

Initiation of the Numerical Investigation of the Hybrid Plasma Thruster TIHTUS

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The two-staged hybrid plasma thruster TIHTUS consists of an arcjet as the first stage and an inductively coupled plasma generator as the second stage. In order to numerically simulate the thruster, a discharge solver has to be implemented in a simulation code. The existing models for the inductive discharge use the magnetic vector potential to simplify Maxwell's equations. These models neglect the Hall effect in Ohm's law. When the Hall effect is considered, the resulting discharge equation is not independent in time and therefore much more complex. Hence, the dimension of the Hall current density is roughly estimated by an analytical model of the radial distribution of the magnetic field. The results show that the Hall effect should not be neglected and has to be further investigated.

Nomenclature

a	coefficient
\vec{A}	magnetic vector potential
\vec{B}	magnetic induction
\vec{E}	electric field
F	function
G	function
\vec{H}	magnetic field
I	current
J_0	Bessel-function
\vec{j}	current density
l_{coil}	length of the coil
n	number density
n_{coil}	turns of the coil
R	radius of the coil
r	radial coordinate
t	time

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\vec{v} plasma velocity
 V_j ratio of current densities
 z axial coordinate

Subscripts

coil induction coil
e electrons
eff effective
h hall effect
i ions

Symbols

β hall parameter
 ϵ_0 electric permittivity
 μ_0 magnetic permeability
 ω frequency
 ω_e Larmor frequency of electrons
 ρ charge density
 σ electric conductivity
 τ_e mean collision time of electrons
 θ azimuthal coordinate

HERTA High Enthalpy Radiation Transport Algorithm

HIPARC High Power ARCjet

IPG Inductively heated Plasma Generator

IRS Institut für Raumfahrtssysteme (Institute of Space Systems)

SINA Sequential Iterative Non-equilibrium Algorithm

TIHTUS Thermal-Inductive Hybrid Truster of the Universität Stuttgart

I. Introduction

At the Institute of Space Systems (IRS), a hybrid plasma thruster is currently under experimental¹⁻³ and numerical⁴ investigation. TIHTUS (Thermal-Inductive Heated Thruster of the Universität Stuttgart) is two-staged and consists of an arcjet as the first stage and an inductively heated plasma generator (IPG) as the second stage. The plasma plume downstream the arcjet has high temperature and velocity in the core and lower energies in the surrounding flow. To increase the energy in the plasma flow, the second stage inductively couples energy into the colder region of the plasma near the wall. For better understanding of the interaction of the various physical phenomena and for a comparison of the experimental results, numerical investigations of the thruster are needed. After the validation of the numerical code, efforts in optimizing thrust and efficiency of TIHTUS are intended.

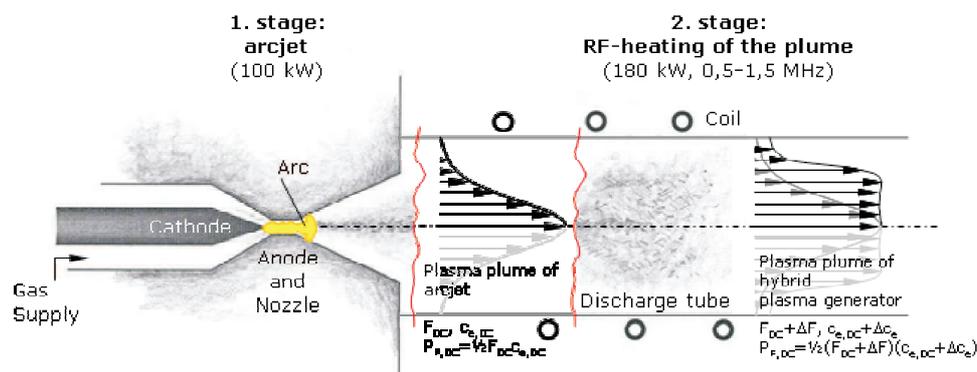


Figure 1. Principle of TIHTUS

Figure 1 shows the principle of the hybrid thruster. The first stage of TIHTUS is a high power arcjet of the 100 kW class, that is called HIPARC. The plasma flow of the first stage is expanded in a nozzle before entering the second stage. The outflow of the first stage has a fast and hot core with steep gradients to the outer regions, as already mentioned. The plasma then enters the second stage, an IPG, where an additional mass flow can be added. Due to the skin effect, mostly the colder regions of the flow near the wall are heated by the inductive discharge. This increases the energy in the plasma and hence the specific impulse and thrust. Moreover, the radial profile of the plasma flow is made more homogeneous by the second stage, which may be interesting for plasma technology processes. For optimization of the thruster there are several interesting possibilities. One is the electric input power of the single stages and their ratio to each other. The geometry of the thruster and its single stages will also be of interest as well as the frequency of the inductive discharge, which is of course depending on the geometry of the coil. This leads to a large amount of experiments needed for an optimization of TIHTUS. Some of these experimental optimizations, especially when the effect of different geometries is examined, take a lot of time and money. This effort is intended to be reduced by numerical simulations.

II. The program system SINA

For the numerical simulation of TIHTUS the program system SINA (Sequential Iterative Non-equilibrium Algorithm) is used. It was developed in order to numerically simulate the complex thermal and chemical phenomena in the plasma wind tunnel facilities at the IRS.⁵ It can also be applied to axisymmetric plasma sources or thrusters with an electric arc. SINA consists of three different semi-implicit and explicit independent solvers which are loosely coupled, as shown in Fig. 2. It is also possible to additionally use the (external) radiation solver HERTA⁵ (High Enthalpy Radiation Transport Algorithm), which computes the radiation transport in the flow field.⁶

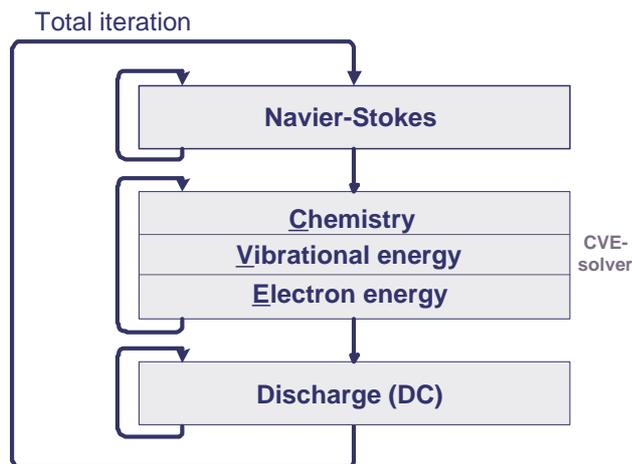


Figure 2. Iteration scheme of SINA with the three different solvers

The flow solver (Navier-Stokes solver) accounts for mass, momentum and total energy conservation. The usage of multi-block grids enables SINA to deal with complex geometries although structured grids are used. SINA also exists in a version, which runs on parallel computers to save time.

The second solver is called CVE solver and determines the **C**hemical composition and solves the conservation equations for **V**ibrational energy and **E**lectron energy in addition. It employs complex thermo-chemical relaxation models for different gases. In order to cope with the conditions in plasma wind tunnels, the transport properties are calculated for partially ionized air in chemical and thermal non-equilibrium employing an 11-component air model. The verification of the modelling of the complex thermal and chemical phenomena was performed by comparison with experimental data for air as flow medium.

The third solver (discharge solver) is used to solve the discharge equation in order to account for ohmic heating and magnetic acceleration in magnetoplasmadynamic plasma generators. At this time only a DC discharge of arcjets can be treated by the discharge solver. Thus, this part of the code has to be further

developed. The new discharge solver has to be able to solve both DC and RF discharge for a simulation of TIHTUS. Since the DC discharge is already implemented, the RF discharge is the actual field of research.

III. Consideration of inductive heating

The energy is coupled inductively into the plasma at the second stage of TIHTUS. The RF fields of the coil induce an azimuthal electric field. This field causes an azimuthal current, that heats the plasma. Due to the skin effect, this heating takes place in an annular zone near the coil. It is important to know the RF fields and the current density in the plasma in order to get the source terms of the ohmic heating $\vec{j}\vec{E}$ and Lorentz force $\vec{j} \times \vec{B}$ for the flow field solver. Several similar models for the inductive discharge in IPGs are already presented in literature.⁷⁻⁹

A. Existing models

The governing equations for electromagnetic fields are Maxwell's equations

$$\nabla \times \vec{B} = \mu_0 \left(\vec{j} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right), \quad \text{Ampère's law,} \quad (1)$$

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}, \quad \text{Faraday's law of induction,} \quad (2)$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}, \quad \text{Gauss' law,} \quad (3)$$

$$\nabla \cdot \vec{B} = 0, \quad \text{absence of magnetic monopoles.} \quad (4)$$

Additionally, Ohm's law

$$\vec{j} = \sigma(\vec{E} + \vec{v} \times \vec{B}) - \frac{\omega_e \tau_e}{B} (\vec{j} \times \vec{B}) + \frac{\sigma}{en_e} \nabla p_e \quad (5)$$

is needed to close Maxwell's equations. Equation (5) is Ohm's law for the presence of a magnetic field.^{10,11} The first term on the right hand side is the current driven by the electric field and the induced electric field $\vec{E}' = \vec{v} \times \vec{B}$ by the motion of the plasma through the magnetic field. The second term is the Hall current density and the last is the current density caused by the electron pressure diffusion.

One of the first to derive a two-dimensional electromagnetic model was McKelliget.⁷ In all existing models quasineutral plasma ($n_e = n_i$) is assumed, which means that the charge density in Eq. (3) is $\rho = 0$. The displacement current ($\epsilon_0 \partial E / \partial t$) is negligible for $\sigma \epsilon \gg \omega_e$,¹¹ which is the case for the frequencies of IPGs. McKelliget uses the very simple form of Ohm's law $\vec{j} = \sigma \vec{E}$, which neglects the plasma movement, the Hall effect and electron pressure diffusion. To the knowledge of the authors, all of the existing models use the magnetic vector potential \vec{A} , which is defined by $\vec{B} = \nabla \times \vec{A}$, to simplify Maxwell's equations.

As an example, the derivation of the model of Sleziona⁸ is shown here, because this model uses the more detailed form of Ohm's law

$$\vec{j} = \sigma(\vec{E} + \vec{v} \times \vec{B}) \quad (6)$$

and it is used in the simulation code ARCHE,⁸ which was developed at the IRS to numerically simulate IPGs. Combining Ampère's law (Eq. 1) with the vector potential ansatz $\vec{B} = \nabla \times \vec{A}$ results in

$$\nabla \times (\nabla \times \vec{A}) = \mu_0 \vec{j} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2}, \quad (7)$$

which can be further simplified by using the Lorenz calibration

$$\nabla \cdot \vec{A} + \frac{1}{c^2} \frac{\partial}{\partial t} \Phi = 0. \quad (8)$$

Neglecting the electrostatic potential Φ leads to an inhomogeneous equation of waves

$$\mu_0 \vec{j} = \frac{1}{c^2} \frac{\partial^2 \vec{A}^c}{\partial t^2} - \nabla^2 \vec{A}. \quad (9)$$

In this equation, \vec{j} can be expressed by Ohm's law without the Hall term (Eq. (6)). The vector potential oscillates with the frequency ω and hence can be expressed as $\vec{A}(\vec{r}, t) = \vec{A}^c(\vec{r}) \cdot e^{i\omega t}$, where the superscript c denotes a complex vector. Then, Eq. (9) becomes a elliptical equation of the complex vector potential

$$\nabla^2 \vec{A}^c - i\omega\mu_0\sigma\vec{A}^c + \mu_0\sigma\vec{v} \times \nabla \times \vec{A}^c + \frac{\omega^2}{c^2} \vec{A}^c = 0. \quad (10)$$

Note that this equation is independant in time. The derivation in time of Eq. (9) is removed by derivating the exponential term of the complex vector potential. The exponential term itself, which represents the time dependancy, is then cancelled down in every term of the equation.

The electromagnetic field quantities are obtained from the solution of Eq. (10) by

$$\vec{E} = -i\omega\vec{A} \quad (11)$$

and

$$\vec{B} = \nabla \times \vec{A}. \quad (12)$$

The boundary conditions are derived from an analytical solution of Maxwell's equations.¹² Some newer models⁹ obtain the boundary values for the grid within the discharge tube from an extended grid outside the tube, where the vector potential equation is also solved. For this extended grid the boundary condition is simply $\vec{A} = 0$ at each boundary.

When trying to express \vec{j} in Eq. (9) by the detailed form of Ohm's law (Eq. (5)), including the Hall term, the time dependancy can not be fully removed. This is due to $\vec{j} \times \vec{B}$. Regarding only this term, using $\vec{B} = \nabla \times \vec{A} = \nabla \times (\vec{A}^c \cdot e^{i\omega t})$ and Ampère's law (Eq. (1)) we get

$$\vec{j} \times \vec{B} = \frac{1}{\mu_0} \nabla \times (\nabla \times \vec{A}) \times (\nabla \times \vec{A}) = e^{2i\omega t} \cdot \frac{1}{\mu_0} \nabla \times (\nabla \times \vec{A}^c) \times (\nabla \times \vec{A}^c). \quad (13)$$

For simplicity, the displacement current in Ampère's law is neglected here. The problem is that the exponential term appears squared in this equation, which shows that the Hall current density oscillates with the frequency 2ω . Hence it can not be fully cancelled down, as it is possible when deriving Eq. (10) without the Hall current density. This means that the vector potential ansatz does not lead to a time independant equation when regarding the Hall effect in Ohm's law.

B. Estimation of Hall current density

As mentioned above, all known models disregard the term for the Hall current density in Ohm's law without explanatory statement. Considering the Hall current density causes problems when deriving a discharge equation using the vector potential, as already shown. Hence the influence of the Hall effect on the current density in the plasma should be estimated in order to see whether it may be neglected or not.

For this estimation we use the model described by Petkow,^{13,14} which derives a radial distribution of the magnetic field $\vec{H}(r)$ from Helmholtz equations. Within this analytic model for a quasineutral plasma, a constant conductivity σ_{eff} is assumed in the simple version of Ohm's law

$$\vec{j} = \sigma_{eff} \vec{E}. \quad (14)$$

With $\epsilon_r = \mu_r = 1$, which is a good approximation for an inductively heated plasma,¹⁵ and using a harmonic description for waves $\vec{F} = F_0 \cdot e^{i(\omega t - \vec{k}\vec{r})}$, one obtains the vectorial Helmholtz equation

$$\Delta \vec{H} - i\mu_0\omega\sigma_{eff}\vec{H} + \epsilon_0\mu_0\omega^2\vec{H} = 0. \quad (15)$$

Note that the Helmholtz equation is independant in time. The second term ($-i\mu_0\omega\sigma_{eff}\vec{H}$) describes the inductive operation mode of the IPG, while $\epsilon_0\mu_0\omega^2\vec{H}$ describes the dielectric mode of operation. The latter can be neglected for the operation frequencies of the IPG.¹⁵ The Helmholtz equation for \vec{H} becomes

$$\Delta \vec{H} - i\mu_0\omega\sigma_{eff}\vec{H} = 0. \quad (16)$$

The equation for the electric field is derived analogously and is

$$\Delta \vec{E} - i\mu_0\omega\sigma_{eff}\vec{E} = 0. \quad (17)$$

The power induction from the coil to the plasma is described with the model of a current transformer. Assuming an infinitely long coil, the one-dimensional model leads to the assumption of an axial magnetic field with no radial or azimuthal component. It is constant in z - and θ -direction, but damped in r -direction due to the skin effect. For the radial damping of the RF fields,

$$\delta = \sqrt{\frac{2}{\mu_0\sigma_{eff}\omega}} = \frac{1}{\sqrt{\pi\mu_0\sigma_{eff}f}} \quad (18)$$

is the penetration depth of the electric field for the assumption of strong damping.¹⁶ Here, δ is the thickness of the annular zone, where 85% of the total power is coupled into the plasma. These assumptions and the substitution $\zeta = (-i)^{1/2} \cdot r/\delta$ lead to

$$\frac{d^2 H_z}{d\zeta^2} + \frac{1}{\zeta} \frac{dH_z}{d\zeta} + H_z = 0, \quad (19)$$

which has the form of a Bessel differential equation of the order of zero. The solution of the one-dimensional Helmholtz equation is

$$H_z(r) = H_{coil} \frac{J_0(ar)}{J_0(aR)} \quad (20)$$

with the Bessel function J_0 . This function can be splitted into the real and the imaginary part according to $J_0 = F + iG$, where the functions F and G are

$$F\left(\frac{\delta}{R}\right) = \text{Re}[J_0(a \cdot R)] = \sum_{m=0}^{\infty} \frac{(-1)^m}{2^{2m}[(2m)!]^2} \left(\frac{R}{\delta}\right)^{4m} \quad (21)$$

and

$$G\left(\frac{\delta}{R}\right) = \text{Im}[J_0(a \cdot R)] = - \sum_{m=1}^{\infty} \frac{(-1)^m 2^{2m}}{2[(2m-1)!]^2} \left(\frac{R}{2\delta}\right)^{4m-2}, \quad (22)$$

with $a = \zeta/r$. These equations are known as Kelvin functions. The independant variable is δ/R . When the radius of the discharge tube R is replaced by r , δ results from a given δ/R and a known R .

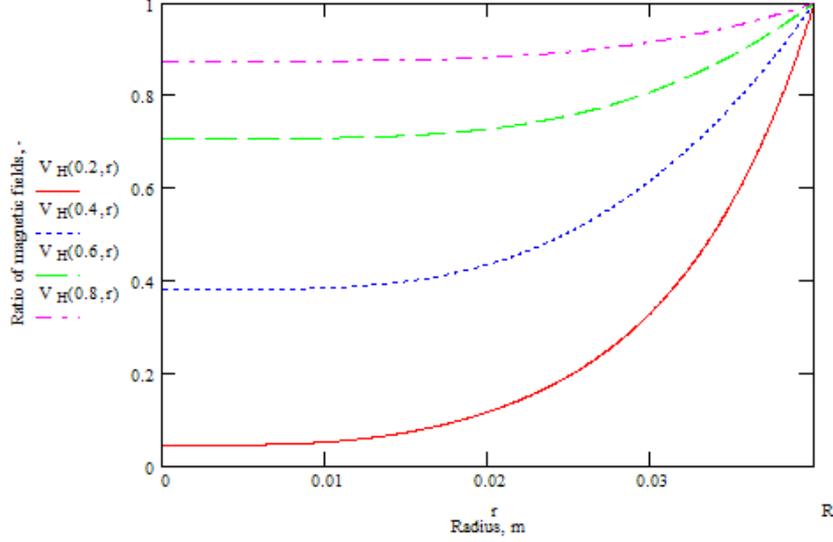


Figure 3. Radial plot of ratio V_H of the magnetic field in the plasma $|H_z|$ and the magnetic field of the coil H_{coil} for different ratios δ/R according to Petkow¹³

Finally, the ratio of the magnetic field $|H_z|$ and the magnetic field of the coil H_{coil} can be obtained from¹³

$$\frac{|H_z(r)|}{H_{coil}} = \frac{\sqrt{[F(r)F(\delta/R) + G(r)G(\delta/R)]^2 + [F(\delta/R)G(r) - F(r)G(\delta/R)]^2}}{F^2(\delta/R) + G^2(\delta/R)}, \quad (23)$$

which is shown in Fig. 3 for a varying ratio of δ/R . Assuming a long coil, the magnitude of the magnetic field $|H_z|$ in the plasma can be obtained by

$$|H_z(r)| = \frac{|H_z(r)|}{H_{coil}} \cdot I_{coil} \cdot \frac{n_{coil}}{l_{coil}}. \quad (24)$$

Here, I_{coil} is the current in the coil, n_{coil} is the number of turns and l_{coil} the length of the coil. This radial distribution of the magnetic field is obtained without considering the Hall effect, since for the derivation of the Helmholtz equation the simplified Ohm's law (Eq. (14)) was applied. Now, the current density caused by the Hall effect can be roughly estimated. The Hall current density is equal to the third term in Ohm's law (Eq. (5))

$$\vec{j}_h = -\frac{\omega_e \tau_e}{B} (\vec{j} \times \vec{B}), \quad (25)$$

The fraction of the electron cyclotron frequency ω_e and the mean collision time for the electrons τ_e divided by the magnetic flux density B can be shortened by

$$\frac{\omega_e \tau_e}{B} = \frac{eB \sigma m_e}{m_e n_e B} = \frac{\sigma}{en_e} = \beta, \quad (26)$$

where β is the Hall parameter. Assuming an azimuthal current density, an axial magnetic field and neglecting the negative direction, the Hall current density becomes

$$j_h = j\beta B \quad (27)$$

To estimate the influence of the Hall effect, the ratio of Hall current density to the azimuthal current density

$$V_h(r) = \frac{j_h(r)}{j(r)} = \beta B(r) = \beta \mu_0 \frac{|H_z(r)|}{H_{coil}} \cdot I_{coil} \cdot \frac{n_{coil}}{l_{coil}} \quad (28)$$

is of interest. The ratio $|H_z|/H_{coil}$ is determined from Eq. (23). For the solution of this equation, seven variables are needed: I_{coil} , f , n_{coil} , l_{coil} , R , σ and n_e . The first five can be taken from experiment, since they

are geometric values or operation parameters. The last two, namely the electric conductivity and the electron number density, are taken from numerical simulations.

For all parameters, the IPG3¹⁵ of the IRS will be the reference with a tube radius of $R = 0.04$ m and $n_{coil} = 3$ to 5 turns of the coil, which has a length of $l_{coil} = 0.15$ m. The operational frequency ranges from 0.5 MHz to 1.5 MHz and the current in the coil ranges from 200 A up to more than 600 A. The electric conductivity σ and the electron density n_e are not yet described by an approximating analytical expression, but results of the operating conditions. Numerical simulations show that σ ranges between 2500 S/m and 3500 S/m in the discharge region and is much lower in the core of the plasma flow. Depending on pressure and deviation from thermal equilibrium, these conductivities correspond roughly to temperatures of up to 12500 K.¹⁷ Electron density is less than $5 \cdot 10^{20}$ 1/m³.

Figure 4 shows the radial distribution of the ratio of the Hall current density and the azimuthal current density for moderate parameters: $I_{coil} = 300$ A, $n_{coil} = 3$, $l_{coil} = 0.15$ m, $R = 0.04$ m and $f = 700$ kHz. The resulting values for $\sigma = 3000$ 1/(Ω m) and $n_e = 3.5 \cdot 10^{20}$ 1/m³ within the discharge region near the wall are approximated as constant over r . According to Fig. 4, the Hall current density is above roughly 20% up to 40% of the azimuthal current density within the discharge region near the coil.

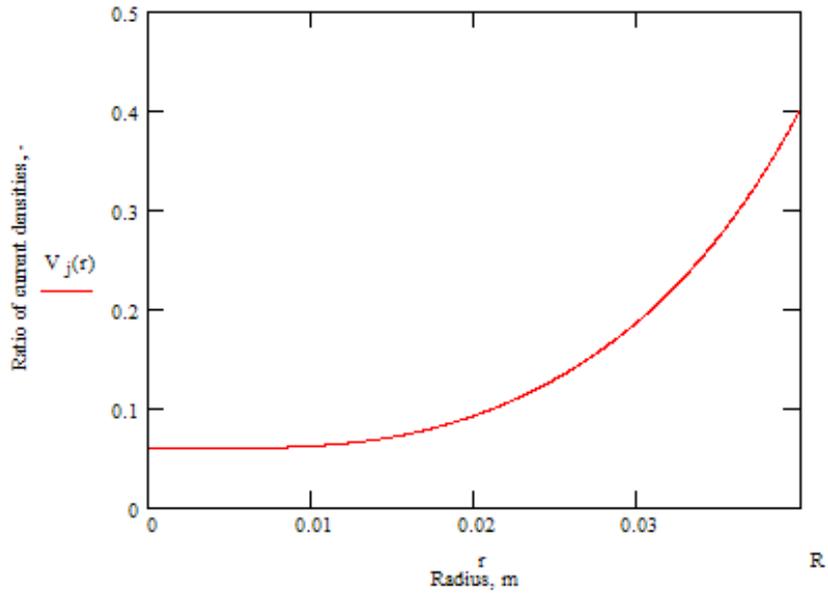


Figure 4. Radial plot of ratio V_j of hall current j_h and azimuthal current density j for moderate parameters

The current density ratio (Eq. (28)) is directly proportional to the number of turns and the current of the coil. It is reciprocally proportional to the length of the coil and the electron number density. In order to get an estimate of the minimum ratio of current densities, $n_{coil} = 3$ and $I_{coil} = 200$ A are set to their lower end values and $n_e = 5 \cdot 10^{20}$ 1/m³ to its maximum. The electric conductivity is set to $\sigma = 2500$ 1/(Ω m), which corresponds to a temperature of about 10000 K. The result is shown in Fig. 5. The Hall current density is still more than 5% to 15% of the azimuthal current density in the discharge region.

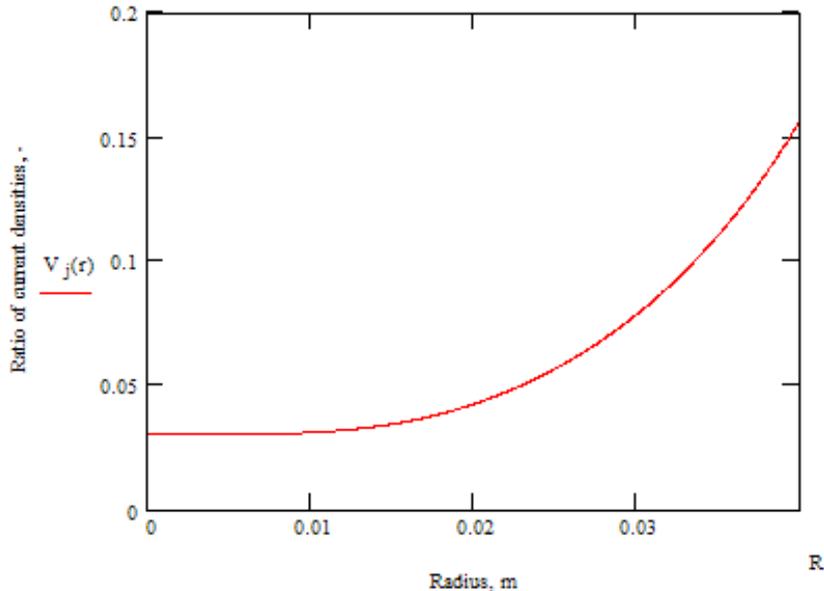


Figure 5. Radial plot of ratio V_j of Hall current j_h and azimuthal current density j for conservative parameters

This rough estimation of the Hall current density in the IPG shows that it is questionable to neglect the Hall effect in the modeling of the RF discharge, at least when the operating conditions are within the previously given range. Looking at Eq. (28), one can see that this statement strongly depends on the coil current and the number of turns of the coil as primary parameters and the electron number density and electric conductivity as resulting variables. The frequency does not change the maximum of the current density ratio, but the minimum (and the slope of the curve), because of a stronger damping of the fields with higher frequency, resulting in a thinner discharge region δ . Note that the Hall current density was not considered within the derivation of the magnetic field distribution, but estimated afterwards. The results show that a more detailed investigation of the Hall effect in inductively coupled plasmagenerators has to be performed.

IV. Summary and Outlook

The modeling of the discharge in the second stage of the two-staged hybrid plasma thruster TIHTUS was discussed above. The existing models for an inductive discharge use the magnetic vector potential to reduce Maxwell's equations. This method, however, does not lead to a time independent equation, when considering the Hall current density in Ohm's law. In order to estimate the proportion of the Hall current density in comparison to the azimuthal current density, a 1D analytic model with radial damping of the electromagnetic fields was used. The results show that the Hall term in Ohm's law may not be neglected for the typical operating conditions of the inductive plasma generator IPG at the IRS.

Due to the fact that the Hall current density in the discharge region may not be neglected, most probably a new discharge solver has to be developed for SINA in order to numerically simulate the hybrid thruster TIHTUS. This solver has to be able to handle both the discharge of the arcjet and the inductive discharge of the second stage, most probably regarding the Hall effect.

Since TIHTUS uses hydrogen as propellant, the module for the computation of the plasmas chemical composition has to be extended. In this process the chemistry module is intended to be converted into a more general formulation which allows an easy adaption to other gases than air and hydrogen.

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