A Hydrodynamic-Based Erosion Model for Hall Thrusters

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Lifetime issues for Hall thrusters are becoming increasingly important as the mission envelopes for such devices continue to expand. A hydrodynamic model of the plasma flow within a Hall thruster acceleration channel is developed to analyze the erosion of the channel walls over time. Comparisons to experimental data are presented. The total erosion rate over the initial 1200 hours is captured fairly well, but results indicate that more erosion mechanisms exist beyond those captured by the model. Changes in the potential drop within the channel and thrust are also observed as the channel widens from erosion.

Nomenclature

\[ B = \text{Magnetic field [T]} \]
\[ E = \text{Electric field [V/m]} \]
\[ Q = \text{Heat (power density) [J/(m}^3\text{ s)}]} \]
\[ T = \text{Temperature [K]} \]
\[ V = \text{Velocity [m/s]} \]
\[ e = \text{Elementary charge [C]} \]
\[ j = \text{Current density [A/m}^2\text{]} \]
\[ k = \text{Boltzmann constant [J/K]} \]
\[ m = \text{Particle mass [kg]} \]
\[ n = \text{Number density [m}^{-3}\text{]} \]
\[ s = \text{Secondary electron emission coefficient} \]
\[ \beta = \text{Ionization rate [m}^3\text{/s]} \]
\[ \mu = \text{Mobility [m}^2\text{/(V s)]} \]
\[ \nu = \text{Collision frequency [s}^{-1}\text{]} \]
\[ \phi = \text{Potential [V]} \]

Subscript
\[ a = \text{Neutral atom} \]
\[ e = \text{Electron} \]
\[ i = \text{Ion} \]
\[ r = \text{Radial} \]
\[ s = \text{Sheath} \]
\[ z = \text{Axial} \]

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I. Introduction

Through high values of specific impulse, electric propulsion offers a means of space propulsion that significantly reduces the propellant mass required for many types of missions. However, this reduction in propellant mass comes with the trade-off of lower thrust when compared to chemical propulsion systems. Lower thrust often translates into longer mission lifetime requirements, especially with the impetus to expand the mission envelope of Hall thrusters beyond Earth orbit. Thus, there is a growing need to ensure that these thrusters will be able to operate in a satisfactory manner after thousands of hours of operation. Experimental validation requires much time and subsequently high costs. A computational simulation to model and predict thruster lifetimes would be an inexpensive and quick way to aid in the design and validation process alongside experimental means.

The main life-limiting factor for Hall thrusters is the erosion of the acceleration channel walls. Exposure of the magnetic coils to the discharge plasma can lead to failure. There have been several attempts to analyze the erosion of the channel walls. However, these models are primarily semi-empirical in nature. The next step is to incorporate more physics into the erosion modeling effort. This will help to provide more insight to the erosion process and its causes. In addition, changes to the thruster performance due to the erosion can also be observed. It will also help to develop erosion models that are more robust in covering a variety of different thrusters and operating conditions.

Another goal in the development of these erosion models is to be able to use them in a predictive design capacity. Thus, a quick turnaround, accomplished through computationally inexpensive codes, is desired. This leads to the choice, in the present work, of a hydrodynamic description of the plasma within a Hall thruster discharge channel, as it is faster to compute compared to particle-based methods such as Particle-in-Cell or Monte Carlo.

The hydrodynamic model used for the research presented here is described below. Sample test cases are run for conditions applying to an SPT-100 Hall thruster, for which experimental erosion data is available. Certain observed trends are discussed as well as areas where more work needs to be done.

II. Models

A. Hydrodynamic model

The hydrodynamic model is based on one developed in a previous work by Keidar et al. The main set of equations under consideration is given below.

\[ \nabla \cdot (n_a V_a) = -\beta n_a n \]  
\[ \nabla \cdot (n V_i) = \beta n_a n \]  
\[ m_i n \left( \nabla (V_i) \right) V_i = e n E - \beta n_a m_i n (V_i - V_a) \]  
\[ 0 = -e n (E + V \times B) - \nabla (n k T_e) - \nu_{eff} m_e n V_e \]  
\[ \frac{3}{2} \frac{\partial (j_e T_e)}{\partial z} = Q_{\text{joule}} - Q_{\text{ion}} - Q_{\text{wall}} \]

The neutral atom and ion continuity equations, Eqs. (1) and (2), both contain a source term relating to the ionization rate, whose coefficient, \( \beta \), is calculated as a function of the electron temperature.\(^7\) The neutral flow is modeled as one-dimensional with a constant axial velocity for simplicity, obviating the need for the atom momentum equation. In the ion momentum equation, Eq. (3), a friction term arising from ionization is included in addition to the force induced by the electric field.\(^8\) The magnetic term is neglected because it is assumed that the ions are unmagnetized since their Larmor radius is much larger than the length of the channel. Finally, the ions are assumed to be cold, negating the need for the pressure term as well as the ion energy equation.

The ion equations are solved within an axisymmetric framework through an implicit two-layer iterative method. These equations are supplemented by the electron momentum and energy equations, Eqs. (4) and (5). The axial and radial electric field components are found from the electron momentum equation. Only the radial component of the magnetic field is considered here and it is assumed to vary only along the axial direction. This will facilitate the computation of the axial and radial components of the electron momentum equation. Since the electrons are highly magnetized within a Hall thruster, they are more constrained to...
move along the magnetic field lines. If the electron temperature is assumed to remain constant along the magnetic field lines, this leads to the formulation of a thermalized potential,\footnote{9} which is used to determine the radial electric field.

\[ E_r = -\frac{kT_e}{e} n \frac{\partial n}{\partial r} \]  \hspace{1cm} (6)

The axial electric field,

\[ E_z = \frac{j_{ee}}{en \mu_e} - \frac{kT_e}{e} n \frac{\partial T_e}{\partial z} - \frac{k}{e} \frac{\partial T_e}{\partial z} \]  \hspace{1cm} (7)

is dependent upon the electron mobility across the magnetic field. The mobility is affected by the effective electron collision frequency, $\nu_{\text{eff}}$, which is the sum of the electron collision frequencies with ions, atoms, and the walls. It also includes anomalous transport, or Bohm diffusion. The electron energy equation, Eq. 5, is used to solve for the electron temperature. It consists of a balance between Joule heating, $Q_{\text{joule}}$, ionization losses, $Q_{\text{ion}}$, and wall losses, $Q_{\text{wall}}$.

The treatment of the near-wall region is an area of considerable research for Hall thrusters\footnote{6,10–12} and is of particular interest in the investigation of the erosion process.\footnote{13} The hydrodynamic model is configured such that the lateral boundaries, and the corresponding conditions, are set at the sheath-presheath interface. The governing equations above, Eqs. (1)-(5), are only valid within the quasi-neutral region of plasma flow and therefore the sheath is not included in the domain. However, sheath effects play an important role and it is important to model them correctly. In this model, the Bohm condition is not applied\textit{ a priori} at the sheath entrance along the channel. At locations where the ion velocity cannot reach the Bohm velocity at the sheath edge, a smooth presheath-sheath patching technique is applied where the electric field and the ion entrance velocity are determined as a part of the solution.\footnote{6,14–16} The effects of secondary electron emission (SEE) are also included in the near-wall region. The SEE coefficient, $s$, is modeled as a linear fit to data obtained by Dunaevsky et al. for boron nitride.\footnote{17}

\[ s \approx 0.54 + (1 - 0.54) \frac{T_e [\text{eV}]}{40} \]  \hspace{1cm} (8)

At a critical value of the SEE coefficient,\footnote{18} $s_{\text{crit}} = 1 - 8.3 \sqrt{\frac{m_e}{m_1}} = 0.983$ (for Xe) \hspace{1cm} (9)

the charge-saturation regime is reached and the physics of the sheath are altered. This has implications on the potential drop across the sheath among other things and will be discussed later.

\section*{B. Erosion model}

The hydrodynamic model is run within the presheath region, which extends across the width of the thruster channel, and along with the proper boundary conditions at the sheath entrance provides the ion flux to the lateral walls. This ion flux is coupled with a sputtering model to determine the erosion rates along the walls.\footnote{13,19} Experiments and simulations show that sputtering is a function of four main parameters: wall material, ion species, ion energy, and incident angle of the ion with respect to the target.\footnote{4,20–23} The sputtering model is obtained from empirical fits\footnote{19} applied to data obtained in experiments performed by Garnier et al. where boron nitride samples are bombarded with a beam of xenon ions at various ion energies and angles of incidence.\footnote{20} It should be noted that low energy ($< 100$ eV) sputter data is scarce and difficult to obtain for insulators such as boron nitride. However, this region is of importance for applications of Hall thruster wall material sputtering since ion energies will typically be less than that of the discharge voltage of the thruster, which for the case under examination (SPT-100) is 300 V.\footnote{24} For the results here, we have assumed a 70 eV threshold for sputtering and applied a logarithmic fit to the energy dependence data.

The calculation of the incident ion energies and angles is dependent on the properties of the plasma sheath. Since the sheath by definition has a potential drop and is no longer quasi-neutral, it will affect the ion flow to the walls. In regions where the charge-saturation regime has not been reached, the potential drop across the sheath, $\Delta \phi_s$, can be given by

\[ \Delta \phi_s = -\frac{kT_e}{e} \ln \left( \frac{1 - s}{V_s \sqrt{2\pi m_e/kT_e}} \right) \]  \hspace{1cm} (10)

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where $V_s$ is the ion velocity at the entrance of the sheath. When the charge-saturation regime is reached, a potential well forms within the sheath and Eq. (10) no longer holds. In those cases, we set the potential drop across the sheath to be approximately equal to the electron temperature at that wall location.$^{18}$

The ions are accelerated across the sheath in the direction normal to the wall surface. The ion energies and incidence angles are accordingly modified. The sputter yield is calculated based on those parameters and multiplied by the ion current to the walls to determine the volumetric erosion rate. From there, the radial erosion rate along the channel walls can be found.

This simulation is run on a regular structured mesh, which introduces some difficulty in trying to implement the changing geometry of the channel as it erodes away. The wall profile is approximated on the mesh, but the smooth profile information is retained. Figure 1 illustrates this. Though the hydrodynamic portion of the code runs on the discrete mesh, the proper wall angles and locations are found from the smooth profile and used in the erosion calculations. It is important to maintain monotonicity and smoothness, in general, of the erosion profiles as instabilities tend to arise otherwise.

![Image of smooth and discretized wall profiles](image)

**Figure 1.** The smooth and discretized wall profiles.

### C. Simulation parameters

The simulation is run for conditions that apply to SPT-100 Hall thrusters. The main parameters are given in Table 1. In addition to those inputs, a prescribed radial magnetic field profile is applied. The code is run on an initial mesh of 2400 by 314 cells until 4000 hours of simulated operation time is reached. At each time step, usually less than ten iterations are run to obtain representative values of the plasma flow properties in the channel. Though it is recognized that the actual plasma flow experiences a number of oscillations of various natures,$^{25}$ due to the slow timescale of the erosion process, average values of the flow properties should be sufficient for these calculations. Overall, the code finishes an erosion simulation on the order of minutes, which satisfies one of the goals of this project in keeping it feasible for design purposes by providing quick turnarounds.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>outer diameter</td>
<td>0.1000 m</td>
</tr>
<tr>
<td>inner diameter</td>
<td>0.0686 m</td>
</tr>
<tr>
<td>channel length</td>
<td>0.024 m</td>
</tr>
<tr>
<td>mass flow rate</td>
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<td>discharge voltage</td>
<td>300 V</td>
</tr>
<tr>
<td>discharge current</td>
<td>4.5 A</td>
</tr>
</tbody>
</table>

**Table 1.** Operating conditions for an SPT-100 thruster.
III. Results and Discussion

A sample plot of the plasma number density calculated by the hydrodynamic model after the walls have eroded is shown in Fig. 2. Erosion data for the SPT-100 Hall thruster were obtained experimentally by Absalamov et al. as well as Garner et al.\textsuperscript{26,27} The total volumetric erosion rate was measured at certain intervals over a testing duration of 4000 hours. Figure 3 displays the results of the hydrodynamic model against the experimental data. The results from the model seem to follow the general trend as given by the experimental data until 1200 hours where zero erosion is calculated by the model. Experiments, however, show that erosion exists even after 4000 hours. In the model, once the walls reach a certain angle of inclination, the ions no longer impact the walls, thus causing no erosion. Essentially the axial acceleration caused by the applied electric field is too high for any sheath acceleration or existing radial velocity to overcome.

![Figure 2. A contour plot of the plasma number density after 600 hours.](image1)

![Figure 3. The total volumetric erosion rate.](image2)
One of the more interesting aspects of the erosion rate profile over time is the contrast between the higher drop in erosion rate towards the beginning-of-life as opposed to the near steady rate after around 1000 hours. Explanations have ranged between a two-mechanism process\textsuperscript{3} to a logarithmic dependence.\textsuperscript{5} In previous work, we have tried to apply a physical explanation to a logarithmic dependence through kinetic theory.\textsuperscript{13,19} The present results of the hydrodynamic code would seem to suggest that perhaps a two-mechanism process is in place. The hydrodynamic code models the ions as monoenergetic and calculates the average flow properties at each point along the channel. Thus, as seen in Fig. 3, the main flow of ions may not be directly impinging on the walls anymore. However, in actuality, there is a velocity distribution, even though the ion temperature may be low, and that would allow for a small ion flux to the walls to continue the erosion. Scattering collisions with the neutral atoms would also contribute to the ion flux to the walls. Finally, curvature of the magnetic field, especially near the walls, will also have an effect on the ion flux to the walls. Thus, the higher erosion rate near the beginning-of-life for the thruster may be due to the plasma flow directly impacting the walls whereas the relatively low erosion later in the lifespan is caused from ions diverted from the main flow.

Looking at the erosion profiles along the channel, as shown in Fig. 4, the model seems to overestimate erosion in the upstream region. Near the downstream edge, the model tends to underestimate, especially at the earlier hours. The overall shape of the erosion profiles appears to be similar. The tapering of the erosion at the upstream edge appears since the ion energy at a particular axial location decreases as the channel becomes wider. This is due to a lower calculated potential drop within the channel as the channel width increases, as will be shown later. The results of the hydrodynamic code also show the erosion profiles approaching a linear result after 1000 hours. This corresponds to the wall angle where the ions, as calculated by the hydrodynamic model, do not impinge on the wall anymore. Also notice that, experimentally, the upstream data points continue to erode whereas the model shows that they stop eroding after a while. This may also be indicative that there is erosion occurring at these locations due to a velocity distribution, scattered ions, or a curved magnetic field.

Figure 5 shows a closer look of the erosion at the exit plane of the thruster over time. It should be noted that the outer wall is only 5 mm thick, and thus has completely eroded away at the exit plane by 1000 hours. The two sets of experimental data show very good agreement with one another. The model results also agree fairly well for the first 1200 hours. After 1200 hours, zero erosion is calculated and the output would thus be just a flat line across the rest of the graph. Here, there does not seem to be any indication that a two mechanism erosion process is in place. Rather, the data trend is fairly steady towards 4000 hours, though a slight decrease in the rate can be detected. Since this is at the exit plane, however, the expansion of the plume beyond the channel may affect the erosion at this location.

In the course of this erosion study, we noticed that several other parameters are affected as the walls erode away. One that was alluded to above is the potential profile. Figure 6 shows the potential drop within the channel, as well as the ion current, as a function of time. At the beginning-of-life, the potential drop within the channel is nearly 200 V, but by 1200 hours it has decreased to almost 130 V. This potential drop decrease affects the ion velocities and therefore energies. This in turn helps to account for some of the reduced erosion as the thruster continues to operate. The reason behind this phenomenon is not entirely clear, though it may be due to changes in electron mobility across the magnetic field.\textsuperscript{28} The electron-wall collision frequency as calculated by the hydrodynamic code used in this project is inversely proportional to the channel width. Anomalous, or Bohm, diffusion is also included in a semi-empirical form, however, it is kept constant throughout the channel and through time. It would be interesting to discover the exact connection between electron mobility and the potential profile, but that is beyond the scope of this paper. Another possibility may be from a drop in the overall discharge voltage as observed by Garner et al.\textsuperscript{27} However, this drop in the discharge voltage over the first 1000 hours was measured to be only about 5 V.

Finally, thrust as a function of time is shown in Fig. 7. It is seen to drop monotonically by about 15 mN over the course of 1200 hours. This drop in thrust, though to a lesser extent, has been seen during lifetime testing of the SPT-100.\textsuperscript{27,29,30} Interestingly, however, this drop has only been observed experimentally for the initial 1000 to 1500 hours. It then rises a little again to a stable value. This initial observed drop may be due to the relatively rapid erosion of the walls near the beginning-of-life. Once the walls have relatively stabilized, perhaps the thrust does as well. Experiments have also shown that other parameters, such as the mass flow rate and the discharge current, vary with time as well.\textsuperscript{27,29} However, these are set as constant for this simulation.
Figure 4. Erosion profiles along the outer and inner walls.
Figure 5. The erosion at the channel exit for the inner and outer walls.

Figure 6. The potential drop within the channel and the ion current as a function of time.
IV. Conclusions

A hydrodynamic model of the plasma flow within a Hall thruster acceleration channel was developed to calculate and predict erosion along the outer and inner walls over time. The overall erosion over the initial 1200 hours as calculated by the model matched fairly well with experimental data. However, after 1200 hours, the model predicted zero erosion. This is clearly incorrect and suggests that direct impingement upon the walls by the ion flow is not the only source of erosion. Divergent or scattered ions will also contribute to the erosion process and perhaps can account for the low, but steady, erosion rate observed later in the thruster lifetime. The effects of a fully two-dimensional magnetic field also need to be investigated further. The sheath model can still be improved upon, which would also affect the results as it plays a significant role in the near-wall processes. Better low-energy sputter yield data or knowledge would also improve the erosion model.

The erosion profiles along the channel are less satisfactory when compared to experimental data. Though it is good to be able to match the overall erosion rates, one of the goals of this project is to provide a tool that will be useful in thruster design. Accurate prediction of the erosion profiles is important as it allows for designers to know when critical components of the thruster, such as the magnetic coils, are exposed to the discharge plasma and can possibly lead to failure. This aspect has the most direct effect in determining the lifetime of Hall thrusters.

A few other phenomena are observed as the channel erodes away. The potential drop within the channel, as well as the thrust, are affected as the channel expands radially. It is still uncertain what mechanisms exactly are causing these effects. A more careful investigation of the electron mobility may shed some light. A thruster simulation that includes the near-field plume would also be beneficial as this also helps to determine the plasma properties in the channel.

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References


