2.2m, the wave amplification factor $e^{-jkx} = e^{154}$. This suggests an enormous amplification, but in practice this is never achieved due to the nonlinearity and the system saturation effects.

Table 1. Parameters appropriate for wave amplification (UK 10 Ion Thruster)

Rf signal (f) Hz	**Rf signal (ω) Rad/s**	**ω_{ep} Rad/s**	**ω_{ip} Rad/s**	**$W=\omega/\omega_{ep}$**	**$R=(\omega_{ip}/\omega_{ep})^2$**	**u_e m/s**	**u_i m/s**	**u_i/u_e**
600MHz	$36x10^8$			**0.06**				
2GHz	$12x10^9$		**$3.6x10^9$**	**0.2**				
		$60x10^9$	**($N_i=10^{15}cm^{-3}$)**		**0.003**	**$4.4x10^6$**	**$5x10^4$**	**0.01**
4GHz	$24x10^9$			**0.5**				
10GHz	$60x10^9$			**1.0**				

However, the AC current in the beam cannot exceed the DC loading (≈0.5 Amps). The wave voltage and current flow are related through the surge impedance $(\mu_o \mu_r /\varepsilon_o \varepsilon_r)^{1/2}$. Taking μ_r =1 for non-magnetic materials and dielectric constant for plasma ε_r = 0.8, gives the expected voltage maximum approximately of the order of 210Volts.

Nergaard[10] using an algebraic approach to the solution of Eq.(21), has shown that the values of the wave vector depends on parameters involving the ratio of the DC velocities of the two beams and the ratios of the propagation constants in the two beams. The power gain of the wave is given as

$$G = 8.686\sqrt{\frac{\omega_{ep}\omega_{ip}}{u_e u_i}}X \quad \text{decibels per cm} \qquad (28)$$

and the value of the gain factor X is determined from the computed results in Fig.8.

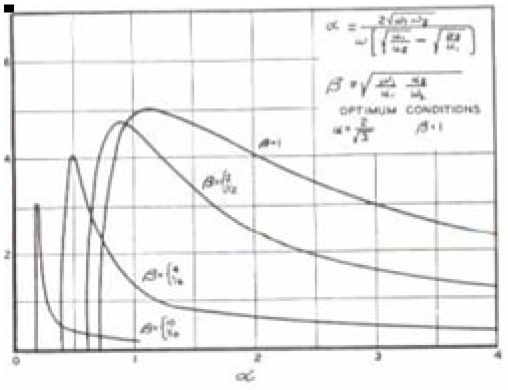


Figure 8. Plot of wave Gain factor X against α

where
$$\alpha = \frac{2(\sqrt{\omega_{ep}\omega_{ip}})}{\omega\left[\sqrt{u_e/u_i} - \sqrt{u_i/u_e}\right]} \tag{29}$$

$$\beta = \sqrt{(\omega_{ep}/u_e)(u_i/\omega_{ip})} \tag{30}$$

Nergaard[10] however did not taken into account the beam space charge effects. The neglect of these effects lead to anomalously high gain values. It is suggested that the space charge effects may be included, evaluating the respective plasma collision frequency parameters i.e. ω_{ep} and ω_{ip} in Eq.(28) in terms of the space charge limited current flows j_{en} , j_{in} within the beams, and the beam accelerating grid voltages V_{en} and V_{in} as:

$$\omega_{ep} = 1.85\times10^{10}\sqrt{j_{en}/V_{en}^{1/2}} \tag{31}$$

and for xenon ions

$$\omega_{ip} = 1.85\times10^{7}\sqrt{j_{in}/V_{in}^{1/2}} \tag{32}$$

For the electron beam, the space charge limited current flow is

$$j_{en} = 2\times10^{-5}\,V_{en}^{3/2}/d_e^2 \tag{33}$$

where d_e is the keeper to hollow cathode inter-space and V_{en} is the applied keeper voltage

For the xenon ion beam, the space charge limited current flow is

$$j_{in} = 5\times10^{-9}\,V_{in}^{3/2}/d_i^2 \tag{34}$$

where d_i is the accelerating grid electrode inter-space, and V_{in} is the accelerating voltage.

For UK10 electrostatic ion thruster, according to the above formulation the Eq.(28) for the power gain reduces to: G= 3.2 X. Taking appropriate value for X= 0.48 from Fig .8, gives approximately the gain G= 1.54 dB / cm length of the beam at 5GHz.

V. RF Radiation from Electron/Ion Beam System

The spatial development of the charged particle beams is shown in Fig..9. 'AB' is the zone of 'bunching' where both local density and velocity gradients exist and interaction of wave-particles takes place, leading to amplification. 'BC' is the intervening plasma zone. Here a decreasing spatial electron density gradient may exist. The length of distances AB and BC is difficult to evaluate. CD is the distance over which the neutral gas particles degenerate into vacuum. Fig.10 illustrates diagrammatically the amplification of a modulating signal ω, leading to the generation of RF radiation into free space.

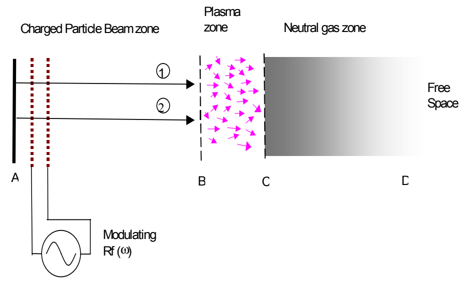


Figure 9. Various two beam charged particle system zones

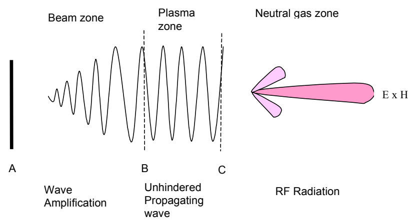


Figure 10. Spatial progression of modulated signal

With reference to Fig.10, the injected rf wave is first amplified over a finite part of the distance AB only. No amplification takes place over the plasma zone BC and the neutral gas zone CD. Provided the frequency ω of the propagating wave into the plasma zone BC is greater than the electron plasma frequency of this zone, the zone BC behaves as a perfect dielectric and the wave propagates unimpeded.

With increasing distance x, the beam continually expands and likewise the number of electrons per unit cross-sectional area of the beam continually decreases. In plasma the refractive index decreases with an increase in the number density of the electrons. Thus an incident wave continually propagates and the amplification ceases when the ion-electron beams are moving at the same velocity i.e. the system is neutralised. But as discussed in Sect.4, amplification does not take place if the velocity ratio of the two beams is considerably shifted away from the optimum gain velocity ratio at a particular given value of W, the ratio of the frequency to the electron plasma frequency (see Fig. 7).

A travelling wave of voltage V along a conductor is normally accompanied by a proportional wave of current i = V (Y) , where Y is the surge admittance equal to $(\varepsilon_r \varepsilon_o / \mu_r \mu_o)^{1/2}$. The direction of current is the same as the direction of travel if the voltage is positive. However the surge susceptance is different for the two zones 'AB' and 'BC' in Fig.9. In both cases it is dictated essentially by the relative permitivity or the dielectric constant

$$\varepsilon_r = 1 - j\frac{\sigma}{\varepsilon_o \omega} \tag{35}$$

Where σ is the conductivity of the medium

A propagating current wave gives rise to radiation and the Ponyting vector (ExH) may be evaluated using the vector potential A alone, which in turn is determined by the current wave.

The Ponyting vector i.e. the energy flux is

$$ExH = \varepsilon_o c E^2 \tag{36}$$

where the velocity

$$c = \frac{1}{\sqrt{\mu_o \mu_r \varepsilon_o \varepsilon_r}} \tag{37}$$

Again here, the relative permittivity is involved. The electric field intensity

$$E = -\nabla V - \frac{\partial A}{\partial t} \tag{38}$$

Or in terms of the vector potential A, the field

$$E = -\frac{j\omega}{k^2}\nabla(\nabla \bullet A) - j\omega A \tag{39}$$

Where for a travelling current wave e^{-jkx} the vector potential

$$A = -\mu \int \frac{i e^{-jkx}}{\pi x} d\overline{V} \tag{40}$$

Ramo[8] and Stratton[9] give a solution for the radiated power for a progressive current wave along a straight wire, and it is shown that for the velocity of the propagating wave approaching the velocity of light, the main radiation lobe is in the direction of the wave propagation with subsidiary lobes at 90 degrees. In the present case the phase velocity in the beam and plasma zones (Eq.37) differ from the velocity of light and thus a different pattern of lobes is expected. The lobe pattern however depends on the relative permittivity that in turn depends on the plasma conductivity (Eq.35), controllable to a point by the beam parameters.

Taking the portion of the beam system with a finite conductivity supporting a current flow i.e. the length of the ion/electron beam zone and the plasma zone (AB+BC) in Figs. 9 and 10, as an equivalent length ℓ of a dipole antenna, the resulting radiated power is proportional to $(\tilde{\ }\ell/\lambda)^2$. Here ℓ due to the two beam system is large approaching some 20m. This is a distinct advantage of using the discharge beams as an antenna, as the conventional antenna of similar dimensions would be cumbersome to install aboard a satellite.

VI. Conclusion

1. Both, the internal main discharge plasma of an electrostatic ion thruster and the internal plasma of the neutraliser hollow cathode, exhibit negative conductance properties in a diode mode. For UK10 ion thruster, this occurs at 60MHz and its second harmonic 120MHz .

This feature of the plasmas may be used as a two terminal oscillator concept to sustain RF oscillations in an oscillatory LC or a tuned crystal circuit.

2. The combined two beam system comprising an ion beam from the electrostatic ion thruster and the electron beam from the neutraliser hollow cathode, may be used to amplify a modulating RF signal. The theoretical wave amplification in such a system is high; 1.54dB/cm length of the beam at 5GHz. It is expected that in practice the upper limit is set by the beam nonlinearity and the current saturation effects and cannot exceed the DC loading of the beams.

3. RF radiation into free space arises due to the propagation of an amplified wave along the length of the charged particle beams and the adjoining plasma zone. The composite length of such a linear conductor relevant for radiation into free space is exceedingly long (few tens of metres) for all frequencies exceeding the electron

plasma frequency of the plasma zone (10 GHz). Thus the electron/ion beam system of an electrostatic ion thruster provides a convenient long wire-like radiation antenna aboard a satellite.

4. Utilising the plasma reflection properties at specific frequencies, the two beam system may even be used as a regenerative system for the scientific data transmission.

5. Future work is needed to develop a suitable beam modulation technology, and also to ascertain the pattern of the expected radiation lobes.

Appendix A

Theoretical Derivations for Wave Propagation

Using the sign convention for the various parameters as shown in Fig.A1:

<u>For electrons:</u>

The mean charge density (–en) is -ve

Current density due to pressure gradient (-env) is -ve

Current density in the direction of the applied E-field is -ve

<u>For ions:</u>

The mean charge density (+en) is +ve

Current density due to pressure gradient (env) is +ve

Current density in the direction of the applied E-field is +ve

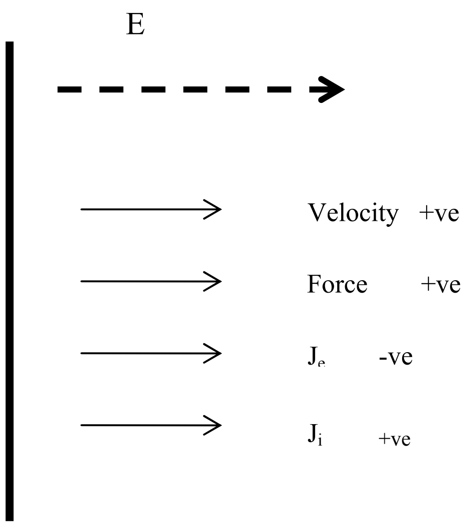


Figure. A1

The charge density ρ and the velocity of a beam with an average charge density ρ_o moving with velocity u_o, and with a superimposed space charge wave may be described as

$$\rho = \rho_o + \tilde{\rho} e^{jwt-\Gamma x} \tag{A1}$$

and

$$\upsilon = u_o + \tilde{\upsilon}\, e^{j\omega t-\Gamma x} \tag{A2}$$

Here the subscript 0 and the superscript ˜ indicate the DC and the fluctuating components.

The current density (J= ρυ), neglecting the product $\widetilde{\rho}\widetilde{v}$ (i.e. retaining the linear 1st order equations only) is

$$j = u_o \rho_o + (\rho_o \widetilde{\upsilon} + u_o \widetilde{\rho})e^{j\omega t - \Gamma x} \tag{A3}$$

The fluctuating particle density components are relevant for the estimation of the wave propagation constant Γ, determined essentially by substituting particle density components into the Poisson's equation (15). The fluctuating density components in turn are evaluated using basically the current density, continuity and the force equations for the particles involved as shown in the following sections:

For electrons:

The spatial variation of the electron current density from Eq.(9) is

$$j_e = -(u_e \widetilde{\rho}_e - \widetilde{v}_e \rho_e)e^{-\Gamma x} \tag{A4}$$

Where the propagation constant Γ= kx, and k is the wave vector. The terms within the bracket represent the fluctuating current density component and the exponential its spatial variation.

Continuity equation

$$\frac{d\rho_e}{dt} = -\nabla . j_e \tag{A5}$$

Using $j_e = -j_{eo} e^{-\Gamma x}$ in conjunction with Eq.(A5) and taking jω = d/dt

$$j\omega\rho_e = -\Gamma j_e \tag{A6}$$

Substituting Eq.(A4) into Eq.(A6) (the exponential term understood) gives

$$j\omega\rho_e = (u_e \widetilde{\rho}_e + \widetilde{v}_e \rho_e)\Gamma \tag{A7}$$

Or

$$\widetilde{v}_e = -\frac{\widetilde{\rho}_e}{\rho_e \Gamma}(j\omega - \Gamma u_e) \tag{A8}$$

Force Equation

$$m_e \frac{d\upsilon_e}{dt} = -eE \tag{A9}$$

taking

$$\frac{d\upsilon_e}{dt} = \frac{\partial \upsilon_e}{\partial t} + \frac{\partial \upsilon_e}{\partial x}\frac{\partial x}{\partial t} \tag{A10}$$

$$v_e = u_e e^{-\Gamma x} \tag{A11}$$

$$V = V_o e^{-\Gamma x} \tag{A12}$$

and $$j\omega = \frac{d}{dt} \tag{A13}$$

Eq.(A6) reduces to:

$$(j\omega - \Gamma u_e)\tilde{v}_e = -\frac{e}{m}\Gamma V \tag{A14}$$

Substituting for $\tilde{v}_e$ from Eq. (A8) into Eq.(A14) gives

$$(j\omega - \Gamma u_e)^2 \frac{\tilde{\rho}_e}{\Gamma \rho_e} = -\frac{e}{m}\Gamma V \tag{A15}$$

Multiplying by $1/j^2$ throughout Eq.(A15) may be expressed as:

$$\tilde{\rho}_e = \frac{\Gamma^2 V \omega_{ep}^2 \varepsilon_o}{(\omega + j\Gamma u_e)^2} \tag{A16}$$

where the electron plasma frequency

$$\omega_{ep} = \left(\frac{e^2 n}{\varepsilon_o m_e}\right)^{1/2} = \left(\frac{e\rho_e}{\varepsilon_o m_e}\right)^{1/2}$$

(A17)

Taking $\Gamma = jk$, Eq.(A16) may also be expressed as

$$\tilde{\rho}_e = \frac{\Gamma^2 V \omega_{ep}^2 \varepsilon_o}{(\omega - ku_e)^2}$$

(A18)

For ions

The spatial variation of the ion current density from Eq.(9) is

$$j_i = (u_i \tilde{\rho}_i + \tilde{v}_i \rho_i) e^{-\Gamma x}$$

(A19)

The terms within the bracket represent the fluctuating component of the current density and the exponential its spatial variation.

Continuity equation

$$\frac{d\rho_i}{dt} = -\nabla \bullet j_i$$

(A20)

Using $j_i = +j_{io}e^{-\Gamma x}$ in conjunction with Eq.(A20) and taking jω = d/dt

$$j\omega\rho_i = +\Gamma j_i \tag{A21}$$

Substituting Eq.(A19) into Eq.(A21) (the exponential term understood) gives

$$j\omega\rho_i = (u_e\tilde{\rho}_i - \tilde{v}_i\rho_i)\Gamma \tag{A22}$$

Or

$$\tilde{v}_i = \frac{\tilde{\rho}_i}{\rho_i\Gamma}(j\omega - \Gamma u_i) \tag{A23}$$

Force Equation

$$m_i\frac{dv_i}{dt} = +eE \tag{A24}$$

taking

$$\frac{d\upsilon_i}{dt} = \frac{\partial\upsilon_i}{\partial t} + \frac{\partial\upsilon_i}{\partial x}\frac{\partial x}{\partial t} \tag{A25}$$

$$v_i = u_i e^{-\Gamma x} \tag{A26}$$

$$V = V_o e^{-\Gamma x} \tag{A27}$$

$$\text{and } j\omega = \frac{d}{dt} \tag{A28}$$

Eq.(A24) reduces to:

$$(j\omega - \Gamma u_i)\tilde{v}_i = \frac{e}{m}\Gamma V \tag{A29}$$

Substituting for $\tilde{v}_i$ from Eq. (A23) into Eq.(29) gives

$$(j\omega - \Gamma u_i)^2\frac{\tilde{\rho}_i}{\Gamma\rho_i} = +\frac{e}{m}\Gamma V \tag{A30}$$

and multiplying by $1/j^2$ throughout Eq.(A30) may be expressed as:

$$\tilde{\rho}_i = -\frac{\Gamma^2 V\omega_{ip}^2\varepsilon_o}{(\omega + j\Gamma u_i)^2} \tag{A31}$$

where the ion plasma frequency

$$\omega_{ip} = (\frac{e^2 n}{\varepsilon_o m_e})^{1/2} \quad \text{(A32)}$$

Taking Γ = jk, Eq.(A31) may also be expressed as

$$\widetilde{\rho}_i = -\frac{\Gamma^2 V \omega_{ip}^2 \varepsilon_o}{(\omega - k u_i)^2} \quad \text{(A33)}$$

Eqs (A18) and (A33) for the electron and the ion densities in conjunction with the Poissons Eq.(17) determines the wave propagation constant Γ.

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