Advanced Electric Propulsion Concepts Based on Magnetic Flux Compression and Expansion

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Abstract: High energy missions to the outer planets and beyond that involve a human crew require both high specific impulse and high thrust. Such missions may represent the first real need for controlled fusion power sources. Conventional concepts for fusion power typically result in designs with specific power values far too low for space propulsion interest. A novel form of electric propulsion based on magnetic flux compression and expansion offers opportunities for high specific power and direct conversion of fusion energy to exhaust kinetic energy at relevant speeds. The present paper reviews the concept of Magnetized Target Fusion (MTF) and the earlier development of the LINUS concept for stabilized, repetitive liner implosion technology. A fast transfer, crewed mission to the Jovian System is considered.

Nomenclature

a_c	= acceleration of spacecraft
c	= sound speed in liner material
C	= radial compression ratio of inner surface of liner
d	= distance traveled along trajectory
H	= heat per pulse due to fusion neutrons and losses
f	= various fractions of plasma energy associated with energies denoted by subscripts in text
$F(\beta_r)$	= factor (less than unity) to compare fusion gain from plasma-field mixture to pure plasma
g_o	= standard earth gravity
K	= number of atoms relative to atoms of He^3, added to reduce exhaust speed
M	= various masses denoted by subscripts in text
N	= various particle numbers denoted by subscripts in text
n	= various number densities denoted by subscripts in text
p_f	= pressure at peak compression
$P(\zeta)$	= energy production parameter for fusion gain with compressible liners
Q	= fusion energy gain relative to plasma energy
r_f	= radius of inner surface of liner at final (peak) compression
r_o	= initial radius of inner surface of liner
T	= plasma temperature
u_e	= exhaust speed
v	= various speeds for trajectory analysis as denoted by subscripts in text
w_s	= specific energy [J/kg] for energy storage
α	= specific power [W/kg]
α_J	= jet-specific power [W(thrust power)/kg(power system)]
α_R	= specific power of radiators
β_E	= plasma energy divided by total energy in plasmoid
η	= efficiency of converting external power to thrust power
ζ	= compressibility factor $p_f/\rho c^2$
ρ	= mass density of liner material

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I. Introduction

Over the years and decades, we experience cycles of interest in regard to the most important missions for space exploration and the role that electric propulsion concepts can play. In the last few years, enthusiasm developed for a large mission to Jupiter's icy moons, perhaps using a nuclear-fission and electric propulsion combination. More recently, however, there has been a re-dedication instead to the classic sequence of missions to the Moon followed by a mission to Mars. For such missions, we might expect electric propulsion techniques to offer several capabilities, ranging from auxiliary propulsion for spacecraft control to the perennial electric propulsion space-tug to various nuclear-electric schemes for a significant Mars voyage. In the intervening years between cycles of interest, substantial progress has been made both in demonstrating the space-worthiness of electric propulsion and in developing several types of thruster. For the limited specific impulse (< several thousand seconds) and total power (< few megawatts) needs of the nearer-term lunar and Mars missions, ion engines, Hall thrusters and possibly applied-field MPD thrusters are quite acceptable. A focus on further refinement and application of such thrusters for the Moon-Mars program may occur in the present cycle of interest. The Universe still beckons, however, so consideration of advanced electric propulsion concepts for higher power, higher specific impulse missions remains a useful activity for future interest.

A remarkable result from the NASA Fusion Propulsion Workshop (MSFC, November 2000)[1] was the recognition that propulsion to the outer planets represents perhaps the first and best application of controlled fusion. Other applications, such as terrestrial power production or near-earth missions, have more economical alternatives, including solar and nuclear fission sources. The high specific impulse needs of outer planet missions favor power supplies that operate at the highest temperatures, far from the Sun. While fission-electric propulsion systems certainly can develop high exhaust speeds, the total thrust is obviously limited by the allowable size of the fission reactor, (as set, for example, by launch considerations). Nuclear fusion systems, on the other hand, do not start with built-in radioactive materials. They tend to require very high total powers (> 100 MW) in operation, which may be a good feature in the present context, and sometimes can claim high specific power. Thus, fusion propulsion might naturally connect to very high energy, long range missions.

The problem, of course, for several decades now, has been to achieve a successful fusion power system, short of repeated, thermonuclear bomb blasts. The further step of utilizing such a system for space propulsion might be deemed a bridge too far. Nevertheless, studies of fusion propulsion have been made since the 1950's. More recently, detailed system considerations have been given to fusion propulsion designs based on both tokamak and inertial-confinement fusion concepts. Typically, however, such "conventional" fusion notions suffer from both the size and complexity of the subsystems needed to attain fusion conditions and to sustain continued operation. This situation arises in large measure from the relatively low energy and power densities associated with component technologies. For terrestrial applications, size and complexity may be acceptable burdens, but space propulsion systems should strive for simplicity and compactness.

The present paper considers an approach to controlled fusion called LINUS[2] that was developed during the 1970's at the Naval Research Laboratory, Washington, DC. In part because of its compactness and ability to combine several reactor features in a single element, LINUS was rated the most desirable reactor concept in an extensive review of alternatives to conventional, magnetic confinement fusion approaches. The reduced size and complexity of LINUS derives from the use of repetitive, stabilized implosions to create near megagauss magnetic fields that compress and confine the fusion plasma. Such implosions were successfully demonstrated at NRL in a series of hydrodynamic model experiments. At the time, plasma targets for these implosions were poorly understood. As with space exploration, there has been a new cycle of interest in attaining fusion conditions by implosion of so-called 'compact toroid' plasmas at megagauss field levels. Considerably better understanding of plasma stability and transport has developed over the intervening thirty years. We therefore examine the use of LINUS stabilized implosions in the context of space propulsion for very high energy missions. In particular, the earlier reactor concept is extended to allow the compressed plasma and magnetic field configuration to expand to high exhaust speed, with the implosion system returning to its initial state for repeated operation. Without fusion energy gain, the system acts as an electromechanical device for concentrating large amounts of energy in small masses for high specific impulse. With fusion gain, from either D-T reactions or advanced fusion-fuels, a portion of the fusion energy directly contributes to exhaust kinetic energy. Thus, a development strategy for a family of advanced electric propulsion devices is possible based on increasing levels of fusion energy gain.

II. Brief Review of Some System Goals

Before discussing particular technical schemes, it is useful to establish some goals for the propulsion system. With human exploration of the outer solar system deferred indefinitely, we might as well adopt a sporting approach that attempts to address some concerns for a crewed voyage, namely, the consequences of prolonged weightless conditions, the accumulation of radiation dose and the risk of high dose events during the trip. While there are undoubtedly many solutions to these problems, a minimal, but useful, level of artificial gravity achieved by continual thrust at high power-density would reduce trip time and thereby ameliorate all three concerns simultaneously. Apparently[†], a level of acceleration of only a few percent of earth's surface gravity is adequate to mitigate deterioration that would otherwise occur due to prolonged weightlessness. Such acceleration would result in total trip times of less than a year to the Jovian system and could imply useful maneuver capabilities at the destination.

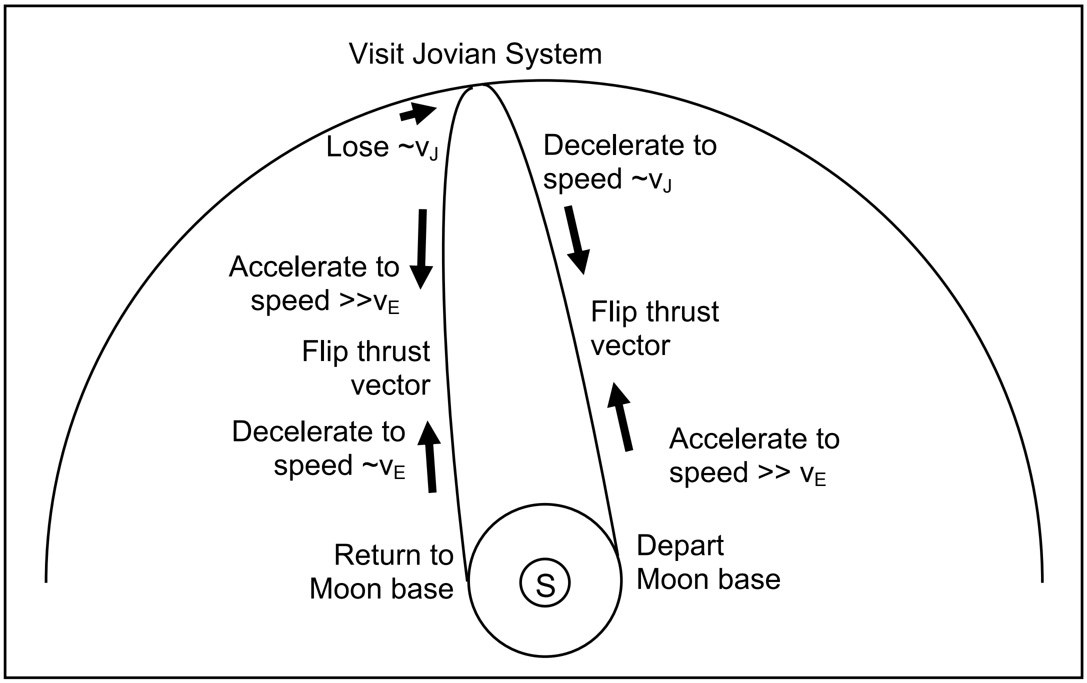

Figure 1. Sketch of fast mission to Jovian System in which continual acceleration or deceleration supplies effective gravity for the crew. Cargo of equipment and propellant precedes mission by more efficient, slower trajectory.

In anticipation of a relatively short duration voyage, suppose we use an acceleration of $0.01g_o = 0.098$ m/s^2 and estimate the trip time and effective Δv. For accelerations much higher than the local solar gravity (< 0.006 m/s^2), trajectories can be simplified to nearly straight lines. The outward trip would then consist of two segments: acceleration at a value a_c from an initial speed v_o to a maximum speed v_1, in time t_1; followed by deceleration at a_c to a speed v_2, in time t_2. If we deploy from the Advanced Space Propulsion Laboratory site on the Moon [which is itself an unstated justification for the Moon-Mars mission], then v_o would be about the orbital speed of the earth (v_E = 29.8 km/s). To match the orbital speed of Jupiter v_J, v_2 = 13.1km/s. The distance traveled can be about 5 AU (7.5 x10^{11} m) in order to allow a modest stay in the high radiation environment of Jupiter and still catch the earth quickly (Fig. 1). The time for the first segment of the outward trip is then

$$t_1 = [\,(v_E/a_c)^2 + V\,]^{1/2} - (v_E/a_c) \quad (1)$$

$$= 2.47 \times 10^6 \text{ s} \quad (28.6 \text{ days})$$

[†] F. Chang-Diaz, personal conversation, NASA Marshall Space Flight Center, Huntsville, AL, c. 2000.

where $V = [(d + a_c t_o^2/2) – v_E t_o]/a_c$, with $t_o = (v_E – v_J)/a_c$. The speed increment for this segment is $\Delta v_1 = a_c t_1 = 242$ km/s. The time for the second segment, in which the thrust direction is reversed and the spacecraft decelerates at a_c, is

$$t_2 = (v_E + \Delta v_1 - v_J)/a_c \tag{2}$$

$$= 2.64 \times 10^6 \text{ s}, \quad (30.6 \text{ days})$$

and involves a speed increment $\Delta v_2 = a_c t_2 = 259$ km/s. The total trip from the orbital radius of the earth to that of Jupiter requires under 60 days, and a total speed change of 501 km/s. This speed change is, of course, much higher than the value needed by a quasi-circular spiral for which $\Delta v \approx v_E – v_J = 16.7$ km/s. Presumably, supplies, propellant and equipment that can stand the radiation dose and are immune to weightlessness can travel on such an economical path. (For a few hundred kW-class ion engine at 10,000 s, ten tons of cargo could be delivered this way in a few years.) The very fast transit time of the present scenario could allow a couple of weeks of exploration and study near Jupiter and still permit earth rendezvous before the earth has traveled more than half its orbit since departure (i.e., 134 days vs 182 days).

To achieve the desired acceleration, we need a high value of specific power α (in W/kg, following Stuhlinger[3]). For the estimated values of Δv, however, we must employ high exhaust speed in order to arrive with adequate payload mass. These requirements compete, indicating an optimum exhaust speed, u_e, to minimize the specific power for a desired acceleration

$$a_c = 2P\eta / M(t)u_e \tag{3}$$

$$= 2 f_s\alpha\eta/u_e \, exp(\Delta v/u_e)$$

where P is the total power for operating the propulsion system. The total spacecraft mass M(t) comprises the propellant and the final mass M_f delivered with the speed increment Δv, of which a fraction f_s is the power system mass $M_s = P/\alpha$. The total efficiency of converting power to thrust power is denoted by η, which would be assembled from the various component processes of the propulsion and power system. To maximize the acceleration for a given specific power and desired final mass, $u_e = \Delta v$. Thus, the necessary specific power is

$$\alpha = a_c e \Delta v/2 f_s\eta \tag{4}$$

(Note this is the specific power needed to have the desired acceleration at the beginning of the voyage, when the entire propellant mass in onboard. By the end of the first segment of the trip, the acceleration could increase to 0.0162 g_o, and could continue to increase up to $0.0271 g_o$ near the end of the outward trip.) If we suppose that the crewed spacecraft will rendezvous near Jupiter with the cargo vessel, in order to re-fuel, then the speculated mission has the previously estimated Δv of 501 km/s. With $\eta = 0.5$ and $f_s = 0.75$, the assumed acceleration of $0.01 g_o$ then demands a specific power $\alpha = 1.8 \times 10^5$ w/kg. Such an extraordinary value provides two particular mandates for the power and propulsion system, high specific energy and minimum thermal power.

For a pulsed thruster arrangement, the specific energy w_s [J/kg] of energy storage must strive for very high values and long life (i.e., many repetitions); it had been pointed out[4] for capacitive systems that the product of α and thrust time τ that occurs in many mission-analysis formulas[3] equals the product of specific energy and shot-life of the capacitors. An additional feature of the present study is the absolute mass required for systems that require large total amounts of energy to achieve significant fusion gain. Table I provides a list of some energy storage media and suggests the attractiveness of pneumatic and inductive mechanisms. For a combination of pneumatic (i.e., high pressure gas) and inductive storage techniques, the necessary value of specific power would require operation at repetition frequencies on the order of a several hertz. Capacitive technologies, on the other hand, would require upwards of a few kilohertz and need lifetimes of ten billion shots.

The other mandate derives from the limiting value of specific power incurred because of heat rejection. For a radiator areal mass density of 1 kg/m^2, an emissivity of 0.5 and a surface temperature of 600 C, the specific power of the radiators would be $\alpha_R = 16$ kW/kg. Thus, we would use up most of the budget toward the necessary total specific power, if we have to reject heat at more than about ten percent of the operating power of the propulsion system. This suggests that the power and propulsion system operate nearly as an open cycle in which that fraction of the exhaust enthalpy not converted to directed kinetic energy remains in the exhaust flow. For most electric propulsion devices, all the flow energy is delivered from an external source, so the normal concatenation of inefficiencies, even for ion engines, would preclude satisfying the present thermal mandate. Addition of energy in the exhaust by fusion reactions, if accomplished with sufficient gain over the energy required for plasma preparation and thruster operation, may convert the electric propulsion system into a largely open-cycle arrangement similar to conventional rocket engines.

Table I: Specific Energies of Several Energy-Storage Technologies

Type	[J/kg]	
Capacitive		
Present	150	
Future	2500	
Rotational	8×10^4	
Inductive	5×10^5	(transient)
Pneumatic	5×10^5	
Explosive	4×10^6	

III. Some Fusion Technology Considerations

Presently, the two main (significantly funded) approaches to controlled fusion represent particular embodiments of magnetically-confined and inertially-confined schemes. The former approach initially attempted to hold plasma in a "magnetic bottle", discovered numerous MHD instabilities, and transitioned to arrangements in which the energy-density of the plasma is a small fraction of the magnetic energy-density. To the extent that convective energy losses are controlled, such arrangements are limited by diffusive processes, so plasma containment time improves as the square of the plasma size. The inertial confinement of plasma accepts convection as the basic mechanism reducing the plasma density, but seeks to operate at the highest possible densities in order to achieve adequate gain in energy and temperature before disassembly of the plasma. The two approaches to fusion result in two separate concerns that dominate progress and application, total energy for attainment of magnetic fusion and power density [W/m^2] for inertial fusion. These concerns become represented in terms of cost of research and development, and weight, volume and entry-levels of economical power generation for future applications.

To achieve the necessary gain, Q, of fusion energy over the plasma energy supplied requires the product of particle density and containment time exceed a minimum value. The necessary value for "breakeven" of the so-called $n\tau$-product is commonly quoted as 10^{14} s/cm^3 (the *Lawson criterion*[5]), which actually assumes a deuterium-tritium plasma temperature of 10 keV and a thermodynamic cycle efficiency of 1/3 for generated heat to replace plasma energy. (Thus, "scientific breakeven", in which the local fusion energy gain equals the local plasma energy, can occur for about a third of the Lawson value.) Within the inertial-confinement fusion community, the goal is often expressed in terms of mass density and radius as the ρr-product. Once ignition occurs, and the temperature of high-density fuel increases rapidly, the actual gain from a so-called thermonuclear capsule depends on details of the design.

For both magnetic and inertial fusion, the relationships of size and energy or power density indicate the scaling of their respective concerns. Thus, for a given value of Q, the $n\tau$-product combines with diffusive loss in magnetic fusion to provide a scaling of energy W with magnetic field B. At a given plasma temperature $n \sim B^2$ and $\tau \sim x^2$, so

$$W \sim B^2 x^3 \tag{5}$$

$$\sim Q^{3/2} / B$$

This suggests that the energy and size of a magnetic fusion device would decrease, if higher magnetic fields are applied. Similarly, on the inertial fusion side (where the equivalent magnetic fields corresponding to high plasma densities would be extreme), we have the power density S scaling as the energy divided by area available for irradiation and the containment time, so

$$S \sim W/r^3 \tag{6}$$

$$\sim [Q^3/\rho^2]/r^3$$

$$\sim \rho$$

In terms of an equivalent magnetic field, at a given temperature, $B \sim \rho^{1/2}$, and the concerns of inertial fusion increase with (equivalent) magnetic field.

These oppositely trending concerns for magnetic vs inertial confinement fusion approaches suggest there may exist an optimum regime between the two arrangements. This intermediate regime[6] has been termed *Magnetized Target Fusion (MTF)*[7], and comprises a variety of concepts ranging from inclusion of magnetic fields within thermonuclear capsules to operation of magnetically-confined and/or insulated plasmas at magnetic field levels much higher than possible with conventional steady-state magnetic design, i.e., megagauss vs 100 kG. Such high fields require the dynamic technique of magnetic-flux compression whereby a hollow, electrically-conducting cylinder, known as a *liner*, implodes onto an appropriate magnetized-plasma "target". Present programs call for combining the implosion of an aluminum liner, (previously demonstrated on the Shiva Star capacitor bank at the Air Force Research Laboratory[8]), with a field-reversed magnetized-plasma target under development at Los Alamos National Laboratory[9], in order to test the ability to attain fusion temperatures by this technique. For repetitive operation needed by space applications, however, we need to return to the stabilized cyclic implosions of the LINUS program at the Naval Research Laboratory[2], c. 1979.

IV. LINUS

The notion of using imploding liners to compress and confine a magnetized-plasma considerably predates the current interest in MTF. A line of effort, instigated by A.D. Sakharov[10] (c. 1952) at a secret Soviet laboratory, and continued by the late V.K. Chernyshev and his colleagues, is called MAGO[11,16] (for *MAGnitnoye Obzhatiye*, magnetic compression) and involves liner compression of plasma for which the magnetic field supplies enhanced thermal insulation, rather than mechanical support (Fig. 2). The "theta-pinch with liner" concept was promoted publicly, c. 1970, by E.P. Velikhov[12,16] and his colleagues at the IAE Kurchatov, Moscow, USSR. This concept comprised compression of a simple straight theta-pinch plasma with a cylindrical metal liner (Fig. 3), and also shaped-liners for compression of so-called *compact toroid* plasmas that were a form of field-reversed theta-pinch[16] (Fig. 4). In these arrangements, the magnetic field supported the plasma away from the inner wall of the liner. In 1971, the Naval Research Laboratory began the LINUS program, inspired by the work of Velikhov's group.

A particular focus of the NRL activities was the need to stabilize the liner motion against Rayleigh-Taylor instabilities that would destroy the fusion plasma during the final stages of plasma compression. This classic problem in using a high mass-density material to compress a relatively low mass-density target has been encountered since the earliest days of the nuclear age and was solved at NRL by means of liner rotation. Basically, the inner surface of a rotating liner spins up as its radius decreases, resulting in a centripetal acceleration that reverses the direction of the effective gravity at the surface in favor of stability. The first demonstration[13] of rotational stabilization of a liner compressing magnetic flux involved electromagnetically-driven liners of liquid sodium-potassium alloy (NaK at its eutectic). Use of electromagnetic drive, however, subjected the outer surface of the liner to Rayleigh-Taylor instability during the initial stages of implosion and also as the liner returned from peak compression. This problem was solved by eliminating the free outer surface of the liner[2]. The liquid liner material was instead driven and recovered by a free-piston moving axially. The resulting implosion and re-expansion of the liner was demonstrated in a series of hydrodynamic experiments (as displayed in Fig. 5), which obtained repetitive, cyclic behavior of the stabilized implosion system. In these experiments, the liner material was water and the driving-gas was high-pressure air. Subsequent experiments used high-pressure helium driving liners of liquid NaK.

The ability to control the liner motion allowed the conceptual design of a fusion reactor[2,14] in which the liner material served multiple functions. In addition to providing the basic pulsed power system for achieving high magnetic fields and fusion plasma temperatures by adiabatic compression, the liner material acts as the reactor blanket and site for tritium production (in DT systems). This also enables the nuclear energy associated with neutrons (and their reaction with a lithium-bearing liner) to deposit immediately in a liquid medium for heat transfer and processing. The "first-wall" of the reactor is also the liner material and is automatically replenished, with heat deposition from plasma radiation and magnetic diffusion directly adding to the other heat generated in the liner by neutrons. More importantly, the stabilization of the liner motion allows the additional pressure in the plasma due to alpha-particle deposition to perform work on the liner. This work pays for losses associated with operation of the liner implosion system without having to process heat through a thermodynamic cycle. The necessary value of Q can therefore decrease from greater than ten for conventional fusion reactor designs to values on the order of two or three. Such a decrease greatly reduces the energy, size and associated output power of a LINUS reactor compared to

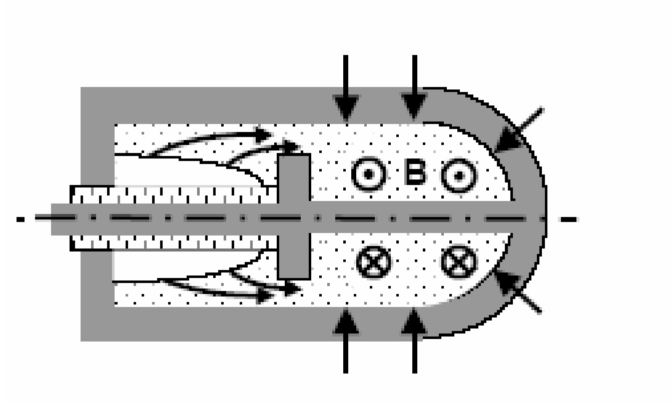

Figure 2. Schematic arrangement[16] of MAGO configuration[11] in which an inverse-pinch discharge pushes plasma magnetized with azimuthal field into an implosion chamber. The azimuthal field provides thermal insulation for plasma supported by walls.

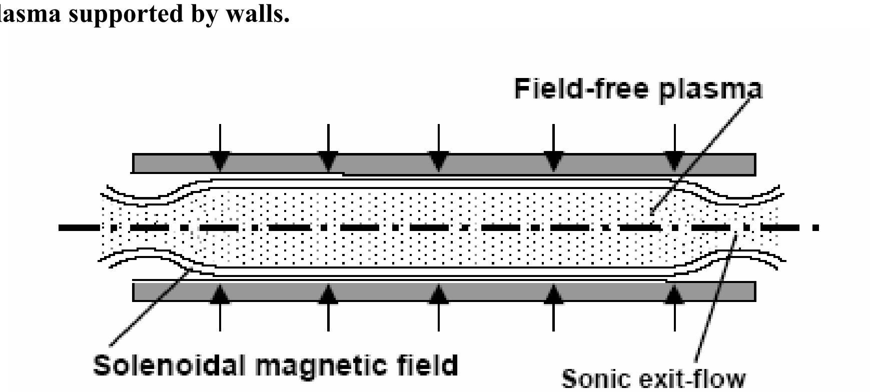

Figure 3. "Theta-pinch-with-liner" concept[16] for imploding liner compression of plasma to fusion conditions. (Magnetic "nozzles" that occur at open ends due to supersonic flow to vacuum are not MHD stable.)

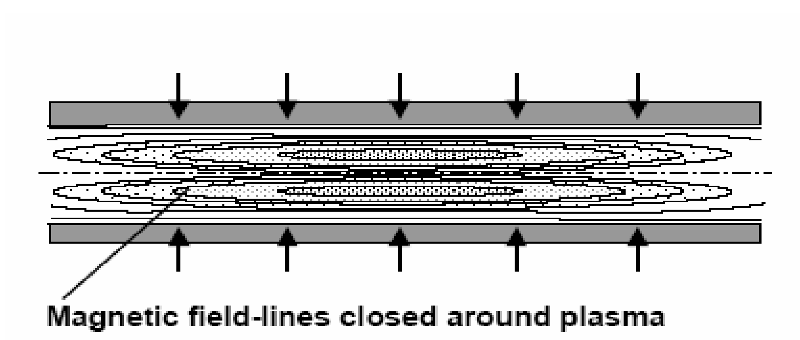

Figure 4. Field-reversed configuration (aka, *compact toroid*) within imploding liner provides distribution of closed-field lines containing plasma, for both thermal insulation and mechanical support[16].

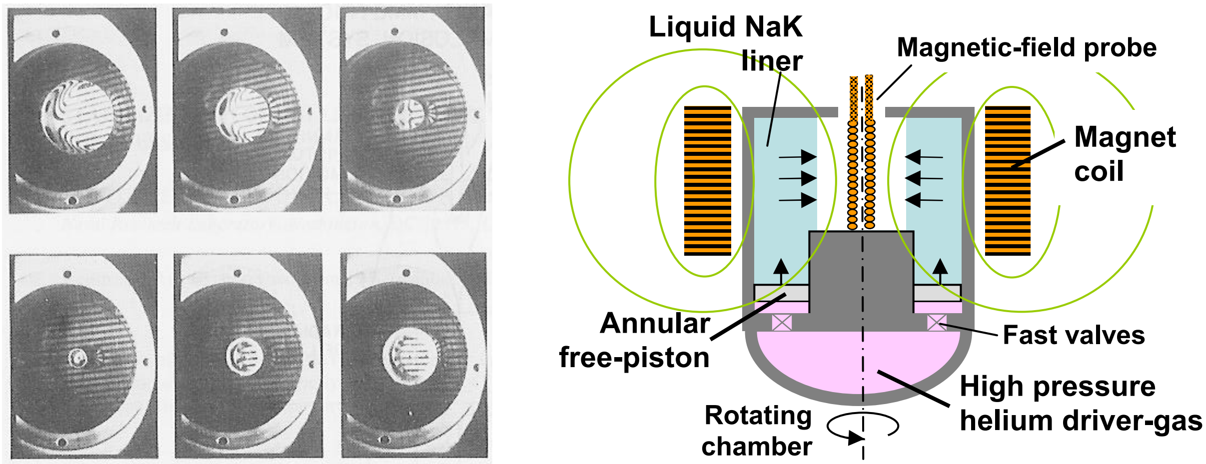

Figure 5. Frames from high-speed movie of stabilized liner implosion and re-expansion in hydrodynamic experiments[2]. The liner material here is water and the free-piston pneumatic power system uses high pressure air. The liner and its chamber rotate to prevent Rayleigh-Taylor instability of the inner surface as it compresses trapped air. Similar experiments were performed with liners of NaK, driven by helium and compressing argon and magnetic flux, shown schematically on the right.

conventional designs. The combination of reactor features and demonstrated liner implosion technology resulted in LINUS achieving the highest rating for reactor desirability in the so-called Science Court assessment by DOE of a dozen alternative concepts for controlled fusion.

V. Application to Advanced Space Propulsion

For the LINUS reactor design, it was merely necessary to create heat in an economical (i.e., small and low power) arrangement. Thus, we employed deuterium-tritium as the fusion fuel, absorbed the 14.1 MeV neutrons into a liner of lead and lithium, and generated additional energy in the liner while creating tritium. Diffusive losses associated with viscosity and skin-currents on the inside surface of the liner during magnetic-flux compression contributed to the total heat deposited in the liner material. The α-particle energy added to the plasma energy and replaced losses of mechanical energy by the liner implosion system without passing this energy through the thermodynamic cycle. All the other energy, however, was processed thermodynamically to supply the needs of reactor operation, such as plasma preparation, and to provide a net output of electricity. (Even with the sophistication of controlled thermonuclear fusion, the power plant would look very much like a conventional system with large cooling towers.) For space propulsion purposes, we need to minimize heat production by neutrons and maximize extraction of work from charged-particles to power the implosion system and provide the exhaust flow. We therefore shift the design to so-called "advanced fusion fuels", such as D-He^3, and adjust the operating parameters to make use of the resulting increase in charged-particle energy.

In line with the earlier speculation of deployment from the Moon, and the notion that substantial He^3 has accumulated there, in addition to water ice, we suppose the use of the D-He^3 reaction[15]

$$D + He^3 \rightarrow He^4\ (3.6\ \text{MeV}) + H\ (14.7\ \text{MeV}) \tag{7}$$

To access this reaction efficiently, we need higher plasma temperatures than normally invoked for fusion reactors using D-T (e.g., 100 keV vs 10 keV). Thus, while the averaged cross-section-speed product $\langle \sigma v \rangle$ for D-He^3 at 100-150 keV exceeds that of D-T (at 10 keV) by a factor of more than two, the higher energy of the reactants requires more particle density to achieve the same (scientific) gain, Q; (the additional electron for helium further adds to the plasma energy.) Also, we must allow for neutron and charged-particle products of D-D that occur at equal rates

$$D + D \rightarrow He^3\ (0.82\ MeV) + n\ (2.45\ MeV) \quad (8a)$$

and

$$D + D \rightarrow T\ (1.01\ MeV) + H\ (3.02\ MeV) \quad (8b)$$

At a temperature of 150 keV, the values of $\langle\sigma v\rangle$ for D-He^3 and D-D are 2.2 x10^{-16} cm^3/s and 0.4 x10^{-16} cm^3/s, respectively, a factor of 5.5 in reaction rates, if we replaced a helium ion with a deuterium ion. For a 50-50 D-He^3 mixture, however, the relative reaction rates would differ by a factor of 11. Furthermore, only half of the D-D reactions produce neutrons. All told then, for each 2.45 MeV neutron, we would obtain 407.45 MeV of energy in charged-particle energy. This suggests that we must manage neutron heating equal to less than 1% of the thrust power, which is important in attempting to meet specific power goals within the constraint of radiator mass.

Another aspect of choosing the reactants and their operating temperature concerns the specification of exhaust speed. From the estimates made earlier, an optimum value of exhaust speed for the proposed mission is u_e = 501 km/s. While this is an extraordinary speed, for the mixture of D-He^3 at 2.5 AMU, it corresponds to a plasma temperature (including energy of the electrons) of only 1.09 keV. This is much less than the necessary reactant temperature, which is a typical situation for fusion rocket design. Unless we are going to the stars, fusion provides too much energy per reaction product. In the present case, we can mitigate matters by extracting energy as work done on the liner material. The factor of over a hundred between the "stagnation" temperature and the reaction temperature, however, indicates that we would incur substantial penalties for simply operating the liner implosion system at a power far greater than the thrust power. Instead, we need to add material to the fusion plasma shortly after adequate thermonuclear gain has been attained. In the context of LINUS, this may be readily achieved thanks to the opportunity for Rayleigh-Taylor instability of super-heated vapor expanding from the liner surface just after peak compression.

As the liner compresses magnetic flux, skin-currents heat the liner surface. At any temperature, we may calculate the equilibrium-vapor pressure p_v and compare this with the magnetic pressure difference across a layer of vapor of electrical conductivity σ_v that is in parallel with the heated liner material of conductivity σ_L. This comparison permits an estimate of the thickness of the vapor layer[16]

$$\delta_v = p_v / j_v B \quad (9)$$

$$= \delta_L\ (\sigma_L / 2\sigma_v)[\ p_v / (B^2/2\mu)]$$

where the total magnetic pressure is $B^2/2\mu$ and the vapor thickness scales with the skin-depth δ_L in the liner. During the rise of magnetic pressure, the temperature and pressure of the vapor increase, but substantial evaporation of the heated liquid surface is avoided. After peak compression, however, the super-heated liquid can explode. The vapor layer will attempt to maintain equilibrium, but this will not be stable, causing material to mix with the fusion plasma. Normally this would be considered detrimental to operation of a LINUS reactor because fusion reactions would be quenched, In the present context, however, it is essential to mix the high energy density plasma with additional material, maintaining pressure, but greatly reducing the temperature to values corresponding to the optimum exhaust speed. The original LINUS reactor design included provision for spraying the inner surface of the Pb-Li liner with a coating of lithium to reduce the average atomic number of residual vapor from the surface prior to plasma injection. Here the exact thickness of such a coating would be specified to achieve the proper mass addition for propulsion purposes.

The number of additional atoms depends on the value of Q and the amount of work ΔW needed to re-supply the implosion and plasma systems. For Li^6 and other values as before

$$(\ M_{Li} + M_D + M_{He})u_e^2 /2 = [Q + (1 + f_{rot})/\beta_E\ ]\ [\ 2N_D + 3N_{He}\ ]\ (3kT/2) - \Delta W \quad (10)$$

where

$$\Delta W = [\ 2N_D + 3N_{He}\ ]\ (3kT/2)\ \{\ [\ 1 + f_{rot} + f_d\ ](\ 1 + f_L) + f_p\ (1 - \eta_p\ )/\eta_p\ \}\ /\beta_E \quad (11)$$

with the total masses and numbers of atoms denoted by M and N, respectively, and subscripts indicating lithium, deuterium and helium. Fractional increments of energy are included for rotational stabilization, f_{rot}, diffusional losses, f_d, and the initial plasma energy, f_p, (produced with efficiency η_p). A plasma-energy beta, β_E, scales up the energy requirements from the plasma energy to the total plasma and magnetic-field energy of the target. The factor f_L accounts for the mechanical energy loss during implosion and re-expansion of the liner material. If we define the lithium atom addition relative to the helium atomic content by $N_{Li} = KN_{He}$, then we can rearrange Eq. (11) to obtain

$$K = (5/6)\{(3kT/m_p u_e^2)(Q - [f_d (1+f_L) + f_L(1+f_{rot}) + f_p(1-\eta_p)/\eta_p]/\beta_E) - 1\} \quad (12)$$

where m_p is the proton mass. The following values are appropriate based on experimental tests and other experience or calculations: $f_d = 0.05$, $f_L = 0.025$, $f_{rot} = 0.5$ (for stabilized liners that are thick compared to the inner surface radius at peak compression), $\eta_p = 0.5$, and $\beta_E = 0.65$ (derived from a pressure-averaged β for compact toroids[17]). For a compact toroid without internal azimuthal magnetic field, the plasma length decreases as $r^{2/5}$, so the plasma energy and temperature increase during compression as $r^{-1.6}$. A radial compression of 15:1 therefore would provide an increase of plasma energy by 76, so $f_p = 1/76 = 0.013$. For Q = 1, as an example, we obtain K = 120. Note that the minimum value of Q occurs when there is no energy left after we pay for re-setting the implosion and plasma system. For the assumed parameters, this value is $Q_{min} = 0.156$, and corresponds in a terrestrial fusion reactor design to a circulating power fraction of 100% (i.e., no power output for your investment).

An additional constraint exists for the space propulsion application, namely, the Q value must be high enough to insure that the overall system is not unduly burdened by the lower specific energy of the thermal radiators. The total heat H generated per pulse scales with the energy handled by the system

$$H = [2N_D + 3N_{He}](3kT/2) \{ f_n Q + [(1+f_{rot}+f_d)f_L + f_p(1-\eta_p)/\eta_p]/\beta_E \} \quad (13)$$

where f_n is the relative energy generated by neutrons (which equals 0.006, based on our earlier accounting for D-He^3). The mass of the thermal radiators is then $M_R = H\nu/\alpha_R$, where ν is the pulse repetition frequency. The corresponding energy W_{ps} delivered by the power system (including the plasma preparation subsystem) is

$$W_{ps} = [2N_D + 3N_{He}](3kT/2) [(1+f_{rot}+f_d)(1+f_L) + f_p/\eta_p]/\beta_E \quad (14)$$

To supply this energy, requires a mass $M_{es} = W_{ps}/w_s$. The radiator mass relative to the energy storage mass is then

$$M_R/M_{ps} = (w_s\nu /\alpha_R) \{ f_n Q + [(1+f_{rot}+f_d)f_L + f_p(1-\eta_p)/\eta_p]/\beta_E \}/ \{[(1+f_{rot}+f_d)(1+f_L) + f_p/\eta_p]/\beta_E\} \quad (15)$$

A combination of pneumatic and inductive storage could provide w_s = 250 kW/kg (reduced from values in Table I to allow for more than just energy storage). For the lowest value of Q (= 0.156), and other values as before, a repetition rate of one hertz results in $M_R/M_{ps} = 0.5$. This is rather insensitive to Q because of the low value of f_n using D-He^3. Thus, for Q values characteristic of a LINUS-type reactor, operating on D-He^3, the radiator and energy storage masses can be comparable. Note that lower repetition frequency decreases the relative mass of the radiator because heat is processed at a lower rate. The effective specific power, $\alpha = w_s\nu/(1 + M_R/M_{ps})$, however, improves with higher repetition rate, approaching a limit

$$\alpha_{max} = \alpha_R [(1+f_{rot}+f_d)(1+f_L) + f_p/\eta_p]/ \{ f_n Q\beta_E + (1+f_{rot}+f_d)f_L + f_p(1-\eta_p)/\eta_p \} \quad (16)$$

$$= 29\ \alpha_R \ (= 464 \text{ kW/kg})$$

for Q = 1 and other conditions as before. This result, which exceeds, for the moment, the goal set earlier for the proposed mission (180 kW/kg), is again rather insensitive to Q because of the relatively small contribution of neutrons to the system heating ($f_n \ll 1$).

In the context of the earlier mission analysis, the need for higher values of Q enters through the conversion efficiency η for generating thrust power, with

$$\eta = W_k/W_{ps} \quad (17)$$

$$= \{ Q\beta_E - [f_d (1+f_L) + f_L(1+f_{rot}) + f_p(1-\eta_p)/\eta_p]\}/[(1+f_{rot}+f_d)(1+f_L) + f_p/\eta_p]$$

so

$$Q = \{\eta [(1+f_{rot}+f_d)(1+f_L) + f_p/\eta_p] + f_d (1+f_L) + f_L(1+f_{rot}) + f_p(1-\eta_p)/\eta_p\}/\beta_E \quad (18)$$

To obtain the previously assumed value $\eta = 0.5$, we need Q = 1.4. As noted in the discussion of $n\tau$, the actual size and energy content of the system depends critically on the value of Q. While such scaling arguments suggest the relationship of Q to system size, quantitative statements require actual calculations of the liner dynamics in compressing the target plasma.

Extensive analysis of liner dynamics for LINUS was performed during the late seventies, using unsteady, one-dimensional, compressible fluid codes. Figure 6 displays some typical results[18] in the form of the trajectory of the inner surface of the liner and the associated fusion reaction rate (for D-T) as the plasma density and temperature increase and then decrease during rotationally-stabilized implosion and re-expansion of the liner. A set of curves from this analysis provide a relationship of the minimum radius of the liner surface, r_f and the relative gain Q, in terms of the radial compression ratio, C, plasma (reaction) beta, β_r, and properties of the liner material

$$Q/\rho c r_f = P(C, \zeta) F(\beta_r)/\varepsilon(C, \zeta) \quad (19)$$

where ρ and c are the mass density and sound speed of the liner material and $\zeta = p_f /\rho c^2$ compares the peak pressure p_f to a characteristic dynamic pressure of the liner material in order to account for compressibility effects. The efficiency factor $\varepsilon(C, \zeta)$ adjusts for the use of a Q value in the present paper that is compared to the plasma energy. Integration of the reaction power density through the mixture of plasma and magnetic field for a field-reversed theta-pinch plasma[19] provides $F(\beta_r) = 0.3$, which diminishes the fusion gain compared to a field-free plasma. For a D-T plasma in the vicinity of T = 10 keV, $P(C, \zeta) \approx 2 \times 10^{-6}$ m-s/kg, with a radial compression ratio C = 15 and $\zeta \approx 0.18$. The variation of $\langle\sigma v\rangle$ with temperature near 150 keV for the D-He^3 reaction is similar to the variation for D-T near 10 keV, so we may adjust the value of P in accord with the higher value of $\langle\sigma v\rangle$ for D-He^3 (an increase by 2.2) and the higher operating temperature and particle number (a decrease by 15 and by 1.25, respectively). Also, the output energy used for D-T (including neutrons on lithium in the liner) was 22.4 MeV vs the value of 19.2 MeV for the charged-particle products of D-He^3. Thus, for our purposes here, we may estimate P = 2 $\times 10^{-7}$ m-s/kg and $\varepsilon(C,\zeta) = 0.35$, at C = 15 and $\zeta = 0.18$. For Q = 1.4 and using a Li liner ($\rho = 0.47 \times 10^3$ kg/m^3 and c = 3400 m/s), the minimum radius is $r_f = 5.1$ m. At the value of C = 15, the outer diameter of the implosion system would exceed an American football field. Even within the adventurous spirit of the present paper, this is unreasonable and demands an additional modification of the concept.

Principal features in applying LINUS to advanced fusion propulsion include the use of adiabatic compression to attain the necessary temperatures for D-He^3 (and thereby greatly ameliorate neutron problems) and the use of a dynamic conductor to achieve operation at the highest possible magnetic-field level by flux compression, in combination with pneumatic/inductive storage for high α. In 1979, the study of compressible liner dynamics had already influenced the LINUS reactor design by indicating design optimums for a stabilized-implosion fusion system at much lower fields than the multi-megagauss values initially envisioned. Indeed, the LINUS reactor design had a peak field of just over half a megagauss for Pb-Li liners. Use of a lithium liner (with $\rho c^2 \approx 5.4 \times 10^9$ nt/m^2) would optimize at lower magnetic fields. This suggests that the target plasma could be compressed by liner implosion and then be held at high density in a static chamber long enough to achieve the desired value of nuclear gain. The plasma could then shift to the interior of a second stabilized-liner system that would accept enough work to operate the reactor before releasing the plasma through a shaped nozzle for the rocket exhaust. A carbon-fiber based chamber, at a stress level of 300,000 psi (~20 kbar), could permit confinement of the plasma at equivalent fields up to half-megagauss levels (10 kbar). Such magnetic pressure corresponds to $\zeta = 0.18$ for lithium, which is well into the compressible regime. Thus, the chamber could be capable of pressure higher than would be needed to match an efficient compression of plasma by a lithium liner. Note that use of D-He^3 creates the need for an intermediate chamber, but permits its survival by greatly reducing problems due to neutrons.

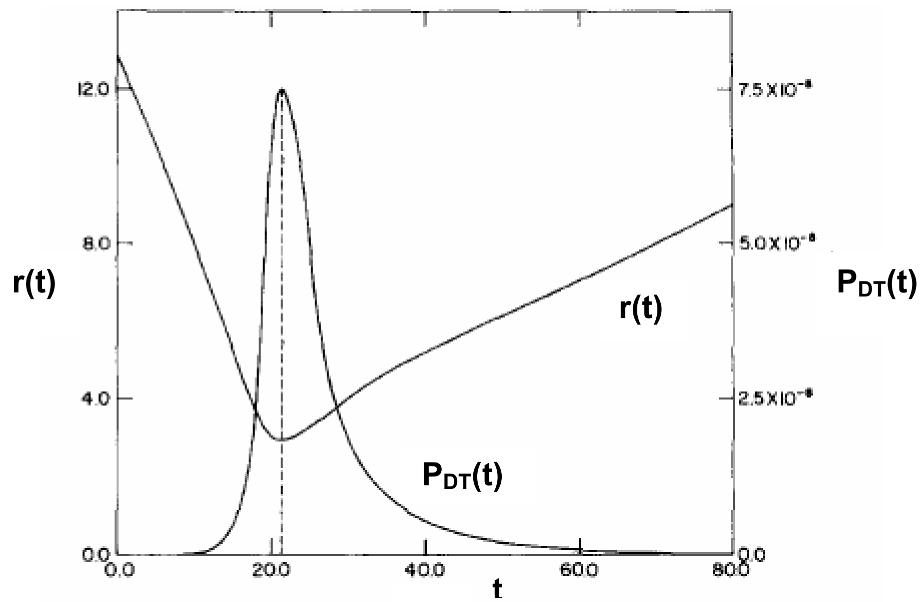

Figure 6. Calculated[18] trajectory of inner surface of liner and D-T reaction rate vs time for a case of substantial liner material-compression (ζ = 0.794), displaying asymmetry of trajectory around minimum radius due to liner compressibility

We may estimate a dwell time in the static chamber needed to achieve the desired value of Q and treat the derived containment time as a goal for plasma development. At the very high temperatures of the present plasma and very high magnetic fields, the electron and ion densities can readily support drift speeds corresponding to the magnetic field distribution in the plasma, even at dimensions of a few cms. Also, the classical diffusion times for magnetic flux are extremely long. Stability and containment thus tend to involve MHD and gyrokinetic processes, favoring fewer ion gyro-radii within the radius of the plasma and plasmoids that are long compared to their diameter[20]. Our understanding of containment time, however, is still an active research activity[9].

At half a megagauss and 150 keV, the particle densities for deuterium and helium are both 8.3 $x10^{21}$ m^{-3}. The necessary effective dwell time τ is then given by

$$n_D n_{He} \langle \sigma v \rangle W_N \tau = Q(2n_D + 3n_{He})(3kT/2) \qquad (20)$$

where W_N = 19.2 MeV is the total nuclear energy released in charged-particles (including, proportionately, from D-D reactions). For $\langle \sigma v \rangle$ = 2.2 $x10^{-22}$ m^3/s and Q = 1.4, τ = 45 ms. For a minimum radius of 10 cm, and a compression ratio C = 15, the initial diameter of the liner surface is 3 m. The elongation (length-to-radius ratio) of compact toroid increases during compression as $r^{-3/5}$, which improves plasma stability. If the initial elongation is two, the value at peak compression would be about ten, corresponding to a length of a meter. The energy content of the compressed plasma is then 47 MJ, so the nuclear energy per shot is about 65 MJ. From Equation (14), the energy per pulse stored in the power system is 117 MJ, which at 250 kJ/kg represents an energy storage mass of about half a metric ton. The mass of imploding liner material M_{liner}, however, is about ten tonnes. For a pair of liners operating to add and then recover work, this number doubles, $M_L > 2M_{liner}$, which is much greater than the mass for energy storage. To achieve the specific power indicated by Eq. (16), we therefore need a repetition rate

$$\nu >> \alpha_R/[W_{ps}/M_L] \qquad (21)$$

$$> 2.7$$

It may be convenient to choose a repetition rate based on the time for an implosion/dwell/expansion event. The implosion time is approximately the initial surface radius divided by a characteristic speed $u_{cI} \approx (2W_{ps}/M_L)^{1/2}$. For the present values, u_{cI} = 153 m/s; the expansion speed should be about the same. Thus, the time to accomplish implosion is roughly ten milliseconds, so the event time totals to about 65 ms. This suggests a maximum repetition rate of 15 Hz, for which the power supply operates at 1.76 GW. Heat must be rejected, according to Eq. (13), at a rate of 0.06 GW, corresponding to a radiator mass of 3.9 tonnes. With a total mass of (at least) 24.3 tonnes, the effective specific power of the power supply (including energy storage) and radiators is only 72 kW/kg, which does not achieve the mission goal for the assumed efficiency of conversion of power supply output into thrust power.

In conventional electric propulsion, the maximum value of system power is that of the external supply. Various processes absorb energy into forms that do not produce thrust, and indeed burden the system. Values of conversion efficiency η on the order of a half, or so, are quite reasonable. For the present fusion system, however, our electrical technology accesses additional energy. We may therefore invoke values of η well above 0.5, in fact, above unity, as long as we can generate sufficient Q. Higher values of η reduce the required α to achieve the desired acceleration and also increase the specific power of the propulsion system.

It is appropriate under these circumstances to specify the jet-specific power $\alpha_J = \alpha\eta$, which for the proposed mission is 89 kW/kg. In the preceding analysis, we have chosen the plasma and liner dimensions and merely specified the necessary containment time. The mass of the liner implosion system and power supply are, therefore, fixed, the repetition rate is fixed and the mass of the radiators only weakly depends on Q (for $f_n << 1$). The jet-specific power thus depends on Q in an apparently straightforward fashion

$$\alpha_J = W_k\nu \,/ (\, 2M_L + M_{es} + H\nu/\alpha_R \,) \qquad (22)$$

Note, however, that for given plasma conditions, all terms scale with the volume of the plasma, and the repetition frequency tends to decrease with increasing Q because the dwell time largely dominates the total time between pulses. Higher pressure helps to increase the energy and frequency, but introduces problems due to liner-material compressibility. The principal difficulty in achieving the desired jet-specific power is the liner mass which is governed by the geometry of the stabilized-implosion technique needed to recover the liner energy safely and efficiently. We may reduce the liner mass by using lower values of radial compression ratio C because the liner volume scales with the initial plasma volume, which is a factor $C^{2.4}$ larger than that of the compressed plasma. Lower compression ratio improves operation at higher values of ζ, but demands more attention to creation of the initial plasma. (An earlier stage liner-driven adiabatic compression, followed by isothermal expansion into the main compression system, may offer an appropriate method of boosting the temperature of the initial plasma.)

For C = 10 vs 15, the mass of liner material is reduced by a factor of 0.378 to M_L = 3.78 tonnes. The plasma energy fraction f_p increases to 2.5% (from 1.3%) and the characteristic implosion time decreases because more energy is delivered to less mass and the initial radius of the liner is smaller. The event time tends to be dominated by the dwell time, so the frequency may be referenced to the previous example as ν = 15 (1.4/Q) = 21/Q. After some substitutions, Eq. (22) provides

$$\alpha_J \approx \frac{21\,[2N_D + 3N_{He}\,](3kT/2)]\{\,1 - [f_d\,(1+f_L) + f_L(1+f_{rot}) + f_p(1-\eta_p)/\eta_p]/\beta_E Q\,\}}{2M_L + [2N_D + 3N_{He}\,](3kT/2)]\Big\{\,([(1+f_{rot}+f_d)(1+f_L) + f_p/\eta_p\,]/\beta_E\,)/w_s + 21\{f_n + [(1+f_{rot}+f_d)f_L + f_p(1-\eta_p)/\eta_p\,]/\beta_E Q\}/\alpha_R\,\Big\}} \qquad (23)$$

Based on previous numerical values (adjusted for C = 10), we find that Q = 2.5 accomplishes the desired jet-specific power, requiring an increase of dwell time to 81 ms. With a characteristic implosion time of 4 ms, the repetition frequency is about 10 Hz; (the approximation in Eq. (23) uses ν = 21/Q = 8.3 Hz.) The total power system mass comprises the material for both implosion and expansion liners (7.5 tonnes), energy storage (0.47 tonnes) and the radiators (2.7 tonnes), for an average thrust power of about a gigawatt, (~ Saturn F-1 engine). While deuterium and helium supply the energy (and can also serve as the driver gas), the propellant is actually lithium, which is eminently storable. The outward voyage would use 24.4 tonnes of propellant of which only 0.25% is deuterium and tritium. Note that, within the construct of a constant acceleration mission, the repetition frequency decreases by *e* during the course of the voyage. A shorter time to the same Δv is possible, of course, at higher acceleration, using the same total propellant and other masses. Trade-offs between human health and engine health are needed to assess other trajectories.

VI. Variations on a Theme

This has been a brief (and perhaps cursory) study of the possibility of using magnetic flux compression and expansion of a fusion plasma to accomplish a mission of human exploration to the Jovian system. As a simple point design, the values displayed here only offer a representation that a solution could exist. Several variations and alternative choices might provide more attractive or viable results. In particular, larger plasma dimensions would probably help in achieving the necessary containment times. At two meters, the present initial diameter of the liner surface might be doubled without causing undue alarm, especially in view of the enormous thrust powers. Indeed, the total initial (wet) mass of the spacecraft is on the order of forty tonnes, which is rather small for a trip to Jupiter, even though the cargo and propellant for the return flight would be sent separately. A factor of two in plasma diameter and length would increase the mission payload by a factor of eight. Diffusion times increase by four, the number of gyroradii in the plasma radius doubles, and the elongation is constant; but the effects on containment time and stability of a simple field-reversed configuration (FRC) remain unclear.

Alternatives to the FRC include addition of azimuthal magnetic field to improve MHD stability for a magnetically-supported plasma, and a MAGO-like approach in which the plasma has only azimuthal field[11] providing magnetic insulation, but not mechanical support, thereby avoiding an immediate MHD stability problem. The former alternative attempts to increase containment time in the face of lower values for both β_E and $F(\beta_r)$. The latter approach increases both β_E and $F(\beta_r)$ toward unity, but involves some challenges for manipulation of the plasma. For example, a shaped, annular duct channeling the liner material provides three-dimensional compression of plasma and an auxiliary magnetic field controls the plasma release. With $\beta_E = 1$, the necessary value of Q to obtain the desired jet-specific power becomes 1.8, reducing the dwell time for this arrangement to 59 ms, which is limited by diffusion, not MHD stability. For classical diffusion[21] in the limit of high magnetic fields, the dwell time allows a characteristic diffusion depth of a few ion gyrodiameters (<< plasma dimensions).

A separate category of variation involves a different choice of mission. At the same (or higher) acceleration, a trip to Mars would be less demanding because of the factor of five reduction in distance. (~1.1 vs 5 AU, for tangential departure from earth orbital-radius). This decreases the trip time and associated Δv to 25 days and 210 km/s, respectively, with a corresponding reduction in the necessary jet-specific power to 37 kW/kg. From the same analysis for FRCs, the necessary Q value is then 0.59, which reduces the required dwell time by a factor of five to 19 ms. Thus, a fast mission to Mars might be performed by a system with less advanced plasma containment. While other technologies may be adequate for a Mars mission, pulsed fusion propulsion could be attractive for Mars and evolve for greater distances within the continuing constraints of frail humanity.

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