Electron Kinetic Effects and Beam-Related Instabilities in Hall Thrusters

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Recent analytical studies and particle-in-cell simulations suggested that the electron velocity distribution function in a Hall thruster plasma is non-Maxwellian and anisotropic. The electron average kinetic energy in the direction parallel to walls is several times larger than the electron average kinetic energy in direction normal to the walls. Electrons are stratified into several groups depending on their origin (e.g., plasma discharge volume or thruster channel walls) and confinement (e.g., lost on the walls or trapped in the plasma). Practical analytical formulas are derived for wall fluxes, secondary electron fluxes, plasma parameters and conductivity. The calculations based on analytical formulas agree well with the results of numerical simulations. The self-consistent analysis demonstrates that elastic electron scattering on collisions with atoms and ions plays a key role in formation of the electron velocity distribution function and plasma-wall interaction. The fluxes of electrons from the plasma bulk are shown to be proportional to the rate of scattering to loss cone, thus collision frequency determines the wall potential and secondary electron fluxes. Secondary electron emission from the walls is shown to enhance the electron conductivity across the magnetic field, while having almost no effect on insulating properties of the near-wall sheaths. Such a self-consistent decoupling between secondary electron emission effects on electron energy losses and electron crossed-field transport is currently not captured by the existing fluid and hybrid models of the Hall thrusters. Electron emission from discharge chamber walls or cathodes is important for plasma maintenance in many thrusters. The electrons emitted from surfaces are accelerated by the sheath electric field and are injected into the plasma as an electron beam. Penetration of this beam through the plasma is a subject of the two-stream instability, which tends to slow down the beam electrons and heat the plasma electrons. The two-stream instability occurs if the total electron velocity distribution function of the plasma-beam system is a non-monotonic function of electron energy. For correct description of the two-stream instability and, hence, penetration of emitted electrons through the plasma, the accurate kinetic description is necessary for both the plasma and the beam. It is also found in one-dimensional particle-in-cell simulations that the two-stream instability depends crucially on the velocity distribution function of electron

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emission. Numerical studies also show that there are regions of parameter space when steady state solution can not be reached and quasi-periodic oscillations occur in plasma parameters.

Nomenclature

x	= coordinate normal to the walls and for the applied magnetic field
z	= coordinate parallel to the walls and for the applied electric field
t	= time
$v_{x,y,z}$	= velocity components of an electron
w	= kinetic energy of an electron
$w_{x,y,z}$	= kinetic energy of electron motion in x,y,z direction respectively
m	= electron mass
M	= ion mass
e	= elementary charge
H	= width of the plasma slab
$E_{x,z}$	= components of the electric field intensity (the self-consistent field is normal to the wall)
B_x	= induction of the applied magnetic field
Φ	= electrostatic potential relative to the dielectric wall at x = H
n_a	= neutral gas density
n_e	= electron density
ν_{turb}	= frequency of "turbulent" collisions
ν_{en}	= frequency of electron-neutral collisions
λ_c	= electron mean free path between two collisions
μ_c	= collisional electron mobility across the magnetic field
r_L	= electron Larmor radius
ω_c	= electron cyclotron frequency
$\Delta w_{\|\|}$	= energy gain/loss parallel to the walls after a single "turbulent" or electron-neutral collision.
Γ_1	= total primary electron flux towards a wall
Γ_2	= total flux of secondary electron emitted from a wall
Γ_i	= ion flux to a wall
γ	= total secondary electron emission coefficient
γ_{cr}	= critical value of the secondary electron emission coefficient for space charge saturated sheath regime
T_{cr}	= critical electron temperature for space charge saturated sheath regime with Maxwellian electrons
γ_b	= partial emission coefficient of a secondary electron beam
γ_p	= partial emission coefficient of plasma electrons
w_b	= average energy of a secondary electron beam when it impinges on the wall
w_p	= average energy of plasma electrons when they impinge on the wall
Γ_b	= primary electron flux towards one wall due to the electrons emitted from the opposite wall
Γ_e	= primary electron flux towards one wall due to the collision-ejected electrons from the plasma bulk
α	= coefficient of penetration of the beam of secondary electrons through the plasma
$u_{y,z}$	= components of flow velocity of a secondary electron beam in y and z directions respectively
J_z	= electric current density along z axis in simulations
T_{ex}	= the effective electric current density along z axis in simulations

I. Introduction

There is a reliable experimental evidence of the wall material effect on operation of a Hall thruster.[1,2] The existing fluid theories explain this effect invoking a strong secondary electron emission (SEE) from the channel walls. The SEE is predicted to weaken insulating properties of the near-wall sheaths and, thereby, (i) to cause cooling of plasma electrons and (ii) to enhance the electron conductivity across the magnetic field. From a practical standpoint, a strong SEE from the channel walls is expected to cause additional inefficiencies due to enhanced power losses in the thruster discharge, and intense heating of the channel walls by almost thermal electron fluxes from the plasma[3]. Moreover, because the SEE may lead to lower values of the sheath potential drop, ion-induced erosion of the channel walls can be also affected. Although these predictions can be certainly applied for plasmas with a Maxwellian electron velocity distribution function (EVDF), there is no consensus between the existing fluid [2,4,5,6,7] and kinetic models[8,9] on how strong the SEE effects on the thruster plasma are. According to kinetic simulations[10,11,12,13] the EVDF in a collisionless plasma is depleted at high-energies due to electron-wall losses. Under such conditions, the electron losses to the walls can be hundreds of times smaller than the losses predicted by the fluid theories. A similar depletion of EVDF at high energies was also reported for other kinds of low-pressure gas discharges.[14,15,16] Note that the deviation of the EVDF from a Maxwellian does not necessarily mean that the SEE cannot play a significant role in the thruster discharge. In experiments with a Hall thruster operating at high discharge voltages, the maximum electron temperature and the electron cross-field current were strongly affected by the SEE properties of the channel wall materials[17,18].

In recent particle-in-cell (PIC) simulations [10,11,12] and in the kinetic analytical study,[14] we showed that the SEE effect on power losses in a thruster discharge is quite different from what was predicted by previous fluid and kinetic studies. In simulations, the EVDF was found to be strongly anisotropic, depleted at high energies, and in some cases, even non-monotonic. The average kinetic energy of electron motion in the direction parallel to the walls is several times larger than the average kinetic energy of electron motion in the direction normal to the walls. Secondary electrons form two beams propagating between the walls of a thruster channel in opposite radial directions[10,11] (also predicted in Ref. 19 in the modified fluid approximation). In this paper, we highlight results of recent kinetic studies of HT (Refs. 10-14) which are summarized in the form of convenient analytical formulas for predicting kinetic properties of HT plasma. It is shown that for a typical high-performance Hall thruster, the electron fluxes to the walls are limited by the source of electrons, overcoming the wall potential and leaving the plasma. The flux of these electrons is determined mainly by the frequencies of elastic electron collisions with atoms and ions. The sheath insulating properties depend on the electron fluxes to the walls and, therefore, on the rate of elastic scattering of plasma electrons.

In previous kinetic studies, Meezan and Cappelli[8] developed a kinetic model based on the so-called nonlocal approach. The non-local approach (described for example in Ref. 20) was developed for large gas discharges with the distance between walls of order tens of centimeters and at pressures above 10mTorr, where the electron mean free path is much smaller than the discharge gap $\lambda_c << H$. In Hall thrusters the characteristic distance between walls is given by the channel width. Because of the smallness of the electron mean free path in these gas discharges, the EVDF is isotropic even for electrons with energy high enough to overcome the wall potential. However, the traditional nonlocal approach is not applicable to Hall thrusters, which operate in the opposite limit $\lambda_c >> H$. Because the electron mean free path in Hall thrusters is much larger than the channel width, the EVDF has been shown to be anisotropic[11]. Moreover, the anisotropy of the EVDF strongly affects the electron flux to the wall, as shown below. Practical analytical formulas are derived for wall fluxes, secondary electron fluxes, plasma parameters and contribution to the electron current due to SEE. The calculations based on the analytical formulas agree well with the results of numerical simulations.

An important implication of the present work is that future theoretical and experimental studies need to determine the influence of these kinetic effects on the thruster performance, heating and erosion of the channel walls. For instance, the reduction of the gas density in the thruster channel might significantly reduce the electron fluxes to the walls because in xenon plasmas of Hall thrusters the electron collisions with neutral atoms is the major scattering process while the Coulomb scattering off the ions gives a small contribution.

II. Particle fluxes to the walls in Hall thruster channel

As shown in Ref. 14 the electron flux to the wall in the limit of the large electron mean free path $\lambda_c >> H$ is reduced by a factor of order H/λ_c compared with the calculation assuming an isotropic EVDF. For typical thruster conditions $H/\lambda_c \sim 1/100$, and the reduction is considerable. The electron flux to the wall can be written as

$$\Gamma_e = \frac{\kappa H}{8\lambda_c} n_e \sqrt{\frac{8T_{ez}}{\pi m}} \exp\left(-\frac{\Phi}{T_{ez}}\right). \qquad (1)$$

Here, n_e is the plasma density in the center, see e.g., Ref. 21 for details, and $\kappa \sim 1$ is a correction factor close to unity, which can only be obtained by a comparison of the approximate estimations with the *exact* result of PIC simulations; in the following this factor is assumed to be unity. For a Maxwellian isotropic EVDF, the flux to the wall is equal to $1/4n_w\sqrt{8T_e/\pi m}$, where n_w is the density at the wall, which relates to the central density through the Boltzmann relationship $n_w = n_e\exp(-e\Phi/T_e)$. Equation (1) has two major differences from the fluid model: the electron temperature T_{ez} enters equation, and there is an additional small factor, $H/2\lambda_c$, which accounts for strong reduction of the electron flux due to the depleted EVDF for electrons with the kinetic energy high enough, so that they are able to leave the plasma.

III. Penetration coefficients of secondary electron emission beams

The secondary electrons emitted from the opposite walls are accelerated in the near-wall sheaths towards the plasma and form counter-streaming beams. For a quasi-stationary symmetric plasma, the wall potentials at the opposite walls are the same. When the beam electrons penetrate through the plasma bulk, they may gain enough energy (due to the E×B motion) to induce the SEE from the opposite wall. Refs. 10,11 and 19 introduced a phenomenological coefficient (α) to describe the penetration of SEE beam from one wall to the opposite wall. The scattering of SEE beams can occur due to collisions with atoms or bulk plasma electrons. However, the probability for such scattering to occur is small, (about a few percents) because the electron mean free path is very large for typical thruster conditions. Another mechanism of scattering involves the high-frequency electric field oscillations with a period shorter or comparable with time of flight of an electron from one wall to another. A possible candidate of high-frequency electric field oscillations is the two-stream instability between the SEE beam and bulk electrons. Such instability excites the plasma oscillations with the frequency close to the electron plasma frequency.

The two-stream instability occurs if the total electron velocity distribution function of the plasma-beam system is a nonmonotonic function of electron speed. Note that if the plasma electrons are described by a Maxwellian EVDF, the combination of plasma and emitted electrons results in a nonmonotonic total EVDF leading to the two-stream instability. However, in low-pressure discharges, the EVDF is not Maxwellian, it is depleted at energies above the plasma potential relative to the wall. Therefore, the development of the two-stream instability in low-pressure discharges is very different compared to Maxwellian plasmas. Hall thruster plasmas are depleted of electrons with energy above the plasma potential. We performed systematic studies of the two-stream instability and found that the pattern of its development depends crucially on the shape of the velocity distribution function of electron emission (VDFEE)[13]. One type of VDFEE considered in the present paper is a monotonically decaying function of electron energy, which starts from a positive value at zero emitted energy. The total EVDF consisting of the plasma EVDF and the VDFEE accelerated by the plasma potential is a monotonically decaying function of speed if the emission current is below some threshold. In this case, the two-stream instability does not occur. If the emission current is above this threshold so that the total EVDF is a nonmonotonic function of speed, then the two-stream instability does occur but quickly vanishes. This happens because the two-stream instability forms a plateau on the velocity distribution function of electrons confined by the plasma potential (i.e., the plasma EVDF), then the total EVDF becomes a monotonic function of speed and the beam propagates through the plasma without perturbations.

On the other hand, the VDFEE may be equal to zero at zero energy of emitted electrons and grow as a function of energy for a few electron volts. Such a nonmonotonic VDFEE is a feature of secondary electron emission from metals. At low pressures, the total EVDF of the plasma-beam system near the emitting wall has a gap of a few electron volts at the energy corresponding to the wall potential. This gap is responsible for the development of the two-stream instability, which is confirmed by simulations with a nonmonotonic VDFEE. In these simulations, the two-stream instability reaches the nonlinear saturation stage and exists for as long as the emission lasts. As a result, the plasma electrons accelerate while the emitted electrons decelerate, which leads to the partial trapping of emitted electrons in the plasma. In our simulations with immobile ions and constant emission current, about 50% of emitted electrons become trapped in the plasma during their first flight between the walls. However, the two-stream turbulence accelerates these electrons back to an energy above the plasma potential so that they leave the plasma after several bounces between the walls. In fact, during a steady state, the sum of wall fluxes of emitted electrons that reach the wall after multiple bounces and those that cross the plasma directly is close to the emitted electron flux. It is therefore expedient to assume that the two-stream instability does not affect the beam propagation and that *the effective penetration coefficient is close to unity.*

IV. Analytical estimate of the wall potential and the electron temperatures

The ion flux can be estimated from the Bohm criterion and the fact that for a planar geometry the plasma density approximately decreases twice from the plasma center to the plasma sheath boundary in a collisionless case (when ion mean free path is large compared with the channel), see, for example, Ref. 21

$$\Gamma_i = \frac{1}{2} n_e \sqrt{T_{ex}/M} \ . \tag{2}$$

Because the SEE beams do not contribute to the current balance at the walls, the ambipolarity criterion implies that the ion wall flux is compensated by the collision-ejected electron flux $\Gamma_i = \Gamma_e$. Under such condition, the plasma potential at the center with respect to the wall (i.e. the potential drop in the sheath and pre-sheath) can be determined from Eqs. (1) and (2), and reads

$$\Phi = \frac{T_{ez}}{e} \ln\left(\frac{H}{\lambda_c} \sqrt{\frac{T_{ez}}{T_{ex}}} \sqrt{\frac{M}{2\pi m}} \right) \tag{3}$$

For the typical thruster conditions, the contribution from the sheath potential gives 5.3, the potential drop in the plasma gives 0.70 and the reduction due to empty loss cone gives -5.1 totaling the value of the wall potential being of order T_{ez}/e:

$$\Phi \approx \frac{T_{ez}}{e}\left[\sqrt{\frac{M}{2\pi m}} + \ln 2 - \ln\left(\frac{2\lambda_c}{H} \sqrt{\frac{T_{ex}}{T_{ez}}} \right) \right] = \frac{T_{ez}}{e}\left(5.3 + 0.7 - 5.1 \right) \approx \frac{T_{ez}}{e} \ .$$

The first term is the sheath potential; the second is due to the potential drop in the plasma; and the last term accounts for reduction of the electron flux due loss cone. Note a big contribution of the term describing the reduction of the electron flux due to the loss-cone effects, not described in the current fluid and kinetic theories. Let us emphasize here that the result of Eq. (3) is only superficially similar to the result obtained by the fluid theory for the sheath potential drop in the space-charge-limited regime of the sheath[4,5,2,3]. The physical meaning of Eq. (3) is fundamentally different because the SEE's contribution to the flux balance is self-canceled and, therefore, the plasma potential with respect to the wall does not depend on the SEE.

From energy balance equation the approximate expression for the electron temperature in the direction of the electric field is

$$T_{ez} \approx k\left(1 + \frac{\nu_{turb}}{\nu_{en}} \right) m_e \left(\frac{E}{B} \right)^2 \left(1 + \ln\left(\frac{H}{\lambda_c} \sqrt{\frac{T_{ez}}{T_{ex}} \frac{M_i}{2\pi m_e}} \right) \right), \tag{4}$$

where, k is the correction coefficient, which can be obtained by a comparison of the approximate temperature estimations with the *exact* result of PIC simulations. The correction coefficient k is varied between 1.4 to about 2.

The electron temperature in the direction perpendicular to walls can be obtained from analysis of loss cone, which gives[14]

$$T_{ex} \approx \frac{e\Phi}{e\Phi + T_{ez}} T_{ez} \ , \tag{5}$$

where the ratio $e\Phi / T_{ez}$ can be obtained from Eq.(2).

Finally, the contribution of SEE electrons to the total current[14] reads

$$J_{bz} \approx \frac{m}{H}\frac{\gamma_p}{1-\gamma_b} n_e \sqrt{\frac{T_{ex}}{M}} \frac{E_z}{B_x^2}, \tag{6}$$

where γ_b and γ_p are the partial emission coefficients due to the electrons of the beam and the plasma bulk, respectively.

V. Temporal oscillations of plasma parameters

In most studies it is assumed that plasma can reach a steady state. However numerical studies show that there are regions of parameter space where a steady state solution can not be reached and quasi-periodic oscillations of plasma parameters occur[22,23]. The oscillations occur due to a sheath instability followed by drastic loss of nearly-trapped electrons with energy of motion in the direction normal to the walls just below the plasma potential. This leads to considerable electron cooling. In the following stage, the plasma heats up and population of nearly-trapped electrons grows as well as their energy until plasma reaches an unstable region where the plasma potential suddenly drops and the nearly-trapped electrons escape to the walls and get substituted by cold secondary electrons inside the plasma. Then the process repeats. Further details will be available in our future publications[24].

VI. Conclusions

The plasma potential, the wall electron flux, and the electron temperatures calculated making use of these formulas agree well with the values obtained in particle-in-cell simulations. The SEE effect on power losses in a thruster discharge is shown to be quite different from what was predicted by previous fluid and kinetic studies. Kinetic calculation gives the values of the electron flux of a few orders of magnitude smaller than the values obtained using the fluid approach. The difference is attributed to the presence of a large depleted loss cone in the electron velocity distribution function. The EVDF in the loss cone is determined by elastic scattering of electrons due to collisions with atoms and Coulomb collisions. Our results suggest that even in the presence of a strong SEE from the walls, a contribution of the wall energy losses to the electron energy balance is much smaller than predicted by fluid theories and is proportional to the elastic scattering of electrons on collisions with atoms and ions and not inversely proportional to the electron time of flight to the walls, as is commonly assumed. It means that the wall flux is proportional to the gas density and is independent on the channel width (as long as $H << \lambda_c$). This is very different from plasmas with the isotropic electron EVDF, including Maxwellian and non-Maxwellian EVDFs.

Another important result of these kinetic studies is that the SEE contribution to the current balance at the walls is self-canceled and, therefore, the plasma potential with respect to the wall and the electron energy losses on the walls are almost insensitive to the SEE. Secondary electrons emitted from the walls form two counter-streaming beams. The effective coefficient for penetration of the SEE beams from one wall to the opposite wall is equal to unity. One may assume the complete penetration of the emitted electrons because the beam electrons, which lose energy due to the two-stream instability and cannot leave the plasma in one pass between the channel walls, will eventually gain energy and escape the plasma. The SEE beams may carry a considerable portion of the cross-field electron current due to their cycloid trajectory in ExB field. This effect should depend on SEE properties of the channel wall material.

In most studies it is assumed that plasma can reach a steady state. However numerical studies show that there are regions of parameter space when steady-state solution can not be reached and quasi-periodic oscillations occur in plasma parameters. The oscillations occur due to a sheath instability followed by drastic change in electron velocity distribution function. Therefore these oscillations are yet another kinetic effect which can not be described by a Maxwellian EVDF.

Finally, the results of these theoretical studies may explain the influence of wall material on the thruster operation and plasma parameters observed in experiments[1,2,18] as well as influence of the channel width on the electron temperature[25] by the enhancement of the electron conductivity due to contribution of the SEE electrons, rather than the enhancement of the energy losses to the walls. This conclusion is in agreement with the analysis of experimental data in Ref. 17. Future studies should be focused on generalization of this model to the two-dimensional geometry.

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References

[1] Y. Raitses, J. Ashkenazy G. Appelbaum and M. Guelman, Proceedings of the 25th International Electric Propulsion Conference, Cleveland, OH, August 1997, Electric Rocket Propulsion Society, Cleveland, OH, 1997 IEPC paper No.97-056.
[2] S. Barral, K. Makowski, Z. Peradzy´nski, N. Gaskon, and M. Dudeck, Phys. Plasmas **10**, 4137 (2003).
[3] E. Ahedo and D. Escobar, J. Appl. Phys. **96**, 983 (2004).
[4] G. D. Hobbs and J. A. Wesson, Plasma Phys. **9**, 85 (1967).
[5] V.A. Rozhansky and L.D. Tsendin, *Transport phenomena in partially ionized plasma* (London ; New York : Taylor & Francis 2001).
[6] E. Ahedo, J. M. Gallardo and M. Martinez-Sanchez, Phys. Plasmas **10**, 3397 (2003).
[7] M. Keidar, I. Boyd and I. I. Beilis, Phys. Plasmas **8**, 5315 (2001).
[8] N. Meeazan and M. Cappelli, Phys. Rev. E **66**, 036401 (2002).
[9] O. Batishchev and M. Martinez-Sanchez, Proceedings of the 28th International Electric Propulsion Conference, Toulouse, France, March 2003, Electric Rocket Propulsion Society, Cleveland, OH, 2003, IEPC Paper No. 2003-188.
[10] D. Y. Sydorenko, and A. I. Smolyakov, Bull. Am. Phys. Soc. 261 (2004).
[11] D. Sydorenko, A. Smolyakov, I. Kaganovich, and Y. Raitses, Phys. Plasmas **13**, 014501 (2006).
[12] D. Sydorenko, A. Smolyakov, I. Kaganovich, Y. Raitses, IEEE Trans. Plasma Sci. **34**, 815 (2006).
[13] D. Sydorenko, A. Smolyakov, I. Kaganovich, Y. Raitses, Phys. Plasmas **14**, 013508 (2007).
[14] I. Kaganovich, Y. Raitses, D. Sydorenko, A. Smolyakov, Phys. Plasmas **14**, 057104 (2007).
[15] L.D. Tsendin, Sov. Phys. JETP **39**, 805 (1974).
[16] I. Kaganovich, M. Misina, S. V. Berezhnoi, and R. Gijbels, Phys. Rev. E **61**, 1875 (2000).
[17] Y. Raitses, D. Staack, A. Smirnov, and N. J. Fisch, Phys. Plasmas **12**, 073507 (2005).
[18] Y. Raitses, D. Staack, A. Smirnov, and N. J. Fisch, Phys. Plasmas **13**, 014502 (2006).
[19] E. Ahedo, and F. I. Parra, Phys. Plasmas **12**, 073503 (2005).
[20] I.D. Kaganovich and L.D. Tsendin, IEEE Trans. Plasma Sci. **20,** 66 (1992).
[21] I.D. Kaganovich, Phys. Plasmas **9**, 4788 (2002).
[22] D. Y. Sydorenko, Ph. D. Thesis University of Saskatchewan 2006.
[23] F. Taccogna, R. Schneider, S. Longo and M. Capitelli, Proceedings of the 43rd Joint Propulsion Conference, Cincinnati, OH, 2007, paper No AIAA 2007-5211.
[24] D. Sydorenko, A. Smolyakov, I. Kaganovich, and Y. Raitses, PPPL report to be submitted (2007).
[25] Y. Raitses, D. Staack, M. Keidar, and N. J. Fisch, Phys. Plasmas **12**, 057104 (2005).