Low Frequency Instability and Enhanced Transfer of Electrons in Near-Anode Region of Hall Thruster

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Abstract: The paper is devoted to building a theoretical model of relatively low-frequency instability of plasma in the near-anode area of an acceleration channel. In the model, the two-liquid magneto-hydrodynamic approximation is used. The finite temperature of electrons is taken into consideration. The instability is due to non-uniformity of the plasma and magnetic field. The instability can be responsible for enhanced transfer of the electrons towards the anode.

Nomenclature

b = size of plasma volume along magnetic field line
B = induction of magnetic field
e = unit positive charge
E = electric field strength
j_e = density of electron current
k_x, k_y = projections of wave vector on X-axis and Y-axis, respectively
l_N = scale of non-uniformity of electron density
l_B = scale of non-uniformity of magnetic field induction
m = electron mass
M = ion mass
n = number density of electrons (ions)
P_e = pressure of electrons
Q_{ea} = cross-section of electron collisions with atoms
R = radius of curvature of magnetic field lines
T_e = temperature of electrons
u = velocity of electron component
v = velocity of electron
v_T = thermal velocity of electron component
V = velocity of ion component

α = numerical factor of collision efficiency
γ = growth rate of instability
τ_{eff} = effective time of free motion of electron
τ_{ea} = time between collisions of electron with atoms
τ_{ew} = characteristic time of electron collisions with walls
υ_i = frequency of ionization collisions of electron
Φ = perturbation of electrical potential
ω_r = frequency of wave

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ω_e = electron cyclotron frequency
ω_i = ion cyclotron frequency

I. Introduction

The near-anode region of a Hall thruster, defined as the area of an acceleration channel between the anode and the zone of intense ionization, occupies a significant fraction of the channel – up to half of its length. Even for this reason alone, the near-anode region needs detailed study of the physical processes in it. More specifically, the near-anode area performs a function of great importance: It provides transfer of electrons, born in the zone of ionization, to the anode. In the near-anode region, the electrons should be moved to the anode without significant consumption of electrical power in spite of rather strong magnetic field with the induction of 1/2 – 1/3 from maximal in this region. The stability of discharge firing, in all probability, also depends on the electron transportation conditions in this area. An additional motivation to investigate the processes in the near-anode region is the necessity to have uniform ions density at the entrance of the ionization zone.

The processes which occur in the near-anode area are substantially complicated by the fact that the longitudinal fluxes of the electrons, produced by both the electric field and gradient of the electron pressure are comparable in the magnitude.

In recent years, the interesting and important complex of the investigations of the near-anode processes in the Hall thruster, based mainly on the analysis of averaged in time parameters of plasma was performed[1-6]. At the same time, strong non-equilibrium of the plasma in this region, in particularly, an available of large gradient of the electron density, points to the fact that the wave processes can play in the near-anode area the essential role.

The paper is devoted to building the theoretical model of the relatively low-frequency ($\nu_i \ll \omega \ll \sqrt{\omega_e \omega_i}$, where ν_i – the frequency of the ionization collisions of the electron, ω_e, ω_i –the electron and ion cyclotron frequencies, respectively) instability of the plasma in the near-anode area of the acceleration channel. The influence of this instability on the transfer of the electrons is discussed.

II. Some Preliminary Evaluations

We begin the consideration of the low-frequency instability in the near-anode region with the evaluation of the electron transfer due to the collisions of electrons in this area with atoms and walls of the acceleration channel. In the plasma with well-magnetized electrons, the density of the electron current across magnetic field is dictated by the following well-known expression:

$$j_{ex} = \frac{e^2 n}{m\omega_e^2 \tau_{eff}} \left(E_x + \frac{1}{en} \frac{\partial P_e}{\partial x} \right) \quad \text{(II.1)}$$

Where
e – the unit positive charge,
n – the number electron density,
m– the mass of electron,
E_x – the projection of electrical field strength on X-axis,
$P_e = nT_e$ – the pressure of the electrons,
T_e - the temperature of the electrons,
τ_{eff} - the effective time of free motion of the electron between collisions with heavy particles and walls,
$\omega_e = \frac{eB}{m}$,
B – the induction of the magnetic field.

In the expression (II.1), the X-axis is directed along the thruster axis.
The expression (II.1) results from the equation of motion of the electron component with collision term and is so-called generalized Ohm's law.

Assuming that in the near-anode region the temperature of the electrons is constant, then instead of (II.1) we will obtain:

$$j_{ex} = \frac{e^2 n}{m\omega_e^2 \tau_{eff}} \left(E_x + \frac{T_e}{en} \frac{\partial n}{\partial x} \right) \qquad \text{(II.2)}$$

At $\frac{\partial n}{\partial x} \succ 0$, positive values for the electron current in the near-anode region are possible in two cases:

$$\text{1) if } E_x > 0 \qquad \text{(II.3)}$$

$$\text{2) if } E_x \leq 0 \text{ and } \left| E_x \right| \prec \frac{T_e}{en} \frac{\partial n}{\partial x} \qquad \text{(II.4)}$$

The first case requires the rather intensive ionization in the near-anode region to compensate escaping this area by ions under the influence of the electric field. Otherwise the discharge will breakdown because the lack of the ions means the lack of the electrons as a consequence of quasi-neutrality. In Hall thrusters with a great positive gradient of the magnetic field, there is, in general, a well-defined ionization zone, located at some distance from the anode. This condition corresponds to the second case, confirmed by direct measurements of the potential distribution. Therefore, we shall restrict our consideration to this case only.

From (II.2) and (II.4), it follows that:

$$\tau_{eff} \approx \frac{mT_e n}{j_{ex} e B_{0av}^2 l_N} \qquad \text{(II.5)}$$

Where l_N – the scale of non-uniformity of electron density,

B_{0av} – the average value of magnetic field induction in near anode region

Assuming that $T_e \sim 10$ eV, $n \sim 10^{18}$ m^{-3}, $B_{0av} \sim 7.5 \cdot 10^{-3}$ T, $l_N \sim 1.5 \cdot 10^{-2}$ m, and $j_{ex} \sim 10^3$ A/m^2, we obtain $\tau_{eff} \sim 1.1 \cdot 10^{-8}$ s.

The time between collisions of the electron with atoms of Xenon is $\tau_{ea} = \frac{1}{n_a < Q_{ea} v >} \approx \frac{1}{n_a v_T Q_{ea}}$.

Where n_a – the number density of atoms,

v – the velocity of the electron,

v_T – the thermal velocity of the electrons

Q_{ea} – the cross-section of electron collisions with atoms,

$< >$ mean averaging on distribution function.

For Xenon under conditions of the Hall thruster, the following cross-section of the collisions may be accepted[1]: $Q_{ea} \sim 2 \cdot 10^{-19}$ m^2. Then at $n_a \sim 3 \cdot 10^{19}$ m^{-3} and $v_T \sim 10^6$ m/s, we have $\tau_{ea} \sim 1.6 \cdot 10^{-7}$ s. This time is significantly larger than τ_{eff}.

Under the conditions in a Hall thruster, the transfer of the electrons across the magnetic field can be derived from collisions of electrons with the walls of the acceleration channel, that is, by means of the so-called near-wall conductivity, theoretically predicted by A. I. Morozov and investigated in details by A. I. Morozov and A.I. Bugrova with colleagues (See, for example, Ref.1). The role of near-wall conductivity in the transfer of the electrons can be roughly evaluated from the following relation: $\tau_{ew} \approx \dfrac{b}{\alpha v_T}$.

Where τ_{ew} - the characteristic time of electron collisions with the walls,

b – the size of plasma volume along magnetic field line,

α is a numerical factor of collision efficiency (it takes into account, first of all, the ratio of a number of the electrons, penetrated through Debye layer, to all electrons moving to the walls).

Everything is defined by the value of α. In principle, α can vary from 0.1 to 1, depending on the potential jump on the near-wall Debye layer.

Assuming that b is the distance between outer and inner walls and equals $1.5\cdot10^{-2}$ m, and assuming that $v_T \sim 10^6$ m/s, $\alpha \sim 0.1$, we obtain: $\tau_{ew} \approx 1.5\cdot10^{-7}$ s, that is the same order as at collisions with atoms. Authors of Ref.1 assume that in the near-anode region, the potential jump on the Debye layer is small and then $\alpha \approx 1$. In this case, the near-wall conductivity can provide the need transfer of the electrons. But taking into account the uncertainty in magnitude of α, it is necessary to consider the alternative version of the electron transportation in the near-anode region. It is reasonable to suppose that enhanced transfer of the electrons is due to wave processes. As a rule, the most effective transfer of the charge particles in the magnetized plasma is produced by low-frequency waves, having a component of the electric field, which is transverse to the direction of the transfer. The possibility of such waves to be exited in the near-anode region will be now considered.

III. Theoretical Model of Low-Frequency Instability

A. Basic Assumptions and Governing Equations

We use a Cartesian frame with X and Z axes directed in parallel to the axis of the thruster and the applied magnetic field, respectively (Fig.1).The behavior of 2D-perturbations, which are infinitely spread along the magnetic lines, is considered. The perturbations are assumed to be potential, quasi-neutral and non-dissipative. The ionization processes are not taken into consideration. The analysis of the stability of the plasma is carried out under the approximation of two-liquid magneto-hydrodynamic with cold non-magnetized ions and hot magnetized electrons. The temperature of the electrons is assumed to be constant. Taking into account the finite temperature of the electrons is needed because in the near-anode region, as distinguished from the acceleration area, the velocity of the drift motion of the electrons, due to the temperature of the electrons, is comparable in magnitude with the velocity of the electrical drift. The finite temperature of the electrons brings about two drifts: 1) Larmour drift due to the gradient of electron pressure; 2) drift due to the gradient of the magnetic field. The latter deserves special attention. If a magnetized charge particle moves in the curvilinear magnetic field (any non-uniform magnetic field is curvilinear one), it experiences a force. This force is directed to the convexity of the magnetic field lines and transverse to the magnetic field. (See, for example, Ref.7) For electrons, the expression for the force is as follows:

$$|F_B| \approx \frac{m v_T^2}{R} \approx \frac{2T_e}{R} = 2T_e \left|\nabla_\perp \ln B_0\right| \qquad \text{(III.1)}$$

Where R – the radius of curvature of the magnetic field line,

B_0 – the module of induction of the magnetic field.

(We assume that the pressure of the plasma is much less than the pressure of the applied magnetic field.)

Under the effect of the force, the electron drifts transverse to both the magnetic field and the force.

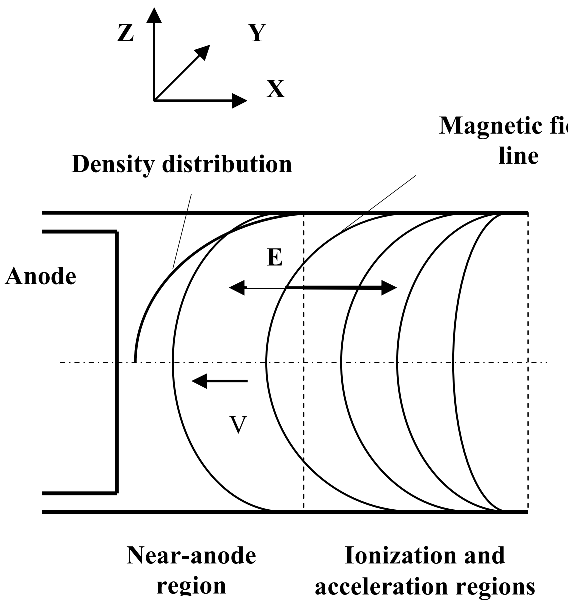

Figure 1. A sketch of the near-anode region

Under the above assumptions, the linearized set of equations of two-liquid magneto-hydrodynamic model for the perturbations takes the following form:

Equations for the ion component

$$\frac{\partial V_x}{\partial t} + V_{x0}\frac{\partial V_x}{\partial x} + V_x\frac{\partial V_{x0}}{\partial x} = -\frac{e}{M}\frac{\partial \Phi}{\partial x} \qquad \text{(III.2)}$$

$$\frac{\partial V_y}{\partial t} + V_{x0}\frac{\partial V_y}{\partial x} = -\frac{e}{M}\frac{\partial \Phi}{\partial y} \qquad \text{(III.3)}$$

$$\frac{\partial n}{\partial t} + n_0\frac{\partial V_x}{\partial x} + V_x\frac{\partial n_0}{\partial x} + V_{x0}\frac{\partial n}{\partial x} + n\frac{\partial V_{x0}}{\partial x} + n_0\frac{\partial V_y}{\partial y} = 0 \qquad \text{(III.4)}$$

Equations for the electron component

$$e\frac{\partial \Phi}{\partial x} + \frac{T_{e0}}{n_0^2}\frac{\partial n_0}{\partial x}n - \frac{T_{e0}}{n_0}\frac{\partial n}{\partial x} - eu_y B_0 = 0 \qquad \text{(III.5)}$$

$$e\frac{\partial \Phi}{\partial y} - \frac{T_{e0}}{n_0}\frac{\partial n}{\partial y} + eu_x B_0 = 0 \qquad \text{(III.6)}$$

$$\frac{\partial n}{\partial t}+n_0\frac{\partial u_x}{\partial x}+u_x\frac{\partial n_0}{\partial x}+n_0\frac{\partial u_y}{\partial y}+u_{y0}\frac{\partial n}{\partial y}=0 \qquad \text{(III.7)}$$

Where V_x , V_y – the projections of perturbed velocity of the ions on the X and Y axis, respectively,

u_x, u_y - the projections of perturbed velocity of the electrons on the X and Y axis, respectively,

M – the mass of ion,

Φ – the perturbation of potential of plasma.

The values with index "0" are non-perturbed.

The non-perturbed velocity of the ions V_{x0} is directed towards the anode.

The non-perturbed velocity of the electrons u_{y0} is directed along Y axis. For finding it, we use the projection of the equation of non-perturbed motion of electrons on X axis:

$$-en_0E_{x0}-\frac{2T_{e0}n_0}{R}-T_{e0}\frac{\partial n_o}{\partial x}-eu_{y0}B_0n_0=0 \qquad \text{(III.8)}$$

From (III.8), it follows:

$$u_{y0}=-\frac{E_{x0}}{B_0}-\frac{2T_{e0}}{eB_0R}-\frac{T_{e0}}{eB_0n_0}\frac{\partial n_0}{\partial x} \qquad \text{(III.9)}$$

Substituting u_y and u_x from the eqns. (III.5) and (III.6),respectively, and u_{y0} from the eqn. (III.9) in the equation of continuity for the electrons (III.7), we obtain the equation of continuity in the following form:

$$\frac{\partial n}{\partial t}+\frac{n_0}{B_0}\left(\frac{1}{B_0}\frac{\partial B_0}{\partial x}-\frac{1}{n_0}\frac{\partial n_0}{\partial x}\right)\frac{\partial \Phi}{\partial y}-\frac{1}{B_0}\left[E_{x0}+\frac{3T_{e0}}{eB_0}\frac{\partial B_0}{\partial x}\right]\frac{\partial n}{\partial y}=0 \qquad \text{(III.10)}$$

In the eqn. (III.10), the approximate equality $\frac{1}{R}\approx\frac{1}{B_0}\frac{\partial B_0}{\partial x}$ is used.

B. Dispersion Equation

We shall use the small-scale approximation. It means, that in the equations (III.2) and (III.4), we must neglect terms $V_x\frac{\partial V_{x0}}{\partial x}$, $V_x\frac{\partial n_0}{\partial x}$ and $n\frac{\partial V_{x0}}{\partial x}$ in comparison with the terms $V_{x0}\frac{\partial V_x}{\partial x}$, $n_0\frac{\partial V_x}{\partial x}$ and $V_{x0}\frac{\partial n}{\partial x}$, respectively. Taking into account this remark, we seek the solution of the set of the eqns. (III.2) – (III.4) and (III.10) in the form:

$$\mathbf{F}(x,y;t)=\mathbf{F}_k\exp(-i(\omega t-k_xx-k_yy)),$$

where $\mathbf{F}$ and $\mathbf{F}_k$ are consequently the vector of perturbed parameters and vector of their Fourier components,

k_x and k_y – the projections of wave vector on the axes X and Y respectively.

As a result, we obtain the following local dispersion equation:

$$\omega^2 - \left(2k_x V_{xo} + \frac{\omega_i k_\perp^2}{k_y \left(\frac{1}{l_B} - \frac{1}{l_N} \right)} \right) \omega + k_x^2 V_{x0}^2 - \frac{e k_\perp^2 \left(E_{x0} + \frac{3T_{e0}}{e l_B} \right)}{M \left(\frac{1}{l_B} - \frac{1}{l_N} \right)} = 0 \qquad \text{(III.11)}$$

Hereafter $k_\perp^2 = k_x^2 + k_y^2$, $l_N = 1 \Big/ \frac{d}{dx} \ln n_0$, $l_B = 1 \Big/ \frac{d}{dx} \ln B_0$, $\omega_i = \frac{eB_0}{M}$.

From the dispersion equation (III.11), it follows:

$$\omega = k_x V_{x0} + \frac{\omega_i k_\perp^2}{2k_y \left(\frac{1}{l_B} - \frac{1}{l_N} \right)} \pm \sqrt{ \frac{k_x V_{x0} \omega_i k_\perp^2}{k_y \left(\frac{1}{l_B} - \frac{1}{l_N} \right)} + \frac{\omega_i^2 k_\perp^4}{4k_y^2 \left(\frac{1}{l_B} - \frac{1}{l_N} \right)^2} + \frac{e k_\perp^2 \left(E_{x0} + \frac{3T_{e0}}{e l_B} \right)}{M \left(\frac{1}{l_B} - \frac{1}{l_N} \right)} } \qquad \text{(III.12)}$$

In the expression (III.12) for the frequency of the perturbations, the ratio of the first term under the root to the third one is ~ $k_x V_{x0} / k_y u_{y0}$. At $k_y \sim k_x$ and a near-anode drop of the potential of about 15 V, the ratio is less than 0.1. The ratio of the second term to the third term under the root and the ratio of the second term to the first term before the root is still less, so that as the first approximation, we can instead of (III.12) write:

$$\omega = k_x V_{x0} \pm \sqrt{ \frac{e k_\perp^2 \left(E_{x0} + \frac{3T_{eo}}{e l_B} \right)}{M \left(\frac{1}{l_B} - \frac{1}{l_N} \right)} } \qquad \text{(III.13)}$$

Expression (III.13) is valid for waves with projections of the wave vector, meeting the condition:

$$\left| k_x V_{x0} \right| \gg \left| k_y \frac{3T_{e0}}{l_B B_0} \right|, \left| k_y \frac{E_{x0}}{B_0} \right| \qquad \text{(III.14)}$$

When analyzing the behavior of the perturbations, described by the dispersion relation (III.13), it is formally necessary to consider several options due to the signs and relative magnitude of quantities l_B and l_N. However, since in a typical Hall thruster, in the near-anode area the induction of the magnetic field increases in the direction from the anode to the acceleration area, we shall consider the positive values of l_B, only. The main ionization of the propellant takes place after the near-anode region, therefore the number density of the electrons should be increased in the positive direction of the *X*-axis, that is, l_N should be positive.

From (III.13) it follows that at the positive values of l_B and l_N , the instability (a negative expression under the root) is possible as at condition that $l_B < l_N$, so at $l_B > l_N$. However, the first case is not of interest, because it can be realized, if only $E_{x0} < 0$ and $\left| E_{x0} \right| > 3T_{e0} \big/ e l_B$. It is difficult to expect that under the condition such that the collision mechanism can be inadequate to provide the transfer of the electrons between the zone of ionization and anode, 1) the gradient of the electron concentration will be comparatively small, 2) the inverse electric field will be so large that it will suppress the collision transfer at all (See (II.2)). Therefore hereinafter, we shall consider only the case: $0 < l_N < l_B$. Then, the instability at $E_{x0} \leq 0$ will arise, if

$$\left| E_{xo} \right| < \frac{3T_{e0}}{e l_B} \qquad \text{(III.15)}$$

The growth rate of the instability is as follows:

$$\gamma \approx k_{\perp}\sqrt{\frac{e\left(\frac{3T_{e0}}{el_B}-\left|E_{x0}\right|\right)}{M\left(\frac{1}{l_B}-\frac{1}{l_N}\right)}} \qquad \text{(III.16)}$$

The frequency of the wave is defined mainly by the averaged velocity of the ions V_{x0} and according to (III.13) equals

$$\omega_r \approx k_x V_{x0} \qquad \text{(III.17)}$$

The instability is strong because the growth rate is of the order of the frequency.

It is necessary to pay attention to the following circumstance. In spite of the small contribution of k_y to the frequency of the instability at the typical values of k_x and k_y , the role of k_y is principal. This is seen from the full expression for ω (III.12). At rather small magnitude of module of k_y, the instability disappears.

Also, it is necessary to note that, strongly speaking, the analysis of the influence of the finite temperature of the electrons on the instability of the plasma in the non-uniform magnetic field of the Hall thruster needs the kinetic approach with the use of the method of integrating on trajectories. Therefore, the application of the significantly simpler magneto-hydrodynamic approach is the first, rather rough, approximation. In particularly, this hold true for a numerical coefficient in the terms with T_{e0}/el_B in (III.10) – (III.16).

The instability by its nature belongs to so-called Rayleigh – Taylor ones and somewhat resembles the flute instability, well-known in the theory of magnetic traps for thermonuclear fusion. As at the flute instability, the "diving force" of the instability in the near-anode region is the magnetic drift of charged particles, but with the difference that instead of predominantly the ion drift, for the instability in the near-anode area, the electron drift is responsible and the ions are not magnetized.

IV. Role of Low-Frequency Instability in Electrons Transfer in the Near-Anode Region

As was noted above, the presence of the *Y*- component of the wave vector is of principal importance for the existence of the considered instability. It means, in turn, that in the wave, exited by the instability, the *Y*-component of the electric field (azimuthal electric field) is always presented. In the radial magnetic field and alternative azimuthal electric field, the electrons drift along the acceleration channel. Owing the fact that the unperturbed concentration of the electrons increases in the direction to the ionization area, the transfer of electrons in the field of the wave towards the anode exceeds the transfer in the opposite direction.

We can outline the development of the instability in the following manner. When the discharge initiates in the thruster, the gradient of the electron pressure in the near-anode region will begin increase. Because the collision mechanism can be inadequate to provide the needed flux of the electrons towards anode, this growth will continue until the relative gradient of the electron density ($1/l_N$) exceeds the relative gradient of the magnetic field induction ($1/l_B$). As soon as this takes place, the considered instability will arise. This will bring about the enhanced electron flux towards the anode. The increase of the electron gradient would be expected to stop at small exceeding $1/l_N$ over $1/_B$ as a consequence of the high effectiveness of the wave mechanism of the electron transfer.

V. About the Possibility to Experimentally Verify the Instability

As is well-known in the plasma of Hall thruster, a broad spectrum of the oscillations is observed. It extends from dozens kHz to several GHz. For the experimental verification of the presented theoretical model, it is necessary to identify the oscillations, which can be excited with the considered instability. With this purpose, the following properties of the waves should be taken into account:

1) The waves, due to the instability, must occupy the range of 50 – 200 kHz;
2) The waves should be localized predominantly in the near-anode region;
3) The waves must necessarily content the azimuthal component of the electric field.

In experimental investigations of the plasma instabilities, the analysis of the threshold properties of the instability is one of the most reliable methods of its indefication[8]. Therefore, it is of special interest to investigate the distribution of the plasma density in the near-anode area and compare it with the distribution of the magnetic field induction. If the amplitude of "suspect wave" grows as the difference between the relative gradient of the density and the relative gradient of the magnetic field grows, this will be an argument of importance in support of the presented theory.

VI. Conclusions

As a result of carried out investigations, one can make the following conclusions:

1. A theoretical model of the low-frequency instability, which must bring about the enhanced transfer of the electrons, has been built.
2. The instability is due to the gradients of both the plasma density and magnetic field induction in the near-anode area.
3. The instability by its nature belongs to Rayleigh-Taylor type.
4. A number of guidelines, which can be used to identify the instability, have been proposed.

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