SMART–1 is the first European satellite utilizing an electric thruster as main propulsion system. In the exhaust plume of the PPS®-1350 Hall thruster collisions lead to concentrations of low energy charge exchange ions. These particles are easily redirected towards the spacecraft where they potentially cause sputtering and electrical charging of the satellite, both leading for example to degradation and malfunction of solar cells and electronics.

Analysis of flight-data has shown a cyclic variation of the spacecraft floating potential with the orbit. ARC Seibersdorf research is investigating the plasma environment and interactions with the solar arrays using the numeric code SmartPIC. A direct correlation to the rotation angle of the two solar array wings could be demonstrated.

Review of plasma physics shows that the quasineutrality assumption used in most plasma simulations does not hold for regions acting as source for backflow currents on SMART–1. This paper describes the enhancements in the model as well as in the numerical methods. A fast multigrid solver working on a specialized semi-adaptive grid in conjunction with domain subdivisioning allows for instantaneous solution of Poisson’s equation at high accuracy in arbitrary distributed high resolution domains. A new kind of electron model enables self-consistent solution of the electron density distribution throughout the large sized domain.
Nomenclature

\( n_e, n_i \) = electron, and ion number densities, \( m^{-3} \)
\( m_e, m_i \) = masses of electrons, and ions, \( 9.11 \times 10^{-31} \text{ kg}, 2.2 \times 10^{-25} \text{ kg} \)
\( k_B \) = Boltzmann constant, \( 1.381 \times 10^{-23} \text{ J/K} \)
\( e \) = elementary charge, \( 1.602 \times 10^{-19} \text{ C} \)
\( \varepsilon_0 \) = electric permittivity of vacuum, \( 8.854 \times 10^{-12} \text{ F/m} \)

I. Introduction

SMART-1, the first mission of ESA’s “Small Missions for Advanced Research in Technology” program, is the premiere European satellite utilizing an electric thruster as main propulsion system. The main objective of the mission was the demonstration of this new technology. Several instruments have been compiled in the Electric Plasma Diagnostic Package (EPDP) and the Spacecraft Potential Electron and Dust Experiment (SPEDE) allowing for determination of plasma parameters such as its potential, temperature, or electron density. The main body is roughly cubic with approximate dimensions \( 1 \text{ m} \times 1 \text{ m} \times 1 \text{ m} \). The PPS\(^{\text{®}}\)-1350 Hall thruster built by SNECMA is situated on the +Z-panel. On each of the ±Y panels of the main body an extension arm of 90 cm length is mounted pivotable around the Y-axis attaching the two solar array wings to the satellite.

The thruster exhausts primary beam ions at energies of approximately 350 eV. The hollow cathode neutralizer causes concentrations of neutral particles at thermal energies around 1,000 K. Charge exchange (CEX) collisions of these neutrals with primary beam ions create a source for slow CEX particles. These are easily redirected by electric fields in the plume and in the vicinity of surfaces. Since the spacecraft can be seen as a floating body immersed in ambient plasma its potential is expected to be negative. Hence the satellite attracts positive CEX ions. The resulting backflow current is responsible for spacecraft charging, surface sputtering, and material deposition. These effects are known to cause degradation of solar arrays, differential charging especially on payload devices, and even sparking. Damage of sensible electronics and short circuits are possible. Hence investigation of the interactions of CEX plasma with the satellite is of vital importance for future spacecraft design.

Analysis of the data provided by plasma diagnostic instruments on SMART-1 has shown a cyclic variation of the spacecraft floating potential \( \Phi_{SC} \) with the orbit. ARC Seibersdorf research is investigating the plasma environment and interactions with the solar arrays numerically with a code called SmartPIC. As has been pointed out previously\(^4,12\) \( \Phi_{SC} \) correlates exactly with the rotation of the solar array which is conducted in discrete steps of 5°. Exemplary data is depicted in Figure 1. SmartPIC has already been able to predict these variations in previous versions qualitatively.

![Figure 1. Cathode reference potential and solar array rotation angle. Data by ESA Plasma Working Group, SMART-1 EPS/PPS\(^{\text{®}}\)-1350, EPDP and SEPTA](image-url)
II. Model utilized in Previous Versions

A. Physical Model

The electric propulsion exhaust and plasma interactions simulation SmartPIC is being developed since the year 2000. It is a hybrid PIC code, meaning that ions and neutrals are treated as particles while electrons are assumed to form an ambient fluid of equal density than the ions. Hence for calculation of plasma potentials $\Phi$ a simplified version of the momentum equation, the Boltzmann relation, is used.

$$ \Phi = \frac{k_B T_e}{e} \ln \left( \frac{n_e}{n_{e,0}} \right) + \Phi_0 $$

where the index 0 indicates a known reference state. The assumption of quasineutrality gives consistent solution of electron densities throughout the computational domain. Such a model requires major changes to the simulation in both, physical modeling, and computational methods.

Collisions are modelled by a statistical Monte Carlo method assuming Markov processes. The probability for a collision, thus, does not depend on the particles history but solely on the current time steps' configuration. The types of collisions included in the current simulation are collected in Table 1. Probabilities $P$ for collisions are computed using the simple model

$$ P(\Delta v) = 1 - e^{-\Delta v \sigma_{\alpha \beta} n_0 \Delta t} $$

where $\Delta v$ is the relative velocity of the two collision partners $\alpha$, and $\beta$, $\sigma_{\alpha \beta}$ is the cross section for the specific collision, and $\Delta t$ is the time step. CEX cross sections are calculated using the model by Miller already introduced in Ref.10 . Elastic collisions use the model by Oh13.

<table>
<thead>
<tr>
<th>Type of collision</th>
<th>Model</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xe$^+$ + Xe $\rightarrow$ Xe + Xe</td>
<td>$\sigma_{el} = \frac{k}{\Delta v}$</td>
<td>$k = 6.42 \times 10^{-16}$</td>
</tr>
<tr>
<td>Xe$^{2+}$ + Xe $\rightarrow$ Xe$^+$ + Xe</td>
<td>$\sigma_{el} = \frac{k}{\Delta v}$</td>
<td>$k = 1.28 \times 10^{-15}$</td>
</tr>
<tr>
<td>Xe$^+$ + Xe $\rightarrow$ Xe</td>
<td>$\sigma_{CEX}(\Delta v) = a - b \ln \Delta v$</td>
<td>$a = 1.71 \times 10^{-18}$, $b = 1.18 \times 10^{-19}$</td>
</tr>
<tr>
<td>Xe$^{2+}$ + Xe $\rightarrow$ Xe$^+$</td>
<td>$\sigma_{CEX}(\Delta v) = a - b \ln \Delta v$</td>
<td>$a = 1.03 \times 10^{-18}$, $b = 7.70 \times 10^{-20}$</td>
</tr>
<tr>
<td>Xe$^{2+}$ + Xe$^+$ $\rightarrow$ Xe$^+$ + Xe$^{2+}$</td>
<td>$\sigma_{CEX}(\Delta v) = a - b \ln \Delta v$</td>
<td>$a = 4.32 \times 10^{-19}$, $b = 1.18 \times 10^{-19}$</td>
</tr>
<tr>
<td>Xe$^{2+}$ + Xe$^+$ $\rightarrow$ Xe$^+$</td>
<td>$\sigma_{CEX}(\Delta v) = a - b \ln \Delta v$</td>
<td>$a = 1.03 \times 10^{-18}$, $b = 2.8 \times 10^{-20}$</td>
</tr>
</tbody>
</table>

B. Computational Model

Exploiting the symmetry of SMART-1 the simulation includes one half of the satellite’s main body, one extension arm and one solar array. The computational domain is sized in a way to provide sufficient space around the spacecraft for CEX plasma flows. The computational grid is of cubic type with a
constant size of $\Delta x = 2.5 \text{ cm}$.

Ions and neutrals are represented by so called “super-particles” representing $10^8 - 10^{11}$ physical particles. Ions are introduced at the thruster exit, neutrals enter the simulation at the thruster, and the hollow cathode position. Background neutrals and electrons are considered as a fluid represented by densities $n_n$ and $n_e \approx n_i$ respectively. Particles move freely through the domain while fields, densities, and potentials are calculated on grid points. According to the PIC scheme the electric forces are projected onto the particle positions by a first-order tri-linear interpolation scheme. Movement is conducted in two half-steps. In the first one energy and thrust are calculated while in the second the positions are updated. This method is usually referred to as the “leap frog scheme”. The simulation runs are done on Dell workstations with 3.0GHz and 1GB of memory under Windows XP. Equilibrium is defined by a stationary plasma flow to the solar array and is reached after approximately 1 ms. Computation takes two days per run.

C. Floating Potentials

In general the spacecraft floating potential $\Phi_{SC}$ is calculated by balancing ion currents against electron currents. SmartPIC implements a model introduced by Samanta Roy\textsuperscript{15}. The surface potential will vary until the net charge flux to the spacecraft vanishes.

The total ion flux $I_i$ is composed of several components: Static thermal flux $I_{TH}$, static ram component $I_{RAM}$ for the spacecraft moving through ambient plasma at speed $v_{SC}$ exhibiting an effective ram area $A_{RAM}$, thruster ion exhaust current $I_{EX}$, and CEX ion backflow $I_B$. Hence

$$I_i = I_{TH} + I_{RAM} + I_{EX} + I_B$$

$$= \frac{en_i A_{SC}}{4} \sqrt{\frac{8k_B T_{i,0}}{\pi m_i}} + en_i v_{SC} A_{RAM} + I_B + I_{EX}$$

Accordingly this current has to be met by a total electron current obtained by the equation

$$I_e = -\frac{en_e A_{SC}}{4} \sqrt{\frac{8k_B T_{e,0}}{\pi m_e}} e^{(\Phi_{SC} - \Phi_{P})} - \int_{A_{SC}} \text{df} \left[ \frac{en_{CEX}}{4} \sqrt{\frac{8k_B T_{CEX}}{\pi m_e}} e^{(\Phi_{SC} - \Phi_{CEX})} \right] - I_N - I_{INT}$$

The components of this current (ordered by appearance of the summation terms) are: Thermal current $I_{TH}$, electron backflow current $I_B$, neutralizer exhaust current $I_N$, and the backflow current additionally collected by interconnector structures on the solar arrays $I_{INT}$. The CEX ion current $I_B$ is calculated by counting ions colliding with surfaces. For reasons of analysis SmartPIC distinguishes currents to different parts of the spacecraft.

III. Analysis and Review of Plasma Physics

SmartPIC has already led to good results regarding the calculation of backflow currents\textsuperscript{19}, prediction of plasma parameters in the plume\textsuperscript{18,10}, and floating potentials\textsuperscript{4}. The physical models the simulation
is based on have been successful and sufficient for previous analysis. Nonetheless for higher accuracy and quantitative correctness the simplifying assumptions leading to quasineutrality and the Boltzmann relation have to be rethought critically.

The theory of plasma sheaths in the vicinity of metallic surfaces is well known. Potentials $\Phi_s$ emerging from surfaces into the plasma are damped until they reach a value of $\Phi_0 = \frac{4}{3} \Phi_s$ at the sheath boundary which is defined by a distance of approximately one Debye length $\lambda_D = \sqrt{\frac{\epsilon_0 k_B T_e}{n_e e^2}}$. Potentials $\Phi \leq k_B T / e$ propagate nearly unhindered deeper into the plasma. In general at the sheath boundary the quasineutrality assumption $n_e \approx n_i$ is valid.

For calculation of the spacecraft floating potential the surface integral in Equation (6) is computed by summation over grid points adjacent to surfaces which defines a constant sheath length of 2.5 cm. This is not correct in terms of the model introduced by Samanta Roy\textsuperscript{15} where $n_{CEX}$, $T_{CEX}$, and $\Phi_{CEX}$ (in Equation (6)) have to be evaluated at the sheath boundary. However determination of the correct values for these three variables is not a simple issue. The Debye length $\lambda_D$ in this simulation ranges from $3 \times 10^{-5}$ m close to the thruster exit up to 1 m in the far regions above the solar array. One main stability criteria for PIC simulations is that the grid size fulfills $\Delta x \leq \lambda_D$. Hence a grid of higher spatial resolution is needed. On the other hand a basic requirement for SmartPIC is to run on a standard workstation PC which has limited memory resources. Filling the complete domain depicted in Figure 2 with a grid featuring $\Delta x = 3 \times 10^{-5}$ m would take $1.7 \times 10^{15}$ grid points which is not feasible. The solution is definitely to use adaptive grid sizes.

![Image of ion density distribution and approximate sheath boundary](image1)

| Figure 3. Debye length and Sheath |

(a) Ion density distribution and approximate sheath boundary (black line)

(b) Theoretical potential distribution in the sheath

IV. Enhancements in the Advanced Version of SmartPIC

A. Semi-adaptive Multigrid

A special grid of rectangular type has been developed allowing for local high resolution at a minimum of total grid points. A modified domain subdivision scheme is used to distribute nested sub-domains of increasing spatial resolution throughout the computational domain. A total of 6 levels provides grid sizes of $8.6 \times 10^{-3}$ m to 0.275 m. Domains are distributed to adapt to the expected density distribution and according Debye lengths. The total number of grid points does not exceed $1.2 \times 10^6$. The finest resolution of $8.6 \times 10^{-3}$ m still doesn’t meet the Debye length in the regions of density $n_e > 10^{14}$ m$^{-3}$ but since microscopic particle movement is not subjected in SmartPIC this is not seen to be a problem. Special treatment is needed for the solar array which is pivotable and features extremely fine interconnector structures and high electric fields. An automatism in the grid generation distributes high resolution domains stepwise on the array to adapt its rotation angle and resolve fine structures. This advanced technique allows further enhancements in modeling of the solar array structures, mainly the interconnectors. These metallic structures have been found\textsuperscript{4} to be the main electron current drain to the spacecraft and, hence play a major role in the calculation of floating potentials. Exact implementation of geometries is an essential requirement for accurate results. The solar array model implemented in the simulation is depicted in Figure 6. Secondary electrons tend to be attracted by the solar array’s positive potentials. On dielectric surfaces they build up space charges that shield out the underlying potentials. In order to simulate this effect a “shielding factor” $\eta_{sh}$ has been introduced. $\eta_{sh} = 100\%$ means total shielding which corresponds to a potential of 0 V. According to Hastings\textsuperscript{7} space charge limitation is not applicable to typical solar array structures in low density plasma. Hence (in contrast to previous
versions\textsuperscript{4}) only glass surfaces are shielded, interconnectors are not.

B. Multigrid Poisson Solver

The method to obtain a solution for the electric potential in most current PIC simulations is based on quasineutrality (for example see Refs.\textsuperscript{1,15,19}). Principally the Poisson equation

\[ \Delta \Phi = \frac{e}{\varepsilon_0} (n_e - n_i) \]  \hspace{1cm} (7)

is solved where the electron density \( n_e \) is obtained by the Boltzmann relation (1). This is a simple and widely used method to include sheath effects at biased surfaces. In fact it is inaccurate for special
Figure 6. Modeling of the solar array structures and potentials. The two solar array wings are each divided into three panels of equal size. Each panel has 6 sections which are split into three strings. The strings again are placed such that the voltage increases meander-shaped from 0 V to a maximum of 50 V along 5 substrings. These are connected at their ends via “interconnectors” of 8 mm width. Interconnectors are metallic and biased to the potential of adjacent solar cells. Supporting structures are metallic and biased to spacecraft ground. Solar cells are covered by non-conductive glass of 150 µm thickness.

configurations. The point is that in applying the Boltzmann relation one states the assumption of a completely uniform density \( n_{i,0} = n_{e,0} \) at the sheath boundaries, and a homogeneous undisturbed plasma of indefinite extension behind. These requirements are necessary to sustain ion and electron currents building up space charges in the sheath. According to Chapman\(^5\), and Chen\(^6\) sheaths take only the known form described by the Boltzmann relation if the currents drawn from the plasma do not represent a serious drain. On SMART–1 the creation of CEX ions can be estimated to be approximately 100 mA. Ion backflow currents are 20 – 30 mA, electron currents are of the same order. According to the sheath theory by Langmuir this is definitely a serious drain. The CEX plasma cloud cannot sustain such high currents to the sheath which results in invalidation of the linear theory. Self-consistent calculation of electron densities is needed.

The multigrid solver is inherently bound to the usage of a multi-grid. The domain subdivision technique requires to adapt the standard scheme found in the literature\(^{21}\). Basically an iterative Gauss-Seidel scheme is utilized to propagate the potential according to Equation (7) on multiple layers gaining a complete solution throughout the computational domain in approx. 5 s. The multigrid scheme is one of the fastest known numeric algorithms for problems of this type. Boundary conditions are of Neumann type at the outer domain boundaries while spacecraft surfaces define Dirichlet conditions.

C. Virtual Instruments

SmartPIC implements various virtual instruments for comparison to real data. Two of these have been utilized for the current investigation: RPA sensors and Langmuir probes.

1. Retarding Potential Analyzer

RPAs measure the energy of incident ions and, thus, provide information about the flow. Positions are defined relative to the thruster exit (see Figure 7(a)). Positioning data for RPAs utilized in ground tests as well as on SMART–1 are collected in Table 2.
2. SPEDE Probes

The two SPEDE probes have already been implemented by Scharlemann et al. Investigation has been conducted without new multigrid techniques. SPEDE consists of two cylindrical shaped metallic probes mounted on 60 cm long booms extending from the $\pm X$ planes of the main body. SPEDE provides two operating modes. First it can be used as an electric field sensor to measure plasma waves. In the second mode the device operates as a Langmuir probe biased at constant or variable voltage to obtain either relative changes in the electron density or electron densities and temperatures.

D. Verification

For verification of the new grid and the multigrid solver several crosschecks of current results with analytical predictions, ground data, in-flight data, and previous results by Scharlemann et al. have been conducted.

1. Poisson solver

Verification of the multigrid solver has been done by comparison to analytical solution. Test objects are point charges immersed in plasma of different density. The potential was measured across several grid domains of different spatial resolution to check for possible instabilities induced at the grid boarders.

2. RPA verification

The ground test campaign by LABEN originally intended to give reference measurements for the STENTOR satellite that was lost in the 2002 Ariane–5 failure. Tests have been conducted with an SPT-100 that was able to simulate the expected PPS®-1350 environment by increased discharge voltage. Explicit documentation of the test campaign and detailed implementation in SmartPIC can be found in an earlier work. The original report by LABEN is unpublished.

In general an offset of 25 V has to be added to experimental results due to the potential configuration in the experiment. A sweep through the beam from $\alpha = -10^\circ$ to $\alpha = +20^\circ$ depicted in Figure 8(c) shows good agreement. In low angle measurements ($\alpha < 42^\circ$) which are taken in the plume SmartPIC’s peak energies comply very well with the measurements again. Full Widths at Half Maximum (FWHM) in the

![Figure 7. Positioning of sensors](image)
simulation are less due to a uniform kinetic exit velocity of the ions according to the discharge voltage. Far angle RPAs are representative for the CEX environment. The absence of energies above \( \approx 25 \text{eV} \) is predicted correctly.

![Graph](image)

(a) Comparison of RPAs 4 and 8 with simulation data at \( \alpha \approx 10^\circ \)

![Graph](image)

(b) Comparison of RPA 10 data at \( \alpha = 60^\circ \)

![Graph](image)

(c) RPA angle sweep

Figure 8. Comparison of LABEN ground test data to simulation results

3. Comparison with Previous Results

The floating potential calculation has been conducted for different rotation angles of the solar array in steps of 45° in the range of 0° to 180°. Shielding was set to \( \eta_{sh} = 70\% \).

In comparison to previous results ion currents to the main body are reduced for a vertical position of the solar array. This can be reasoned by the high positive potentials emerging from the unshielded interconnectors displacing ions from the Y-planes of the main body. The same effect can be seen for the current collecting surface areas in Figure 9(b). In the present SmartPIC version the particle flow to the X-panels of the main body is enhanced due to the satellite being negative with respect to the plasma. Higher collecting areas for the main body are the result. The most obvious change can be seen for the interconnectors. They exhibit only \( \approx 30\% \) of the area in previous versions. Additionally they shield out ions to a far extent due to the much higher potential. This results in an interconnector current much lower than that obtained with previous versions of SmartPIC. Electron currents depicted in Figure 9(c) show great deviations that can again be deduced from the same effects. The plasma flow to the solar arrays is reduced in the vertical position. Hence interconnectors receive less current for this position. Backflow to the solar array (not interconnectors) is increased due to metallic parts between the interconnector lines (see Figure 6) which gain high currents due to the proximity to the high interconnector potentials. Electron backflow to the main body is increased according to the higher collecting surface. This detail is still under investigation.

The total electron currents of previous and current versions match roughly at \( \alpha = 0^\circ \). For greater rotation angles the difference increases until it reaches a factor 3 for \( \alpha = 90^\circ \) and reduces again for...
DDS

For implementation of SPEDE data in SmartPIC the domain size had to be changed to $(\alpha \geq 135^\circ)$. The floating potentials depicted in Figure 9(d) depend linear on the collected currents and exponentially on the potentials according to Equation (6). Since the unshielded interconnectors exhibit much more positive potentials this explains the amplitude of $\Delta \Phi_{SC} = 30 \text{ V}$ compared to $\Delta \Phi_{SC} = 7 \text{ V}$ of previous SmartPIC versions and $\Delta \Phi_{SC} = 14 \text{ V}$ of SMART-1 data. Tests with little shielding of $\eta_{sh} = 20\%$ for interconnectors show a clear reduction of the amplitude which can be seen as argument for interconnector shielding in disagreement to the results of Hastings.\(^7\)

E. Results

1. Current Measurements by SPEDE probes

For implementation of SPEDE data in SmartPIC the domain size had to be changed to $(DDS_x \times DDS_y \times DDS_z) = (3 \text{ m} \times 5 \text{ m} \times 2.2 \text{ m})$ which resulted in approximately $2 \times 10^6$ grid points. Accommodation of the
probes in X-dimension required widening of the domain. In order to keep the total memory consumption down the Y-dimension was cut from 7.5 m to 5 m. For comparison to flight data the virtual SPEDE probes were set to biasing voltages $-3\,\text{V}$, and $3\,\text{V}$.

Measurements on SMART–1 have shown an asymmetry between the two probes. Currents on the -X panel appear to be higher than on the opposite side. Outcomes of SmartPIC confirm this results (Figure 10(b)). Currents on the +X side which is opposite to the neutralizer are lower compared to those on the -X side. The difference of $360\,\text{nA} - 1\,\mu\text{A}$ decreases with rising bias voltage. In conclusion these results emphasize the assumption that the neutralization flow from the hollow cathode affects the CEX flow.

![Figure 10. SPEDE data from SMART–1 compared to simulation outputs](image)

![Figure 11. Comparison of simulation results with SMART-1 EPDP RPA data.](image)

2. **SMART–1 RPA Flight-Data**

In addition to the floating potentials, and SPEDE data comparisons mentioned earlier the EPDP RPA data has been reproduced. Since measured ion energy is relative to the electric ground of EPDP the according floating potential $\Phi_{EPDP}$ for the sensor has to be taken into account. Unfortunately there are no time-correlated data for the EPDP floating potential and RPA measurements. Analysis of available data for $\Phi_{EPDP}$ has shown that the variation, and hence the shift of the RPA curve, is between $-11\,\text{V}$ and $-18.5\,\text{V}$. The resulting upper and lower boundary curves are depicted in dark and light grey respectively in Figure 11. The actual tilting angle $\beta$ of the EPDP RPA is unknown. Figure 11 shows the results for $\beta = 20^\circ$ which represents the best fitting between experiment and simulation. The secondary peak is predicted in perfect agreement to measured data.
V. Conclusion

The plasmodynamic code SmartPIC has been successfully applied in studies of interactions between satellites and their plasma environment. Recent advancements to increase the accuracy of the simulation include a fast multigrid solver working on a modified 6-stage multigrid with domain subdivisioning allowing for exact implementation of spacecraft geometries. For verification of this new version a complete review of flight, and ground test data has been conducted. A comparison of experimentally obtained RPA data with numeric results shows the high accuracy of the simulation in terms of CEX ion energy levels and plasma flow. A measured asymmetry of the flow field due to the neutralizer position could be confirmed by comparison to SPEDE data. Investigation of variations of the spacecraft floating potential on SMART-1 has been conducted. It has been shown that the main cause is an interaction of CEX plasma with the solar array depending on its rotation angle. These results are not only important with regard to the design of future satellites equipped with electric propulsion systems but also because they can be used to understand observations with satellites already in space.

VI. Outlook

Completing the restructuring campaign for SmartPIC a new kind of electron model has been developed. It is dedicated to gain a complete solution for the electron density distribution without relying on the Boltzmann relation. This guarantees conservation of charge and currents. Furthermore the spacecraft potential will be calculated by a new model based on the electric capacity of the satellite immersed in the plasma. This will form a physically complete and accurate model for the calculation of spacecraft charging.

A. Kinetic Density Electron Fluid Model

This model is based on the idea to create a kinetic flow field without the use of discrete particles. Kinetic aspects are necessary in order to permit electrons to escape the CEX flow and follow the potential fields emerging from the solar array to account for electron backflow currents. The basic idea is to calculate flow speeds $v_e$ that apply to densities located at the grid points. In every time step the density is moved virtually by its flow vector $v_e \Delta t_e$ and is then interpolated to the surrounding grid points by a weighting function $f_w$. In order to conserve inertia and energy the weighting has to include the mass and velocity of the density fractions. Starting at the isotropic equation of motion one obtains

$$m_e n_e \frac{\partial v_e}{\partial t} + m_e n_e (v_e \cdot \nabla) v_e + e n_e E + \nabla p = \frac{e n_e j}{\sigma_e}$$

where $p$ depicts the isotropic pressure, $j$ is the electric current density, and $\sigma_e$ depicts the total electron-ion cross section. The collisional term on the right might be omitted. Using a standard forward discretization scheme one can directly convert this equality to a discrete expression for the change in velocity $\Delta v_e$ based on the current set of data $(n_e(x, t), v_e(x, t), E(x, t), T(x, t))$

$$\Delta v_e = \Delta t \left[ v_e^2 n_e \sigma - (v_e \cdot \nabla) v_e - \frac{e}{m_e} E - \frac{k_B}{n_e m_e} \nabla (n_e T_e) \right]$$

Now that the flow speeds are known the move can be performed. In order to respect the continuity equation $\frac{\partial n_e}{\partial t} - \nabla (v_e n_e) = 0$ which guarantees charge and mass conservation a method is necessary which can be represented by an operator normalized to 1. Such an operator is the well known trilinear interpolation. In order to explicitly include Maxwellian velocity distributions in the model the weighting can also be done by a Maxwell-Boltzmann distribution function around the interpolation point. In either case a displacement vector $\Delta x = (v_e + \Delta v_e) \Delta t$ is calculated which gives a virtual position $x_v = x + \Delta x$. Based on this position the density originated at $x$ can be interpolated to the surrounding grid points $p$. Finally the weighting has to include momentum which is guaranteed by

$$v_p = \frac{n_e(p) v_e(p) + n_e(x) f_{w,i}(x)}{n_e(p) + n_e(x) f_{w,i}(x)}$$

Hence charge, momentum, and mass are inherently conserved by the algorithm.

Regarding the electron temperature the standard adiabatic model utilized in previous versions of SmartPIC and other simulations (for example Refs.2,17) is used. Following Boyd2 and Taccogna1 the adiabatic exponent $\gamma$ is set to 5/3. Further refinements in the electron temperature calculation are planned. In
order to save computing time an artificial mass ratio $m_i/m_e = 100$ has been chosen which allows accordingly to use a time step ratio $\Delta t(ion)/\Delta t(e^-) = 10$. This erases microscopic movement like gyration or drifts. Since SmartPIC is a pure electrostatic simulation this is not seen to be a drawback. First tests with the new electron model put on some issues of stability due to numerical noise in electric fields. A method to cut down the field strength in a physically correct way is under investigation. Some aspects of this model might give rise to criticism but it has to be emphasized that in fact no existing model is capable of simulating an electron fluid throughout such a huge domain; this one is intended to be.

B. New model for floating potential calculation

Although there are many interesting approaches in the literature existing models are still based on assumptions. The availability of independent fluxes of electrons and ions in the new electron model will enable to calculate the floating potential in a different way. Returning to the very rudimental physics of spacecraft charging the basic assumption common to all usual models is conservation of currents. This, undoubtable, is a reasonable and valid basis for calculations. But conservation of currents is not the most basic principle in charging. It is the collection of discrete charges, causing electric potentials and, thus, react on the current. Hence the most accurate calculation of a floating potential should include the interaction of potentials and charges. From the theory of electromagnetic fields it is known that for a configuration of $N$ insulated conductors the following relations hold

$$Q_i = \sum_{j=1}^{N} C_{ij} \Phi_j$$

(11)

where $i$ and $j$ are indices of the conductors, $\Phi_i$ depicts the electric potential, and $C_{ij}$ are the electric capacity coefficients depending solely on the geometry of the configuration. If one adapts this picture for a satellite surrounded by plasma in fact the system can be seen as a badly insulated capacitor. The total space charge in the plasma is very low and only effective in the sheaths near surfaces. Hence for a first approximation one can assume the spacecraft to be insulated and floating in vacuum. For such a single conductor Equation (11) simplifies to $Q = C\Phi_{SC}$. The difficulty now is the determination of the exact capacity $C$ of the spacecraft which cannot be obtained analytically. A possible solution is provided by the multigrid solver. If one assumes the spacecraft as just a vacuum filled conductive hull of equal geometry and places a known uniform charge density on the surface one is able to calculate the internal potential for which Faraday states that...
This scheme has already been implemented and results for Smart–1 in a capacity of $1.7368 \times 10^{-10}$ F against vacuum. Bearing in mind that the expected total current from ambient plasma obtained by Equations (5), and (6) for a density $10^8$ m$^{-2}$ and uniform temperature of 0.6 eV initially is of the order $-10^{-5}$ A. Assuming a transient of 0.1 ms one obtains for the spacecraft potential $\Phi_{SC} = -5.8$ V. This correlates excellent with theory and measurements for floating potentials on spacecraft.

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**References**


