Electric Thruster Plume Simulation: Application to MPD and FEEP Thrusters

IEPC-2005-175

Presented at the 29th International Electric Propulsion Conference, Princeton University, October 31 – November 4, 2005

A. Passaro*, F. Nania†, F. Paganucci‡, L. Biagioni§ and L. Priami**
Alta S.p.A. Pisa, Italy, 56121

Abstract: This paper describes the physical and numerical models of the two new codes which are under developing phase at Alta. 2d3vPICFES is bi-dimensional axial-symmetric numerical code able to compute plasma flow in a MPD thrusters with applied magnetic field, while 3dBEMPIC is a three dimensional numerical code able to compute the ion plume of FEEP thrusters. 2d3vPICFES uses a Particle-In-Cell technique only for positive charged particles while a continuous Fluid-Dynamics approach is used for electron simulation. The Flux Vector Splitting scheme, which captures discontinuity phenomena such as shock waves and rarefaction, is applied to solve electrons conservation equations. The ion dynamics module and the electron fluid-dynamics one are separately tested in order to verify the right functioning of the adopted numerical schemes for assigned initial and boundary conditions. 3dBEMPIC is based on a Particle-In-Cell simulation technique for both ions and electrons. The solution of Poisson’s equation is achieved by a hybrid method resulting from the joining of the Boundary Element Method and the iterative Multigrid scheme. Additional computational routines are implemented to address the phenomena related to the formation and evolution of charge-exchange ions.

Nomenclature

\( B_{r,z} \) = radial and axial magnetic induction
\( B \) = vector of magnetic induction
\( B_0 \) = magneto static field vector
\( B_p \) = induced magnetic field vector
\( E_{r,z} \) = radial and axial electric field
\( E \) = electric field vector
\( I \) = current
\( \mathbb{I} \) = unit tensor
\( J_e \) = electron current density vector
\( j_e \) = electron scalar current
\( k \) = Boltzmann constant
\( m_{i,e} \) = ion and electron mass
\( n_{i,e} \) = ion and electron number density
\( n \) = surface normal vector

* Project Manager, Industrial Services, a.passaro@alta-space.com.
† Aerospace Engineer, MSc, Industrial Services, f.nania@alta-space.com.
‡ Associate Professor, University of Pisa, AIAA senior member, f.paganucci@alta-space.com.
§ Chief Executive Officer, l.biagioni@alta-space.com.
** Aerospace Engineer, Industrial Services, l.priami@alta-space.com.
The 29th International Electric Propulsion Conference, Princeton University, October 31 – November 4, 2005

\[ p_e = \text{electron scalar pressure} \]
\[ \mathbf{p}_e = \text{electron tensor pressure} \]
\[ T_e = \text{electron temperature} \]
\[ q = \text{elementary charge} \]
\[ S = \text{surface area} \]
\[ \mathbf{u}_{i,e} = \text{ion and electron velocity vector} \]
\[ \Gamma_{i,e} = \text{ion and electron flux} \]
\[ \Gamma_{qm} \big|_{r,z} = \text{electron momentum flux in radial and axial direction} \]
\[ \Gamma_E = \text{electron energy flux} \]
\[ \varepsilon_0 = \text{dielectric permittivity of vacuum} \]
\[ \phi = \text{electric potential} \]
\[ \mu_0 = \text{magnetic permeability of vacuum} \]
\[ \rho_e = \text{ion and electron charge density} \]
\[ \rho_{m,i,e} = \text{ion and electron mass density} \]
\[ V_{\text{e,CL}} = \text{classical mean collision frequency between ions and electrons} \]
\[ V_{\text{e,AN}} = \text{anomalous mean collision frequency between ions and electrons} \]

I. Introduction

The paper describes the physical and numerical models used to develop two codes able to simulate the plume of two completely different thrusters: MPD thrusters with or without applied magnetic field and FEEP thrusters. The study of plasma dynamics with particle codes (Particle In Cell) was proven to provide better results than two fluids, or MHD or kinetic codes. In fact, the PIC scheme allows to model complex configurations without introducing hypotheses a priori. On the other hand, particle methods are often very expensive from the computational point of view, thus it is preferable to adopt a hybrid approach: to use the particle model for the ions and a fluid-dynamic model for electrons.

Plume simulation in FEEP thruster presents some peculiar aspects due to the linear geometry of the ion emission region. In such a configuration, edge effects, inherent to the operation of the linear slit FEEP emitter, cannot be neglected if the simulation is to provide a realistic approximation of the beam shape at some distance from the thruster exit, making the problem a fully three-dimensional one.

The techniques used and the preliminary results obtained from two computational studies aimed at a 2-D numerical modeling of a MPD thruster and a full 3-D numerical modeling of a FEEP thruster plume are illustrated. The new method of the FEEP model combines the Boundary Element Method and the Multigrid iterative algorithm to improve velocity and flexibility of plume simulation. On the other hand the MPD model represents an innovative way of treating complicated interactions inside this kind of thrusters including the possibility to study magnetic nozzle effects.

II. The 2d3vPICFES Code

A. Introduction

In this section the techniques used to develop the 2d3vPICFES numerical modeling of MPD thruster plume are described. The code belongs to the hybrid 2d-3v methods category. The behavior of the positive ions in the plasma is described with a PIC approach while a fluid-dynamic model (FES) is used to describe electron motion. The choice of different methods to describe the dynamics of ions and electrons is due to the high mobility of electrons: a too small time step should be needed to advance in time the negative particles using a PIC approach. At the same time, electrons are subjected to an extremely larger number of collisions than ions: the simulation can therefore be performed with a smaller computational effort using an electrons fluid dynamic approach.
Although the geometry of problem is considered axial-symmetric and bi-dimensional (2d), all the velocity components of particles are maintained (3v). This is necessary to model the tangential currents which interact with the applied magnetic field and generate the electro-magnetic forces, $j \times B$, which provide the radial plasma confinement in an MPD thruster.

**B. 2d3vPICFES Structure**

The PIC-FES algorithm is based on the typical scheme of a PIC approach\(^1-4\). After having chosen the problem geometry, the domain is divided in a computational grid (problem definition). The ions and electrons are initialized on the injection boundary. The initial boundary conditions are defined and the magneto-static field is computed (problem initialization).

At each code iteration, the fields are computed and the ions and electrons are advanced in time according to the following procedure:

1) Ion charge is transferred on the grid nodes using a suitable weight function (scatter).
2) Poisson’s equation is solved on the grid nodes, and the electric field is calculated (field evaluation).
3) The electric field is associated at the particle position through interpolation (gather).
4) The ions are advanced in time using an ODE equation solver (ion movement).
5) Particles that have exceeded the computational boundary, are removed from the simulation.
6) Electrons are moved of the same time step with a dedicated integration scheme (electrons movement).
7) The magnetic field due to the plasma current (calculation of self-generated magnetic field)
8) The new boundary conditions on the anode potential are finally calculated (calculation of anode potential).

These steps are performed in loop until the statistical convergence of a chosen variable is reached.

**C. Problem Definition**

Figure 1 shows the MPD thruster on which the code is applied and a scheme of the computational domain. The boundaries are defined by: the axis of symmetry (surface #1), the hollow cathode (surface #2), the circular surface made of insulated material (surface #3), the outer insulated cover of the chamber thruster with truncated conical form (surface #4), the truncated conical anode (surface #5). The outer domain, considered as vacuum space, is delimited at two diameter in axial direction and one diameter in radial direction, since experimental data show that plasma out of these zones is almost neutral (surface #6, #7, #8)\(^3\).

**D. Magnetic Field Computation**

In an applied field MPD thruster the magnetic field is due to two contributions: that of the solenoid (magneto-static field, $B_0(r,z)$), and that due to plasma currents (self-generated magnetic field, $B_p(r,z)$):

![Figure 1. Thruster and computational domain scheme.](image-url)
\[ \mathbf{B}(r, z) = \mathbf{B}_0(r, z) + \mathbf{B}_r(r, z) \quad (1) \]

The magneto-static field is computed by dividing the solenoid into \( n_b \) filamentary planar coils (elementary coils) placed along the axis of coil. The base algorithm is that typical Biot Savar Law:

\[ d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{dI \sin \vartheta}{r^2} \quad (2) \]

where the current element \( dI \) provides a contribution at the magnetic field \( d\mathbf{B} \) in orthogonal direction to the plane of \( dI \) and \( r \) (Figure 2).

The magnetic field in every point of the computational domain is then obtained integrating Eq. (2) down the coil length. The radial and axial components of the magnetic field are computed according to the following relations:

\[
\begin{align*}
B_c &= B_0 \frac{1}{\pi \sqrt{Q}} \left[ E(k) \frac{1 - \alpha^2 - \beta^2}{Q - 4\alpha} + K(k) \right] \\
B_r &= B_0 \frac{\gamma}{\pi \sqrt{Q}} \left[ E(k) \frac{1 + \alpha^2 + \beta^2}{Q - 4\alpha} - K(k) \right]
\end{align*}
\]

where

\[
\begin{align*}
\alpha &= r/a, \quad \beta = z/a, \quad \gamma = z/r, \\
Q &= \left[ (1 + \alpha)^2 + \beta^2 \right], \quad k = \sqrt{4\alpha/Q}
\end{align*}
\]

and \( K(k) \) is the complete elliptic integral of the first kind, while \( E(k) \) is the complete elliptic integral of the second kind. \( B_c = i\mu_0/2a \) is the magnetic field in the centre of the coil and \( \mu_0 \) is the magnetic permeability of vacuum. Then the magnetic field generated by each elementary coil is summed to that generated by the other obtaining the total magneto static field in both radial and axial directions.

The model to compute the induced magnetic field, namely the magnetic field generated by the current flowing in the plasma, is still in a developing phase. The base equation is:

\[ \mathbf{B}(P,t) = \frac{1}{4\pi} \int_V \frac{\mathbf{J}(Q,t) \wedge \mathbf{r}_{QP}}{r_{QP}^3} dV \quad (4) \]

where \( \mathbf{J}(Q,t) \) is the current density at the time \( t \), and at point \( Q \), \( Q \) runs over the integration volume \( V \) where the current is non-vanishing and \( \mathbf{r}_{QP} \) is the vector linking \( Q \) to \( P \). Eq. (4) provides a linear relationship between current density and the magnetic field generated by it.

The integral calculus of Eq. (4) can then be performed one only time at the beginning of simulation using a unitary current density at point \( Q \). The result is a matrix \( \mathbf{G}_{kl} (k = r, \vartheta, z; l = r, \vartheta, z) \), that correlates the \( k \)-nth component of magnetic field at point \( P \) and \( l \)-nth component of unitary current density in the \( Q \) point, based on geometry. At each time step during the evolution of the code, the total magnetic field is achieved by the superposition of the magnetic fields generated by the actual value of the current density in each grid element:

\[ \mathbf{B}_k = \mathbf{G}_{ki} \mathbf{J}_i. \]
The computation of the matrix $G_{kl}$ is carried out only when geometrical changes are made, whereas, during the code evolution, only matrix multiplications are needed.

### E. Potential Field Computation

The electric potential is obtained by solving numerically the Poisson’s equation, Eq. (5). Since the field equation is solved at each time step, considering both ions and electrons distribution, the PIC algorithm results in an accurate and self-consistent method.

$$\nabla^2 \phi = -\frac{\rho_i - \rho_e}{\epsilon_0}$$  \hspace{1cm} (5)

The adopted numerical scheme is a Successive Point Over Relaxation (SOR) method, with grid points considered in the order even-odd (Checkerboard Ordering) and acceleration of Chebyschev that modifies the relaxation factor at every cycle, reducing the initial slowing down due to SOR method at fixed relaxation.

The electric field is then computed using the electro-static potential definition (cylindrical coordinates):

$$E_r = -\frac{\partial \phi(r,z)}{\partial r}$$

$$E_z = -\frac{\partial \phi(r,z)}{\partial z}$$

### F. Ion Model

Ions are described with a PIC approach. The particles are advanced in time in accordance to Newton’s law of motion, thus their position and velocity are defined by the Newton-Lorentz equation:

$$m_i \frac{d\mathbf{u}_i}{dt} = q(\mathbf{E} + \mathbf{u}_i \times \mathbf{B})$$  \hspace{1cm} (6)

Eq. (6) is solved by the use of the Leap-Frog scheme which is a stable and efficient algorithm to integrate ordinary differential equations.

### G. Electron Model

A fluid-dynamics model is used to describe electrons motion (FES). To reduce the problem complexity some simplifications have been assumed: 1) Gravitational acceleration is neglected, 2) Negligible viscosity, 3) Neutral atoms are neglected, 4) Only electron-ion collisions are considered, 5) Heat fluxes are negligible, 6) Ionization and recombination phenomena are neglected, 7) The kinetic energy associated with the overall motion of electrons is lower than mean thermal energy, 8) Isotropic distribution of electron random velocity is assumed.

With the above hypothesis the mass, momentum and energy conservation equations may be derived for the electron species.

The mass conservation equation for the electrons can be written in integral form as follows:

$$\frac{\partial}{\partial r} \int_v \rho_{me} dV = -\int_v (\rho_{me} \mathbf{u}_e \cdot \mathbf{n}) dS$$  \hspace{1cm} (7)

where $\rho_{me} = n_e m_e$ and $\mathbf{u}_e$ are the mass density and velocity of electrons fluid respectively.

The momentum equation for electrons is similar to that of Navier-Stokes equation with mass external forces acting on electron fluid. The hypothesis of not viscous fluid involves the lack of friction forces and therefore the presence of just pressure term. Moreover considering an isotropic distribution of electrons random velocity, that is electrons fluid in thermal steady state characterized by a Maxwell-Boltzmann thermal energy distribution, the stress tensor can be written as:
\[ \mathbf{p} = p_e \mathbf{I} \]

\[ p_e = n_e kT_e \] (8)

and then the integral form of the momentum equation becomes:

\[
\frac{\partial}{\partial t} \int (\rho_{me} u_e) \, dV = -\int (\rho_{me} \mathbf{u}_e \cdot \mathbf{n}) \, dS - \int (\mathbf{p}_e \cdot \mathbf{n}) \, dS - \int \frac{qn_e}{v} (\mathbf{E} + \mathbf{u}_e \times \mathbf{B}) \, dV - \int \left( \frac{qn_e}{v} \sigma_{eff} \right) (\mathbf{u}_e - \mathbf{u}_i) \, dV
\] (9)

where \( \sigma_{eff} \) is the effective plasma conductivity which considers anomalous transport phenomena and \( \mathbf{u}_i \) is the ion fluid velocity. The effective plasma conductivity is:

\[
\sigma_{eff} = \frac{q^2 n_e}{m_e \left( \nu_{ei,CL} + \nu_{ei,AN} \right)} = \frac{q^2 n_e}{m_e \nu_{ei,CL} \left( 1 + \frac{\nu_{ei,AN}}{\nu_{ei,CL}} \right)}
\] (10)

where the classical mean collision frequency is estimated by the approximation of Chapman-Enskog \(^8\) while the anomalous collision frequency is computed by the use of relation derived by Choueiri \(^9\).

The general form of the energy conservation equation for an electrons fluid is\(^7\):

\[
\frac{\partial}{\partial t} (\rho_{me} e_T) + \nabla \cdot (\rho_{me} e_T \mathbf{u}_e) + \nabla \cdot \mathbf{q}_e + \nabla \cdot (\mathbf{u} \cdot \mathbf{p}) = \mathbf{j} \cdot \mathbf{E} - \mathbf{u}_e \cdot \mathbf{M}_e - E_l - \dot{N}^K
\] (11)

where \( \rho_{me} e_T \) is total energy per unit volume. Considering that the kinetic energy associated with the overall motion of electrons is littler than mean thermal energy (Maxwell’s distribution of thermal energy is assumed), the total energy per unit volume is assumed equal to the internal energy:

\[
e_T = \frac{3kT_e}{2m_e} = \frac{3p_e}{2\rho_{me}}
\] (12)

\( \mathbf{j} \cdot \mathbf{E} \) is the growth rate of thermal energy of electrons per unit volume due to the electric field; \( E_l \) is the rate of energy loss due to elastic collisions of electrons with heavy particles; \( \dot{N}^K \) is the rate of energy loss due to not-elastic collisions and includes the radiation and neutral atoms ionisation losses. The ionisation phenomenon are not considered and the radiation losses for MPD thrusters are not significant\(^{10}\), then \( \dot{N}^K \) is assumed equal to zero.

The energy transfers between electrons and heavy particles are defined by the following equation\(^8\):

\[
E_{l} = \sum_{r=1}^{2} \frac{2m_r}{m_e} \nu_{er,CL} n_r \frac{3}{2} k (T_e - T_r)
\]

Assuming electron-ion cross section and electron temperature greater than ion one, becomes:

\[
E_{l} \equiv 2\rho_{me} \nu_{ei,CL} \left( \frac{3}{2} kT_e \right) = \frac{3kq^2}{m_e} \left( \frac{T_e n_e^2}{\sigma_{eff}} \right)
\] (13)
Therefore given $E = \frac{3}{2} p_e$ and using Eq. (11) to Eq. (13), the energy conservation equation in integral form becomes:

$$\frac{\partial}{\partial t} \int_V E dV = - \int_{\partial V} (E \mathbf{u}_e \cdot \mathbf{n}) dS - \int_{\partial V} \left( \mathbf{u}_e \cdot \mathbf{p} \cdot \mathbf{n} \right) dS$$

$$- \int_V q \left( \mathbf{n}_e \cdot \mathbf{E} \right) dV - \int_V q^2 \left( \frac{n_e^2}{\sigma_{eff}} (u_e^2 - u_e \cdot \mathbf{u}_i) \right) dV - \int_V \frac{3kq^2}{m_i} \left( Tn_e^2 \right) dV$$

(14)

H. Numerical Solution of the Electron Fluid Equations

Using the axial-symmetry assumption and writing the Eq. (7), (9) and (14) in cylindrical coordinates, the conservative system of equations can be written as:

$$\begin{aligned}
\{ \text{PDE}: \quad & U_r + F(U)_r + G(U)_z = S(U) \\
\{ \text{IC}: \quad & U(r, z; t^n) = U^n
\end{aligned}$$

(15)

where the left term is the convective part of the system that requires the estimation of the variables partial derivatives. The right term is the source part of the problem.

There are two approaches to solve non-linear problems with hyperbolic conservation laws and source terms. The first approach maintains the coupling among convective and source terms, while the second separates these terms at each time step. This last approach is interesting since it allows to use the best numerical scheme for each sub-problem. Therefore to solve the homogenous problem the Flux Vector Splitting scheme is used (FVS), while to solve the source term problem a numerical scheme for ordinary differential equation (ODE) is adopted.

As a result, the complete system of Eq. (15) is split in two homogeneous system is:

$$\begin{aligned}
\{ \text{PDE}: \quad & U_r + F(U)_r + G(U)_z = 0 \quad \Rightarrow \quad \bar{U}^{n+1} \\
\{ \text{IC}: \quad & U(r, z; t^n) = U^n
\end{aligned}$$

(16)

and in the source term system:

$$\begin{aligned}
\{ \text{ODE}: \quad & \frac{dU}{dt} = S(U) \quad \Rightarrow \quad U^{n+1} \\
\{ \text{IC}: \quad & U^{n+1}
\end{aligned}$$

(17)

The initial condition of convective problem, Eq. (16), is the initial condition of whole problem, while the source problem, Eq. (17), uses the solution $\bar{U}^{n+1}$ of Eq. (16) after a time step $\Delta t$ as initial condition. The source problem considers the source term $S(U)$ and is solved for the same time step $\Delta t$. The final solution is an approximation of the solution $U^{n+1}$ of the whole problem at time $t^{n+1} = t^n + \Delta t$.

The numerical solution is obtained using a conservative variables formulation of the mathematical problem, as suggested in Ref. 10. The system of equations Eq. (16) and Eq. (17) are solved using appropriate numerical scheme. The numerical solution of hyperbolic equations is based on Flux Vector Splitting method (FVS) and the PDE integration is carried out by the use of a fourth order Runge Kutta’s algorithm.

An approximated estimation of the stiffness class of the ODE problem (Eq. 17) gives a stiffness ratio of about 1843. For this reason the Gear method for stiffness problems is used to solve the ODE system. Finally, in order to simplify boundary conditions definition, a finite volumes approach is used.
I. Boundary Conditions

Referring to Figure 1 the MPD thruster model has eight surfaces that delimit the computational domain and each surface is different from others. In the following are reported the model used to define the boundary condition on each different surface.

Free flow surfaces

It is assumed that the computational domain is sufficiently large in order to neglect gradients of electrons flow properties at the open walls in orthogonal direction. With the same assumption the boundary conditions on potential require:

\[
\frac{\partial \Phi}{\partial z} = 0 \quad \text{on surface \#8 and \#6},
\]

\[
\frac{\partial \Phi}{\partial n} = \nabla \Phi \cdot n = 0 \quad \text{on surface \#7}
\]

where \( n \) is the outward normal of surface \#7.

Axis of symmetry

The condition of axis of symmetry (surface \#1) requires zero gradient and flow in the radial direction, moreover the components of momentum are zero in radial and tangential direction. The boundary conditions on potential require:

\[
\frac{\partial \Phi}{\partial r} = 0 \quad \text{on surface \#1}.
\]

Solid wall boundaries

The solid wall borders of the thruster are considered coincident with the boundary of the plasma simulation region. The sheath influence is considered in terms of its effects: wall potential alteration and wall losses. Two kinds of solid walls are considered: insulated walls and electrodes.

1) Insulated walls.

The model used to quantify wall losses considers the total current at the dielectric walls equal to zero thus the electrons flux equal to the ions one \((\Gamma_e = \Gamma_i)\). That is, the electrons with enough energy to reach the walls can produce the electrons secondary emission after having impacted with it. The ionic bombardment produces a less secondary emission and is neglected in this model. Defining the electrons secondary emission with \( \delta_e \), thus:

\[
\Gamma_i - (1 - \delta_e) \Gamma_e = 0 \quad \Rightarrow \quad \Gamma_e = \frac{\Gamma_i}{(1 - \delta_e)}
\]

were \( \Gamma_e \) is the electrons flux coming from the plasma region, that is from surface \#3 and \#4.

Surfaces \#3 and \#4 are assumed to coincide with the line sheath-pre-sheath, then it is assumed that the potential gradient is equal to zero at the wall, and the potential trend in the plasma has zero slope at the interface sheath-pre-sheath Figure 3. The boundary condition to solve Poisson’s equation is then

\[
\frac{\partial \Phi}{\partial n} = 0.
\]
Assuming zero electron average velocity in orthogonal direction to the wall, the momentum and energy flux losses at dielectric walls, is estimated by the following relation:

$$\Gamma_e = \frac{n_e}{2\sqrt{\pi}A} e^{-\frac{\sqrt{\pi} \Gamma_{TH} e}{2}}$$  \hspace{1cm} (20)

with $A = \frac{m_e}{2kT_e}$ and $V_{TH}$ minimum velocity needed by the electrons to go through the sheath and reach the wall. Combining Eq. 19 and Eq.20 obtains the threshold velocity:

$$V_{TH}^2 = \frac{1}{A} \ln \left[ \frac{n_e}{2\sqrt{\pi}A} \right]$$

All electrons with velocity greater than $V_{TH}$ reach the wall producing mass, momentum and energy losses. The wall flux losses are then summarized by the following relations:

$$\Gamma_e = \frac{\Gamma_i}{1-\delta_e}; \quad \Gamma_{qm} \bigg|_i = f_1(V_{TH}); \quad \Gamma_{qm} \bigg|_z = f_2(V_{TH}); \quad \Gamma_E = f_3(V_{TH})$$  \hspace{1cm} (21)

2) Electrodes

The cathode (surface #2) is considered as a surface that injects an electron flow equal to ion one in order to obtain neutral plasma. In order to model the electric discharge in the plasma, an electron current equal to that run in the thruster supply was added. The unbalanced electrons and ions introduce a net plasma current not equal to zero.

Then the boundary condition of electrons flux at cathode is:

$$\Gamma_e = \Gamma_i + \frac{I_{cathode}}{qS_{cathode}}$$

Assuming that the electrons have only axial velocity, the momentum and energy fluxes are:

$$\Gamma_{qm} \bigg|_e = 0; \quad \Gamma_{qm} \bigg|_z = \Gamma_e a_{ec}; \quad a_{ec} = \sqrt{\frac{jkT_e}{m_e}}; \quad \Gamma_E = \varepsilon \Gamma$$

where $a_{ec}$ is the electron sound velocity, $\gamma$ the ratio of capacity heat ($= 5/3$) and $\varepsilon$ is the electron temperature.

The formation of sheath is present also on the outer surface of metallic cathode. The cathode is hollow type, thus the sheath is formed also in the inner surface. To avoid the treatment of the interaction between the two sheaths, it is assumed that the surface #2 have a constant potential value equal to the injected plasma one. Thus the boundary condition used to solve Poisson’s equation is:

$$\Phi_{cathode} = \Phi_{plasma}$$

Two type of boundary conditions are necessary also for the anode: one for the potential computation and the other for electron mass, momentum and energy fluxes. Also in this case it is assumed that the separation surface between pre-sheath and sheath coincides with the anode surface #5. Thus the sheath on the anode is avoided and the computational grid nodes are placed on surface #5 (as those on surfaces #3 and #4) belonging to plasma rather than to solid edge. Therefore the anode potential is a plasma potential modified to consider the potential fall produced by sheath.

The anode potential is estimated considering the thruster current equal to current introduced by cathode that is equal to current absorbed by the anode:

$$I_{cathode} = -I_{anode} = q(\Gamma_i - \Gamma_e)S_{anode}$$

and substituting $I_i$ becomes:

$$I_{anode} = \left\{ \frac{qP_{ne}}{2m_e\sqrt{\pi}A} e^{-\frac{\sqrt{\pi} A(V_{TH} - u_\perp})}{2} + u_\perp \sqrt{\pi} A erf\left(\sqrt{\pi}(V_{TH} - u_\perp)\right) - q\Gamma_i \right\} S_{anode}$$

Figure 3: Sketch of electric potential variation near the insulated surfaces.
Considering zero beam average velocity in orthogonal direction to the wall the anode current is simplified in:

\[ I_{\text{anode}} = q \left[ \frac{\rho_{\text{me}} e^{-AV_{\text{th}}}}{2m_e \sqrt{\pi A}} - \Gamma_i \right] S_{\text{anode}} \]  \hspace{1cm} (22)

The threshold velocity can be estimated by an energy balance. The particles go through the sheath and reach the anode wall if their kinetic energy is greater or equal than the potential energy difference between the anodic wall and the plasma at edge of sheath (coincident with surface #5):

\[ \frac{1}{2} m_e V_{\text{TH}}^2 = -q \left( \Phi_{\text{wall}} - \Phi_{\text{Plasma}} \right) \eta_{sh} \]

and therefore it follows:

\[ V_{\text{TH}}^2 = -\left( \Phi_{\text{wall}} - \Phi_{\text{Plasma}} \right) \frac{2q \eta_{sh}}{m_e} \]  \hspace{1cm} (23)

where \( \eta_{sh} \) approximate the anomalous losses due to collision in the sheath, since the model does not consider collisions. In lack of experimental data, this value will be 1. The anode wall potential \( \Phi_{\text{wall}} \) is assumed known, while plasma potential on the sheath edge is computed combining Eq. (22) and Eq. (23):

\[ \Phi_{\text{Plasma}} = \Phi_{\text{wall}} - \frac{m_e}{2qA \eta_{sh}} \ln \left[ \frac{2m_e \sqrt{\pi A}}{\rho_{\text{me}}} \left( \Gamma_i + \frac{I_{\text{cathode}}}{q S_{\text{anode}}} \right) \right] \]

Finally to estimate the mass, momentum and energy fluxes at the anode wall, it is considered that the electron mass flux is different from the ions one, because with the electrons that rejoin with the ions, there are also electrons that exiting from the anode constitute the thruster supply current. Therefore the electrode fluxes are:

\[ \Gamma_e = \Gamma_i + f_1(V_{\text{TH}}) \]
\[ \Gamma_{qe} = f_2(V_{\text{TH}}) \]
\[ \Gamma_{q\text{e}} = f_3(V_{\text{TH}}) \]
\[ \Gamma_e = f_4(V_{\text{TH}}) \]

A detailed description of fluxes computation is reported in Ref. 3.

J. Preliminary MPD Thruster Simulation Results

In order to check the numerical method implemented in the code two kinds of test was performed. Using a simplified electrons model the PIC module of 2d3vPICFEES code was used to investigate plasma plume injected from an allow cathode of the MPD thrusters configuration showed in Figure 1. The electrons model is that described by the Boltzmann isothermal equation:

\[ n_e(r, z, t) = n_{\text{ref}} \exp \left[ \frac{q\phi(r, z, t)}{kT_e} \right] \]

where \( n_{\text{ref}} \) is a reference density.

The magnetic field is generated by the solenoid of thruster (Figure 1) and it is computed at the beginning of the simulation according to the algorithm described in section D. Figure 4 shows the magnetic field line produced by the thruster solenoid.

The potential field in every point of grid is estimated at each time step of PIC module cycle by the use of Eq. 5. Anode potential is fixed, while the injected plasma potential is set equal to the cathode electrode potential. Figure 5 shows the computed electric potential field distribution within and in the near outside of the thruster.

Ion number density distribution is shown in Figure 6 while in Figure 7 is represented the ion temperature distribution.
The FES module of 2d3vPICFES code was tested on a simplified domain without considering ions particles. The lack of ions have implied the introduction of some further hypotheses. The electron conservation equation system is solved without the collision terms. The electric field is fixed constant and equal to that produced by the two electrodes: cathode and anode. The magnetic field is static and produced by the thruster solenoid.

Without ions the rejoicing of electrons at the walls is not possible, here the assumption is that electrons consider the walls as impermeable instead of dielectric, thus all convective flows at the walls are placed equal to zero.

The electron number density distribution within the thruster chamber is shown in Figure 9. Electrons are accelerated by the field forces reaching velocity greater than the local electron sound velocity.
The electron fluid, already supersonic, to be able to see the presence of the impermeable wall, and thus to bend, requires the shock formation. This last phenomenon seems to be the reason that produces the increase of density at wall Figure 9. This type of trend is confirmed by the Mach number distribution Figure 11 and by the electron pressure distribution Figure 10.

An oblique shock is formed near the wall and electron trajectories are bent in order to assume the direction towards the axis of symmetry. Considering that for the axial-symmetric condition the electrons velocity can have only axial component on axis, the electrons can not reach the axis and another oblique shock has to be formed to bring the flow in parallel direction at the axis and increases the density. The rarefaction at the edge is probably owing to flow which after the compression due to oblique shock, has to enter in a region at lower pressure. This situation produces expansion fans which bring the flow to get the outer pressure.

The electron current lines are shown in Figure 12 (the velocity vector is tangential in every point at these lines). The effect of axial and radial components of velocity produced by the shock and expansion system is evident.

---

**Figure 9.** Electrons number density distribution [#/m³].

**Figure 10.** Electron pressure distribution [N/m²].

**Figure 11.** Mach number distribution.

**Figure 12.** Current line distribution.

### III. The 3dBEMPIC Code

#### K. Introduction

In this section the techniques used for the 3d numerical modeling of FEEP thruster plume are described. The mathematical model and the numerical techniques presented here are based on a 3d Particles In Cell (PIC) plume simulation and on an hybrid method to solve the Poisson’s equation. The numerical simulation helps to investigate the shape of the ion beam, the charge-exchange ions and the combined motion of ions generated by the emitter and the electrons generated by the neutralizer.

The computational model is based on the assumption that the magnetic field induced by the plume currents is too low to be of importance. The approach used to study the particles dynamic is an electrostatic one.11-13
L. Poisson’ Solver

In the electrostatic problem the fundamental equation is Poisson’s (Eq. 5). To accurately compute the electric field in the thruster very small inter-electrodes region, which mostly influence plume behavior, the Boundary Element Method (BEM) has been chosen. BEM has the advantage of treating open regions without problems intrinsic in boundary conditions definition, and is able to handle complicated electrodes geometry with extreme aspect ratios.

The BEM algorithm is based on the following equation\(^{14}\):

\[
\phi + \int_{\Omega} \rho \phi^* d\Omega = \int_{\Gamma} q \phi^* d\Gamma - \int_{\Gamma} u q^* d\Gamma
\]

where \(\phi\) is the potential field in a point of the computational domain, \(\phi^*\) is the fundamental solution equal to \(1/4\pi r\) and \(q^*\) is the boundary conditions on the electrodes surfaces \(\Gamma\):

\[
q^* = \frac{\partial \phi^*}{\partial n}
\]

\(\Omega\) represents the whole computational domain. Using a discrete representation of the electrodes and of the computational domain it’s possible to transform the above integral equation in an algebraic system of equations.

BEM computations on 3d domains needs quite long computing time so the BEM algorithm is only used to compute the potential field due to the thruster electrodes and spatial charge distribution on the boundary of the computational domain, in other words the BEM method compute the boundary condition needed to solve the potential field in every other point of the computational domain. A Multi-Grid (MG) algorithm is used in turn to solve the Poisson’s equation on large 3d grids in short computational time.

M. Ions Model

The thruster plume dynamics is obtained using a classical Particles In Cell (PIC) technique. PIC algorithm treats the particles forming the thruster beam like macro particles, each macro particle represents many actual particles. The electric charge of the macro particles is deposited onto each grid node using a weighting function. As stated above the induced magnetic field is negligible, therefore the Newton-Lorentz equation of motion Eq. 6 is integrated using a classical leap-frog method.

N. Thruster Model

A dedicated stand-alone meshing tool is used to allow the representation of complex thruster 3d electrodes in order to perform fast triangular superficial meshing\(^{11}\). The output from this tool provides the simulation code with all the needed geometrical and electrical parameters. In Figure 13 the three dimensional mesh of the three FEEP electrodes used to develop the code is shown.

O. Multi-Block Modeling

In FEEP thrusters the ion beam is generated by a slit of about 2 \(\mu m\) width while the inter-electrode region has linear dimensions of a fraction of \(mm\), the spacecraft/plume interaction region is of the order of 1 \(m\). For these reasons multiple partially overlapping domains of increasing size were used to allow for an accurate simulation over all the length scales.

With this scheme the cell size could be increased where the particle density decrease so that the overall accuracy is maximized and the computational cost is reduced. Figure 14 shows the multi-block scheme adopted to perform the numerical simulations. Figure 14 shows a scheme of the multi-block approach.

P. Ion Emission Model

The primary constituents of the FEEP plume are the singly charged ions that are accelerated by the strong electric field. The initial condition for the emitted particles depends on the physical
processes occurring locally on the slit during emission, including Taylor cone formation, field evaporation of ions in the space charge-dominated region close to the cone apex and spatial distribution of the emitting sites along the emitter slit. Such detailed microscopic modeling presents several aspects which are not yet understood. Therefore a simplified approach is adopted assuming a configuration extrapolated from experimental data.

The ions emission model assumes that the emitter slit and the cesium meniscus are represented by a line. The line is divided in a discrete number of equally spaced points, each point is a source of macro-ion. At each time step, a defined numbers of macro-ions (determined by the total emitted current) is generated from the sources.

**Q. Charge-Exchange Ion Model**

A continuous flow of neutral propellant particles is always present along with the ion beam: the neutral efflux actually consists of neutral atoms vaporized from the liquid metal surface. The code simulates the neutral atoms resulting from thermal evaporation. The number density distribution of the neutral plume is modeled as that of a free molecular flow from a point source located at the middle of the emitter slit, the flux of neutrals is the Knudsen efflux.\(^{15}\) This assumption, less accurate in the regions very close to the slit, is a good approximation at a distance from the thruster greater than the slit length.

The neutral atoms flux through a surface \(A\) can be expressed by the relation: \(n_n = I_i/\gamma(eA,v_n)\) where \(I_i\) is the ions current, \(v_n = \sqrt{8kT_n/(\pi m_n)}\) the mean thermal velocity of the neutral atoms and \(\gamma\) the neutral emission factor.

Slow propellant ions are created downstream inside the beam by resonant charge-exchange collision between the fast beam ions and the slow thermal neutrals:

\[Cs^+_{\text{fast}} + Cs^0_{\text{slow}} \rightarrow Cs^+_{\text{slow}} + Cs^0_{\text{fast}}.\]

The result is a fast neutral that travels in a line of sight and a slow ion that is affected by the radial electric field in the beam.

The spatial volumetric production rate of CEX ions is given by:

\[N_{\text{CEX}}(x, y, z) = n_n(x, y, z)n_{bi}(x, y, z)\sigma_{\text{CEX}}(v_{bi})\]

where the relative collision velocity is taken to be the beam ion velocity. Based on theory, which is in good agreement with available data, the velocity-dependent resonant CEX cross section in square meters is given by:

\[\sigma_{\text{CEX}} = (k_1 \ln v_{bi} + k_2)^2\]

For cesium \(k_1 = -1.4611 \cdot 10^{-10} \text{ m}\) and \(k_2 = 2.6963 \cdot 10^{-9} \text{ m}^{16}\).

A Monte-Carlo collision algorithm is implemented to simulate the CEX behavior using the following scheme: the probability \(P\) for the collision between an ion and a neutral beam particle is calculated for every ion particle using the equation:

\[P = 1 - \exp[-v_{\text{relative}}\sigma_{\text{CEX}}n_n(x, y, z)\Delta t]\]

---

*The 29th International Electric Propulsion Conference, Princeton University, October 31 – November 4, 2005*
where \( v_{\text{relative}} \) is the velocity difference between the ion and the neutral particle and \( \Delta t \) is the time step. Particles relative velocity can be approximated by ions one because the neutrals velocity is three orders of magnitude lower. If the probability \( P \) is greater than a random number \( R_n \) between 0 and 1 (\( P > R_n \)), a collision occurred and a CEX ion is generated in that location.

In order to study the slow ions dynamics, the output CEX data are used to perform a new simulation where the slow ions are injected in the simulation domain with the appropriate production rate while the steady-state ions plume distribution, obtained with the preceding simulation, is left as a frozen background. By this way is possible to simulate the slow ions motion with low computational time cost.

**R. Preliminary FEEP Thruster Simulation Results**

The results reported in this section are for the geometry of the FEEP thruster whose simplified mesh model is shown in Figure 13. The emitter current is of the order of \( mA \) and the applied voltages are of order of \( kV \). Figure 15 shows an example of the multi-block domains used to compute the ions plume dynamics inside the thruster and in its near outside.

The potential field is computed each time step by the use of a multi-grid algorithm. Figure 16 and Figure 17 show the electric potential distribution within the thruster electrodes. The potential values range from a maximum near the emitter to a minimum in the domain region near the accelerator electrode.

The ion number densities within the electrodes region have the maximum value. Outside the thruster the ion number densities reach lower values due to three dimensional expansion of the plume (Figure 18 and Figure 19).

During the steady-state ions simulation in the domain outside the thruster, using the neutral atoms and CEX models described in section Q, the CEX generation points are computed. The left hand side of Figure 20 shows the CEX birth location during a fixed simulation time. By the use of this data the CEX generation rate per unit volume and unit time are calculated and plotted in right hand side of Figure 20. As can be seen the CEX particles production rate ranges from a maximum value in the bulk of the ions and neutral atoms plume (near the thruster), to a lower value in the far regions from the thruster. The CEX generation rate can be used to perform charge exchange ions simulation so that spacecraft surface contamination can be estimated.
IV. Conclusion

The 2d3vPICFES code is an attempt to use efficient PIC algorithms to model heavy particles and a realistic continuum model to simulate electrons in an MPD thruster with and without applied magnetic field. The hybrid code was developed considering two different electron schemes.

The code with the simplified electron model was tested and also used to extensively test magnetic nozzle influence on the performance of a real MPD Thruster, showing how it is possible to obtain significant thrust increase without major disadvantages.

The electron fluid dynamics model was tested using a simplified thruster geometry in order to avoid formal complications. For this geometry, the electron fluid conservation equations have been applied with dedicated boundary conditions. In order to verify the correct working of numerical procedures, the code has been run without the ions. The qualitative analysis of results has shown that the numerical schemes are able to catch the discontinuity of electron fluid like shock wave and expansion fans.

A first attempt to run the complete code was done on simplified configurations and has shown the need of some more work in order to reach a sufficient level of reliability.

The new code represents a significant progress towards the development of complete simulation tools for this kind of thrusters and, at the same time, for the understanding of basic effects like the use of magnetic nozzles on real configurations.
As far as concerns the 3dBEMPIC code, the use of a hybrid method to solve Poisson’s equation in a domain of complex geometry and the use of multi-block technique allow to start the simulation very close to the electrodes and extend it through multiple domains of increasing size. Preliminary studies and simulations of CEX ions creation and generation rate has been presented.

The code is able to perform simulation of electron particles flowing from a cathode neutralizer with a proper neutralizer mesh model. The code is also able to perform combined simulation of ions and electrons although some problems of beam neutralization have to be investigated in depth. Electrons are treated as particles and their motion equation is solved in sub-cycles on a time scale much lower than that used for ions, due to their much higher mobility. On the contrary CEX ions present the problem of long simulation time due to their low velocity compared with ions. A strategy to speed up CEX simulation is under development.

The present model represents a valuable tool which can be useful in the optimization of a FEEP thruster configuration and operating conditions. The effect on plume due to the introduction of additional electrodes for ion beam focusing will be implemented and simulated. Model able to compute CEX ions contamination, as well as that of neutral atoms on object near the thruster, will be implemented in the code.

References