

Investigation of integral characteristics of the low power Hall Thruster for the spacecraft orientation propulsion system during transition to the stationary operation mode after thruster start up

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Abstract: To find ways to satisfy some technical requirements showed to the Hall thruster (HT) for the spacecraft orientation propulsion system (in particular, small transitional time after start) theoretical researches of transient of HT integral characteristics after start were carried out. Processes in elements of the thruster magnetic system, which substantially limit duration of transition to a stationary operation mode have been in detail investigated. Processes in HT discharge interval during transition were modelled and HT integral parameters (thrust, specific impulse and HT efficiency) were calculated.

Nomenclature

B	=	magnetic induction
d	=	thickness
i	=	current
I_{sp}	=	specific impulse
F_T	=	thrust
H	=	magnetic field strength line
L, l	=	length
R, r	=	ohm resistance
t	=	time
U	=	discharge voltage
V	=	velocity
W	=	capacity
ε	=	voltage
Φ	=	magnetic induction flow
η_T	=	thrust efficiency

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I. Introduction

For many years after the first successful flight, Hall thruster (HT) or stationary plasma thruster – SPT is used in structure of propulsion system of spacecraft (SC) for maintenance of SC orbit. Now the opportunity of low power HT application in SC propulsion system is studied. One of the requirements showed to such thruster - small time of transition in a stationary operation mode after start up, which is determined by time of transition processes in rather independent elements of HT: cathode-neutralizer (CN), magnetic system, a discharge interval between CN and the anode. Some results of researches of the transient processes, carried out both in separate elements of the thruster and at their summary account (in their interrelation)¹ are known. To find ways to overcome of inertness of some processes, it is interesting to study in more detail features of transition of these processes since the moment of initiation of the basic discharge - in elements of magnetic system and in HT discharge interval.

II. Problems of processes researching, during HT transitive operation mode

In various Refs.^{2, 3, 4, 5} separate transitional processes of HT operation were investigated and the submitted results can be used for the analysis of features of the HT operation as a whole. The analysis of results of Ref.^{2, 5} specifies that by start it is probable short-term "inrush" of discharge current up to sizes, which ten times and more surpass value of a discharge current in a stationary mode. The nature of this phenomenon is not investigated completely, but the phenomenon can certainly influence on course of transients in HT further. The analysis of Ref.³ results that transients in the cathode-neutralizer are limited to the period duration about 10^{-6} s. Thus, these processes are allowable to take as "instant", comparison to processes in the anode block of the thruster. In Ref.⁴ duration of dissipation of charges in plasma in the discharge chamber (DC) after discharge interruption is estimated as 1-10 mks, and duration of a current increasing in a discharge circuit of the anode and the cathode is determined up to 100 mks. On the base of these data it was supposed that asynchronous processes in a discharge interval - no more than 100 mks. As a whole, the data submitted in these papers have been used to study processes during HT transitive operation mode. The following problems are allocated to detail research. 1) Selection of processes and HT elements, which determine HT transition time to a stationary operation mode. 2) Study of transients in HT magnetic conductor and in an electric circuit of magnetic coils since the moment of start. 3) Study of influence of transition processes in magnetic system on HT integral characteristic.

III. Research of transients in HT discharge interval

A. Periods of transient

Experimental data of transient processes in an HT electric circuit after start (Fig. 1) are shown as dependences:

$U_d(t)$ - voltage between the emitter of cathode-neutralizer and the anode, $I_d(t)$ - a discharge current. Dependences were determined under various conditions: 1) magnetic coils are included in a discharge circuit; 2) magnetic coils feed by a separate source before discharge ignition. It is established experimentally, that for these two various conditions discharge current $I_d(t)$ and voltage $U_d(t)$ in magnitude do not differ. The analysis of data submitted in Fig. 1, allows making the next conclusions. During the period (0 - a) there is an initiation of the discharge and during the period (0 - b) when the discharge voltage is close to zero or is negative, thrust can be neglect. During the period (b - c) when the discharge voltage leaves in a nominal mode - there is an increase of the thrust.

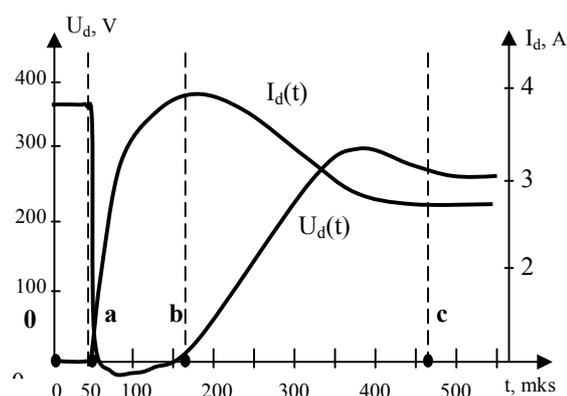


Figure 1 Transient V-A characteristic.

Periods:

0-a - discharge initiation;

a - b - there is no thrust;

b - c - transient in the stationary mode (increasing of $U_d(t)$ and $F_T(t)$).

B. Particles movement in HT discharge interval during increasing of discharge voltage

It is assumed that during time (b - c) the basic processes in HT discharge interval and integral characteristics can be calculated with sufficient accuracy on the mathematical model, developed on the base of the analysis of processes in a stationary operation mode⁶.

It is supposed that measured value $I_d(t)$ includes:

- 1) A flow of electrons from CN, which goes in HT DC;
- 2) A flow of electrons emitted by CN, which goes for ion charge compensation, formed after the moment **b** and moving already outside of DC;
- 3) A flow of electrons emitted CN, which goes for ion charge compensation, formed till the moment **b** and moving already outside of DC. This flow is determined by processes during the period (**0 - a**) and (**a - b**) and not studied in these researches.

To analyze processes the discharge interval in DC is divided into: a zone of ionization of plasma-making gas (it is located in DC closer to the anode) and a zone of ion acceleration (it is located in DC closer to DC edge).

It is supposed that during the period (**b - c**) dependence $U_d(t)$ is determined by the characteristic of the power supply source.

It is supposed that: change of a voltage is a discrete process; while U_d change on some value ΔU_d there is "delay" (backlog in time from new $U_d(t)=U_d+\Delta U_d$) of electrons acting in a zone of ionization (ZI) from DC output for a time $\tau_{de} \approx L_{ZI}/V_{ex} \approx 10^{-2}/2 \cdot 10^3 = 5 \cdot 10^{-6}$ s, where: $L_{ZI} \approx 10^{-2}$ m - extent of a zone of acceleration (ZA); $V_{ex} \sim U_d$, $V_{ex} \approx 2 \cdot 10^3$ m/s - longitudinal velocity of electron movement in ZA for $U_d \geq 100$ V. Process of atom concentration heterogeneities alignment longwise DC (formed because of ion recombination on DC walls) also "delay" from growth of $U_d(t)$. Time of backlog is determined by velocity V_a of atom movement. In view of the factor of ion energy accommodation - $k_a = 0.7-0.8$, after recombination of ions on DC surface, velocity of formed atoms is estimated as $V_a \approx 2 \cdot 10^3$ than $U_d \geq 100$ V, and time of "delay" for atoms $\tau_{da} \approx L_{ZI}/V_a = 10^{-2}/2 \cdot 10^3 = 5 \cdot 10^{-6}$ s, and $V_a \sim (\epsilon_i)^{0.5} \sim (U_d)^{0.5}$.

For ions, moving in ZA to an DC output, time of "delay" τ_{di} (in relation to growth $U_d(t)$) is less than for processes, considered before - $\tau_{di} \approx L_{ZA}/V_i \approx 2 \cdot 10^{-6}$ s, where $V_i \approx 5 \cdot (10^3-10^4)$ m/s - longitudinal velocity of ion in ZA.

Taking into account that: 1) as τ_{di} , τ_{de} , $\tau_{da} \sim 1/(U_d)^{0.5 \dots 1}$ the increasing of $U_d(t)$ do not lead to the strengthening of "delay" effects on process of movement of the charged particles and atoms in ZA; 2) as duration of the transient period (**b - c**) is 200-300 mks and $\tau_{de} \approx 5 \cdot 10^{-6}$ s it is allowable to suppose (as a first approximation) that $I_d(t)$ reflects during each moment of time the established process in HT discharge interval with "delay" in no more than $5 \cdot 10^{-6}$ s.

Thus, at this investigation phase:

- By worked out of mathematical model of processes during the period (**b - c**) (Fig. 1) dependence $U_d(t)$ was taken as known;

- As a first approximation it is neglect the effects caused by "delay" of process of particle redistribution in DC on increasing of $U_d(t)$, at least, than $U_d \geq 100$ V.

IV. Interrelation of transients in magnetic system and HT discharge interval

Among the processes occurring in HT in a transition period are allocated: - processes in magnetic system, including electric circuit and magneto-conductor; - processes of particles movement in a discharge interval. The aprioristic estimation of duration of transient in magnetic system gives values 1-10 ms. Time of probable "delay" in transporting of charges through a discharge interval - about 10^{-5} s. It is supposed that in a transition period reduction of atom concentration in DC "instantly" follows change in electron distribution. Thus, the most inert transitional processes occur in HT magnetic system and discharge current $I_d(t)$ reflects changes in a discharge interval with "delay" about 10^{-5} s. Further, supposing discrete character of discharge voltage process increasing, for calculation of processes in a HT discharge interval, as the base model, the mathematical model of ionization and acceleration of plasma-making gas in HT (HMT) developed (item 6) for a stationary mode has been used.

Analysis of transients processes in HT magnetic system has been resulted as function $i(t)$ - a current in the magnetic coil and $B_{ed}(t)$ - an induction of a magnetic field on a median surface of DC edge. They were separated from processes in a discharge interval. To find these functions magnetic coils were assumed to feed by the independent power supply source.

V. Research of transients in the HT magnetic system, determining value of a magnetic field in the discharge chamber

For research of transients in HT magnetic system the mathematical model created for a long time to study processes in the separate magnetic coil with core (MSC) has been used. MSC developed on the base of the equivalent scheme of an electric circuit (Fig. 2) of the magnetic coil (MC) include: a) the power supply with electro-moving force (EMF) - ϵ_{sou} (it is determined by a voltage of idling U_{idl} and resistance r_{sou}); b) ohm resistance R_{dv} ; c) inductive element L, which was replaced by active resistance r_1 and source EMF - ϵ_{self} (directed opposite ϵ_{sou} when the process after turn-in a circuit by key K is studied). The equation of voltage balance includes (Fig. 2, 3)

$$\varepsilon_{\text{sou}} = U_{\text{rb}} + \varepsilon_{\text{self}} + U_{\text{dv}}, \quad (1)$$

where separate composes was determined as follows: $\varepsilon_{\text{sou}} = U_{\text{idl}} \cdot i \cdot r_{\text{sou}}$; $U_{\text{rb}} = i \cdot r_{\text{b}}$; $U_{\text{dv}} = i \cdot R_{\text{dv}}$; $\varepsilon_{\text{self}} = \frac{d(i \cdot L)}{dt}$.

Resulting EMF circuits

$$\varepsilon = \varepsilon_{\text{sou}} - \varepsilon_{\text{self}}. \quad (2)$$

Inductance of the coil was determined as

$$L = W_{\text{MF}} \cdot 2 / i^2, \quad (3)$$

Where $W_{\text{MF}} = B_{\Sigma} \cdot H_{\Sigma} \cdot V_c / 2 = \Phi \cdot i$ - energy of a magnetic field in volume of magneto-conductor (solenoid) $V_c = l \cdot \pi \cdot r^2$. H_{Σ} and B_{Σ} - resulting intensity of a magnetic field and a magnetic induction are connected by dependence $B_{\Sigma} = f(H_{\Sigma})$ for a concrete material; Φ - a flow of magnetic induction B_{Σ} through the coil; H_{Σ} - is taken as the resulting intensity of a field, created by currents; i_i - a current in a superficial layer (thickness d) of the core, proceeding under action of EMF ε_i ; i - a current in coils of the coil, induced under action resulting EMF ε . Thus

$$H_{\Sigma} = H_p - H_i, \quad (4)$$

where separate composes was determined as: $H_i = \frac{i_i}{2 \cdot r}$

$H_p = \frac{N \cdot i}{l} \cdot K_b$, and K_b - function of radius of coils.

$$i = \frac{\varepsilon}{r_{\text{sou}} + r_{\text{b}} + R_{\text{dv}}}, \quad d = R - r, \quad r_{\text{b}} = \frac{\rho_{\text{Cu}} \cdot N \cdot \pi \cdot R}{D_w^2 \cdot \pi / 4}; \quad N -$$

number of coils; d - thickness of a layer in which the current i_i is concentrated; D_w - diameter of a wire; R - diameter of coils of the coil; $\rho_{\text{Cu}} = 1.7 \cdot 10^{-8}$ Om·m - specific electric resistance of a material of a wire; l - it is long of coils; R_{dv} - active resistance; K_b - the factor which is taking into account a ratio of length and diameter of the solenoid. The valid distribution of a current density on thickness of the core, close to represented in Fig. 4, was replaced with distribution of the current concentrated in a layer by thickness d .

EMF ε_i , generated in the surface core layer (thickness d) of the coil

$$\varepsilon_i = d\Phi_{\Sigma} / dt, \quad (5)$$

where $\Phi_{\Sigma} = B_{\Sigma} \cdot \pi \cdot r^2 = f(H_{\Sigma}) \cdot \pi \cdot r^2$ - a flow of a magnetic induction B . A current in a layer d , arising under action ε_i , was determine as $i_i = \frac{\varepsilon_i}{R_{\text{layer}}}$, where $R_{\text{layer}} = \frac{\rho_{\text{Fe}} \cdot 2 \cdot \pi \cdot r}{l \cdot d}$ -

resistance to a flow of a current i_i , ρ_{Fe} - specific electric resistance of a material of the core (ferromagnetic).

The capacity, spent on heating of a layer of the core, which has magnetic permeability μ and in which the current i_i is flow calculated by

$$\varepsilon_i \cdot i_i = \varepsilon_{\text{sou}} \cdot i - i^2 \cdot (R_{\text{dv}} + r_{\text{b}} + r_{\text{sou}}) - \frac{d(W_{\text{MF}} / \mu)}{dt}. \quad (6)$$

The above-stated Eqs. (1 - 6) are solved in common. While solving it is necessary to choose those values of a current i (decisions of the equations), which give the greatest increasing of entropy of the system.

By calculation on system of the Eqs. (1 - 6) capacity balance was randomly inspected.

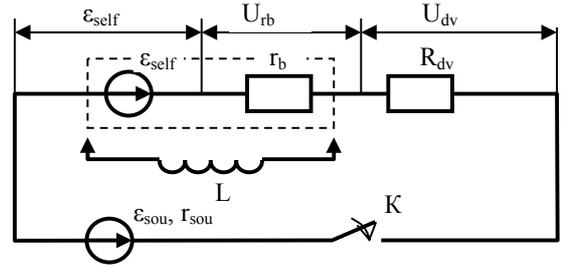


Figure 2. Equivalent schema of electric circuit of magnetic coil.

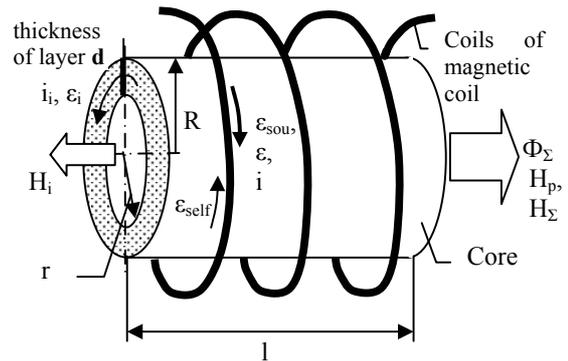


Figure 3. Schema of EMF directions, currents and magnetic flows in magnetic coil.

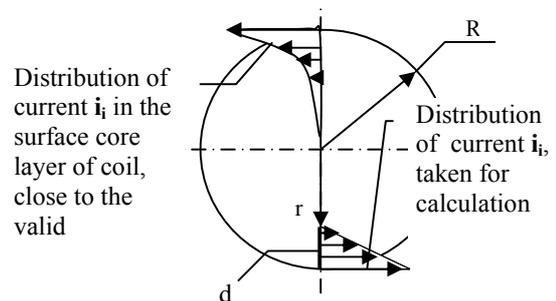


Figure 4. Distribution of current i_i , in the surface layer (thickness d) core of coil.

Checking is carried out according to the requirement of balance Eq. (6) under conditions: $r_{\text{sou}}=0$, $U_{\text{idl}}=\varepsilon_{\text{sou}}=1$ V, $r_b=0.07$ Ohm, $R_{\text{dv}}=0.22$ Ohm, the core with radius $R=0.005$ m, $N=30$ coils (in Fig. 2 experimental results and results of calculations of a current in an electric circuit of the magnetic coil are resulted). Ohm resistance of a layer to a current i_i is $R_{\text{layer}} = \frac{\rho_{\text{Fe}} \cdot 2 \cdot \pi \cdot r}{l \cdot d}$.

The balance of capacity Eq. (6) was used for check as

$$\varepsilon_{\text{sou}} \cdot i = R_{\text{layer}} \cdot i_1^2 + i^2 \cdot (R_{\text{dv}} + r_b + r_{\text{sou}}) + \frac{d(W_{\text{MF}}/\mu)}{dt} \quad (7)$$

The data for checking calculation chosen in various moments of time, are collected in Table 1.

Table 1

d(mm)	i(A)	t(mks)	μ	$\Phi=B \cdot S(\text{wb})$	B(mTЛ)	$i_i(\text{A})$
0.0043	2.92	500	516	0.89E-04	1135.	1.8
0.0036	2.94	520	508	0.10E-03	1274.	1.6

For example, for time $t=520$ mks balance Eq. (7) is thru $2.94 \approx 0.43 + 2.51 + 0.01 = 2.95$ with a relative error within the limits of 1%.

Using described above MSC it has been investigated in details: - generating and change of the current in a surface layer of an elements of magneto-conductor under action of the changing magnetic field; allocation of joule heat because of this current; the phenomenon of magneto-conductor saturation; generating and changing electro moving force in coils of the magnetic coil under action of a changing magnetic field. To check MSC have been calculated and compared to experimental results dependence of a current increasing in the separate coil since the moment of turn-in the electric circuit (Fig. 2) by key K. Similar dependence has been calculated under the well-known formula taken as a constant inductance of the coil $i_{\text{form}} = U/r \cdot (1 - \exp(-r/L \cdot t))$, where r - ohm resistance of an element of a circuit, U - a voltage of the power supply source, t - time, $L \approx \text{const}$ - inductance of the magnetic coil. Result of the analysis of experimental and calculated results (Fig. 4) shows their qualitative conformity that confirms adequacy of the description of investigated processes by MSC.

Further, being based on MSC, the mathematical model of transient in HT magnetic system (MTS) (Fig. 5) has been developed supposed next. Because the surface layer (in which the inductive current is flow) both in polar tips and in a back wall of magneto conductor is extended several times more than this layer in cores of peripheral coils and in central core, losses of energy caused by joule heating of metal body of these element by an induction current was neglected. Thus, energy losses of in a layer with resistance $R_{\text{layer,C}}$ of the central core, through which passes a flow Φ_c of induction B, were determined as $W_{\text{loss,C}} = (d\Phi_c / dt)^2 / R_{\text{layer,C}}$ and is similar - for peripheral cores. By MTS, change of an induction B of a magnetic field on DC edge of HT (Fig. 6) was calculated. Than $i(t) \rightarrow i_{\text{max}}$ and $\Phi(t) \rightarrow \Phi_{\text{max}}$ the induction of a magnetic field $B(t) \rightarrow B_{\text{max}}$ on a median line in an interpolar interval also reaches value 16 - 18 mTl. The experimental estimation of transitional duration of a current $i(t) \rightarrow i_{\text{max}}$ gives value $\approx (1-1.3) \cdot 10^{-3}$ s, that meets to calculate value.

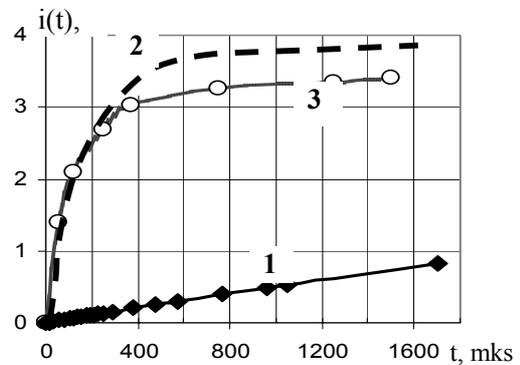


Figure 4. Evolution in time of the magnetic coil current:

- 1 - $i_{\text{form}}(t)$ calculated on the widely known formula (than $L=\text{const}$);
- 2 - $i_{\text{mod}}(t)$ calculated on the mathematical model
- 3 - $i_{\text{exper}}(t)$ determined experimentally.

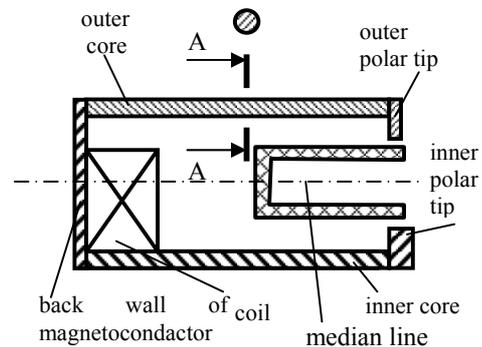


Figure 5. Schema of SPT magnetic system.

VI. 1 D model of ionization and acceleration of plasmamaking gas in HT discharge interval

The general principles of hydrodynamic model development for HT local and integral characteristics calculation (HMT) are stated in Ref.⁶ By development of model some analytical dependences was determine: a) parameters of process of electron transport in an interval from the cathode up to an entrance in DC - velocity of electron axial movement and shares of electrons (emitted by the cathode) which will penetrate in DC; b) negative potential drop in area near to the anode, where return movement of ions to the anode is probably. To check up HMT thrust F_T and discharge current I_d specific impulse I_{sp} and total thrust efficiency η_T for the HT (SPT M-70 type) have been calculated on HMT and compared to the experimental result in range of discharge voltage up to 300 V. Results show it qualitative conformity.

VII. Calculation of HT integral characteristics in a transition period after HT starts up

To calculate time dependences of thrust $F_T(t)$, specific impulse $I_{sp}(t)$, thrust efficiency $\eta_T(t)$ and discharge current $I_d(t)$ the following technique has been used. By mathematical model of HT magnetic system - MTS (item V), setting volt-ampere characteristic of the power supply of the magnetic coil and the characteristic geometrical sizes of HT magnetic system were calculated dependences $B_{r,mod}(t)$ and $i_{mod}(t)$ (Fig. 6). For this calculation voltage of the power supply source, which provides a necessary current and a magnetic induction B at the DC edge median line in value $B=16-17$ mTl was set. Using model of ionization and acceleration - HMT (item VI) and certain dependences $B_{r,mod}(t)$, setting mass flow rate through the anode block and the cathode, value of a discharge voltage and the characteristic sizes of the thruster dependences $F_T(t)$, $I_{sp}(t)$, $\eta_T(t)$, $I_d(t)$ (Fig. 7, 8) are determined. Calculations were carried out under condition that power supplies of the basic discharge (stabilized voltage $U_d=315$ V) and magnetic system of the thruster were turn-in simultaneously.

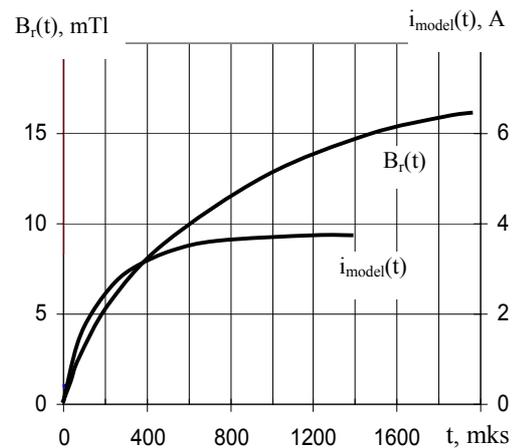


Figure 6. Evolution in time t of the magnetic field radial component $B_r(t)$ at the edge of DC and coil current $i_{mod}(t)$, calculated on MTS.

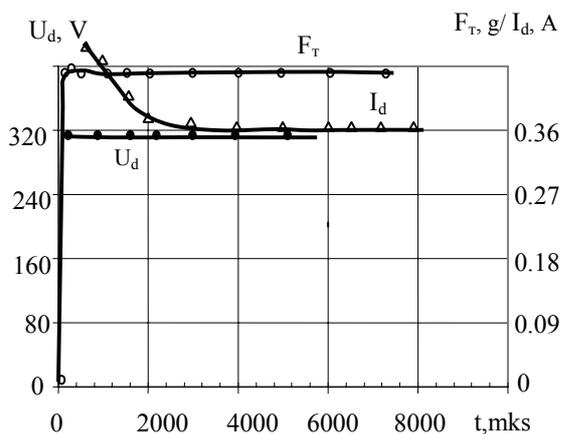


Figure 7. Evolution in time t of thrust $F_T(t)$ and discharge current $I_d(t)$ of the low power HT after start, calculated on HMT while discharge voltage is U_d .

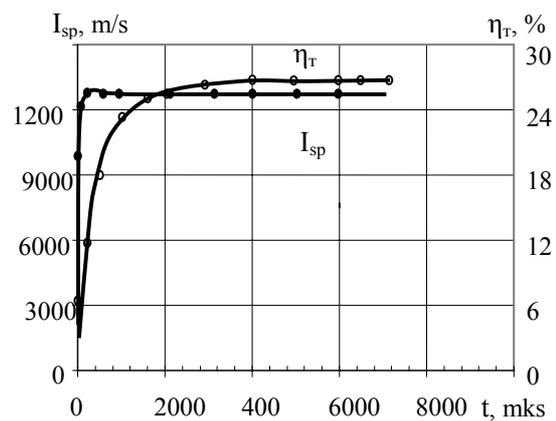


Figure 8. Evolution in time t of specific impulse $I_{sp}(t)$ and total thrust efficiency $\eta_T(t)$ of the low power HT after start, calculated on HMT.

VIII. Conclusion

1) After initiation of the HT discharge, transition to a mode of relative stabilized thrust is possible in time no more than 1.5 milliseconds. Transition in a mode of the stabilized thrust efficiency is possible in time within 2.5 milliseconds. Further, in a case of a “cold” start, a stabilization of HT integral characteristics will be after stabilization of temperature of a HT design elements within approximately 1 hour.

2) On the base of results analysis the following directions of research and design works for HT dynamic characteristics increasing were allocated. To create conditions for fast increasing of a magnetic field (that determines efficiency of ionization processes of a plasmamaking gas) it is necessary to design elements of magnetic system having the big surface and small thickness, and applying magnetic metal having big specific electric resistance - to reduce influence of a current induced in cores, which limits growth of a magnetic field. Besides it is possible to start the thruster then the initial magnetic field (magnetic system is turn-on) induction B does not exceed some threshold value.

Acknowledgments

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