Victor B. Tikhonov’s MHD Channel Theory: a review

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Abstract

MHD channel has been extensively investigated since the late ‘50s to better understand the physical mechanisms which govern an accelerating plasma flow. Here we present the MHD channel analysis as it was performed by Professor Tikhonov. This is one of the most relevant works on this subject, because it deals with the problem in a very general way, relaxing as many simplifying assumptions as possible. Besides, the analysis of V. Tikhonov extends also to the MPD thruster analysis, with special attention to the instability phenomena which take place in this kind of thrusters. We will cover also this second part of his analysis, in order to give a complete overview of his work.

Nomenclature

\[ a \] = sound speed
\[ B \] = magnetic field
\[ c_p \] = specific heat at constant pressure
\[ E \] = electric field
\[ I \] = total current intensity
\[ j \] = current density
\[ M \] = Mach number
\[ m \] = mass flow rate
\[ p \] = pressure
\[ R \] = gas constant
\[ t \] = time
\[ T \] = temperature
\[ v \] = flow speed
\[ \mu \] = vacuum magnetic permeability
\[ \rho \] = gas density
\[ \sigma \] = electric conductivity
I. Introduction

This paper aims at reconstructing step by step one of the most important works by Professor V. Tikhonov: the complete MHD channel analysis and its implications on the MPD thrusters theory (with special regard to the so called “onset problem”).

MHD channel analysis is usually performed assuming the plasma as a conducting fluid and treating the problem as a mono-dimensional one: with these two main assumptions the governing equations are considerably simplified and they can be solved analytically for a simple channel geometry. MHD channel has been extensively investigated since the late '50s. The most relevant works in this field were carried out by Resler and Sears (1957), Jahn, Sutton and by Tikhonov himself. Initially, the main purpose of these studies was to understand the physical phenomena laying behind the electro-magnetic acceleration of a plasma flow inside a duct. Then, after the discovery of MPD thrusters, a special interest for this kind of analysis has risen all through the world of electric propulsion since it was clear that the working principles of these new devices and the MHD channel were tightly connected.

Actually, as we are going to see in the following paragraphs, Tikhonov’s work on MHD channel was just the first step of a more complex analysis regarding the accelerative process inside an MPD thruster (with and without applied magnetic field).

II. Tikhonov’s analysis: an overview

Before presenting Tikhonov’s analysis, it is necessary to remind the main reason which led him to study this subject. It is well known that MPD thrusters show an unstable behavior when the applied current exceeds a limit value (actually there are also other factors which affect the instability occurrence). This phenomenon, usually addressed with the name of “onset”, causes a heavy depletion of the overall performance of the thruster. However, outside the instability region, MPDTs seem to work better for higher values of the applied current. So, it is a key issue for their future development to understand what are the physical factors governing the onset phenomenon, in order to delay its occurrence and have the possibility of raising the applied current intensity.

Tikhonov deeply studied the MHD channel and extended his analysis to MPD Thrusters as well. His purpose was to try to unravel the reasons of the malfunctioning of these devices. He also provided two different criteria for predicting the onset occurrence: the first criterion is relative to self-induced MPDTs, the second one to applied field MPDTs.

With the present work, we have carefully tracked back all the steps carried out by Tikhonov. All the analytical procedures has been implemented again and some of the original diagrams (which are presented in appendix “A”) have been faithfully re-drawn by means of MATLAB and Fortran routines.

In the next three sections we are going to present the core of Tikhonov’s work. This can be distinctly split in three different sections:

1) Analysis of the MHD channel.

2) Analysis of a Coaxial High Current Thruster (CHCT), which is a self-field MPD thruster with a simple geometry.

3) Analysis of a Butt-end High Current Thruster (BHCT) with an externally applied magnetic field.

III. MHD Channel Analysis

Here as follows we present Tikhonov’s analysis of the ideal MHD channel. This kind of analysis has been performed by other authors in the past, but Tikhonov’s work was probably the most exhaustive one. He tried to limit the initial assumptions as much as possible (although a certain amount of simplifying assumptions is always needed when attempting to find an analytical solution to such a complex problem) and in addition he also studied the evolution of other significant processes inside the channel, paying special attention to polytropic processes.
A. General Assumptions and Acceleration Contributions

The basic assumptions are:

1) 1D flow
2) \( E \perp B \)
3) \( j \perp B \)
4) Self induced magnetic field is neglected (only applied field is present)
5) Steady operating conditions: \( \frac{\partial}{\partial t} = 0 \)
6) Number density and temperature of electrons and ions are equal (\( T_e = T_i, n_e = n_i \)).
7) Hall parameter, \( \beta \), much lower than unity (which means highly collisional plasma).
8) Constant specific heat ratio (\( \gamma = 5/3 \), monoatomic gas).
9) The fluid is considered a perfect gas (so the equation of state is simply: \( pv = RT \)).

Under these hypotheses, the governing equations of the plasma flow are the following:

Continuity equation:

\[
\rho v A = \text{constant} \tag{3.1}
\]

Momentum equation along z-axis:

\[
\frac{dv}{dz} = \frac{1}{\rho} \frac{dp}{dz} + \frac{1}{\rho} jB \tag{3.2}
\]

Energy equation:

\[
\left( \frac{j^2}{\sigma} - Q_{\text{loss}} \right) = \rho v c_p \frac{dT}{dz} - v \frac{dp}{dz} \tag{3.3}
\]

Equation of state:

\[
p = \rho RT \tag{3.4}
\]

Ohm’s law:

\[
j = \sigma (E - vB) \tag{3.5}
\]
Combining together the three conservation equations (3.1), (3.2) and (3.3), it is possible to obtain the following expression for plasma flow acceleration inside the channel:

$$\frac{dv}{dz} = \left[ \frac{1}{M^2-1} \right] \left[ \frac{v}{A} \frac{dA}{dz} - \frac{\gamma - 1}{\rho a^2} \left( \frac{j^2}{\sigma} - Q_{\text{loss}} \right) + \frac{jBv}{\rho a^2} \right]$$  \hspace{1cm} (3.6)

From this equation we can see that there are three different contributions to the plasma flow acceleration, and all of them change their sign when the flow switches from subsonic to supersonic.

First contribution: duct shape
$$\frac{1}{M^2-1} \frac{v}{A} \frac{dA}{dz}$$
the flow accelerates in a converging nozzle when it is subsonic and accelerates in a diverging nozzle when supersonic.

Second contribution: ohmic heating
$$- \frac{\gamma - 1}{\rho a^2} \left( \frac{j^2}{\sigma} - Q_{\text{loss}} \right)$$
this contribution accelerates the flow when it is subsonic.

Third contribution: Lorentz force
$$\frac{jBv}{\rho a^2}$$
Lorentz force contributes to the plasma flow acceleration in supersonic regime.

B. Process Representation on a \((M, \Pi)\) Plane for a Constant Area Channel with no Heat Losses

Let us add the following two assumptions to the ones already stated in the previous paragraph:

1) \(A = \text{const}\) (Channel with constant cross sectional area)
2) \(Q_{\text{loss}} = 0\) (no heat losses)

With these two new assumptions, the equations (3.1) – (3.5) become:

\[ \rho v = \text{constant} \]  \hspace{1cm} (3.1')

\[ v \frac{dv}{dz} = -\frac{1}{\rho} \frac{dp}{dz} + \frac{1}{\rho} jB \]  \hspace{1cm} (3.2')

\[ \frac{j^2}{\sigma} = \rho v c_p \frac{dT}{dz} - v \frac{dp}{dz} \]  \hspace{1cm} (3.3')

\[ p = \rho RT \]  \hspace{1cm} (3.4')

\(*\)
\[ j = \sigma (E - vB) \]  \hspace{1cm} (3.5')

Now we introduce the following two dimensionless parameters:

\[ M \text{ (Mach number)} \quad \text{and} \quad \Pi = \frac{vB}{E} \]

Our set of five equations can be rearranged and elaborated in order to obtain a single equation for \( M \) as a function of \( \Pi \). Under our assumptions, this is possible for every acceleration process. So the crucial result is that we can always represent a plasma flow evolution in a \((M, \Pi)\) plane. We will see the great importance of the latter consideration in the next section, while now we are going to give a closer look to the features of \((M, \Pi)\) plane representation.

Combining the equations (3.1') - (3.5') and remembering the definitions of \( M \) and \( \Pi \), we can easily obtain the following two equations:

\[
\frac{dv}{dz} = \frac{\sigma E^2}{\rho a^2} \frac{(1-\Pi)}{M^2-1} \left[ (1-\Pi)(\gamma-1) - \Pi \right]
\]

\[
\frac{dM}{dz} = -\frac{1}{(M^2-1)^2} \frac{\sigma E^2}{\rho a^2} \left[ (1-\Pi)^2 \gamma \Pi \left[ 2(\gamma-1)M^2 + (\gamma-1)M^2 + 1 \right] \right]
\]

According to the first equation we can divide the \((M, \Pi)\) plane in different regions, depending on the sign of \( \frac{dv}{dz} \). From (3.7) immediately descends that:

\[
\frac{dv}{dz} = 0 \quad \text{when:} \quad \Pi = 1 \quad \text{or} \quad \Pi = \frac{\gamma-1}{\gamma}
\]

and

\[
\frac{dv}{dz} \quad \text{changes sign when} \quad M \text{ crosses the unity.}
\]

In fig. 2, regions where \( \frac{dv}{dz} \) is greater than zero are represented in red:

**Fig. 2: Regions of accelerating/decelerating flow on \((M, \Pi)\) plane**
In the same way, considering equation (3.8) we can distinguish regions on \((M, \Pi)\) plane where Mach number is increasing along the thruster, and regions where the opposite happens (fig. 3):

![Fig. 3: Regions of increasing/decreasing Mach number on (M,\Pi) plane](image)

Looking at these last two pictures, we must underline two important things.

1) it is essential to know if our flow is accelerating or decelerating. We are designing thrusters and we want to accelerate the fluid so our processes should always stay in the red-hatched regions illustrated in fig. 2. This means that if we want to cross the line \(M=1\), we have to cross it through the only point linking the subsonic acceleration region to the supersonic acceleration region. This point, addressed as “lower critical point”, is highlighted with a white star in Fig. 2 and in Fig. 3.

2) when a curve is drawn on \((M, \Pi)\) plane, we want to know in which way we are moving on it. This is easy to understand if we know the sign of \(dM/dz\) in every point of the plane (and that derives from equation (3.8) and it is shown in fig. 3).

C. Channel with Constant Cross Section, no Heat Losses and Constant E/B ratio

As we have already stated above, it is possible to represent every acceleration process with a curve on a \((M, \Pi)\) plane. Under the current assumptions, the governing equations are (3.1) - (3.5). If we combine them all and we can obtain the following equation:

\[
\rho v_c dT = \frac{E}{B} dp + \frac{E}{B} \rho v dv - \rho v^2 d\left(\frac{\nu^2}{2}\right)
\]

Integrating this equation from the initial section to a generic section along the channel, we obtain the following relation (all the physical quantities relative to the initial section are indicated with a subscript “0”):

\[
\rho_0 v_0 c_p (T - T_0) = \frac{E}{B} (p - p_0) + \frac{E}{B} \rho_0 v_0 (v - v_0) - \rho_0 v_0 \left(\frac{\nu^2}{2} - \frac{\nu_0^2}{2}\right)
\]

Introducing the non-dimensional parameters \(M\) and \(\Pi\) and reminding that we are working under the assumption of constant \(E/B\) ratio, it is possible to write down the following relation which directly links \(M\) and \(\Pi\):

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All the constant values have been put together in a single factor \((C_1)\), which thus depends on the initial conditions and on the \(E/B\) ratio itself.

From equations (3.9) we can draw the following curves on the plane \((M, \Pi)\). Different curves correspond to different values of \(C_1\), so to different initial conditions for the plasma flow:

![Curves describing acceleration processes in a constant area channel with constant E/B and zero heat losses](image)

From Fig. 4 it is clear that if we want to accelerate our flow from subsonic to supersonic we have to follow a precise path (the black one), in order to pass right through the lower critical point. So we have to properly tailor the initial conditions, otherwise our flow will remain subsonic all through the channel.

**D. Polytropic Processes**

Let us retain the assumptions of constant area duct and absence of losses \((A = \text{const}, \ Q_{\text{loss}} = 0\)\), and let us now examine the behaviour of polytropic processes, i.e. processes obeying a law of the type:

\[
p\rho^{-n} = C
\]  

(3.10)

Where \(n\) (polytropic index) and \(C\) are two constants.

Quite obviously, if in addition to the basic equations (3.11) - (3.5) we also want our flow to obey an equation like (3.10), we are not allowed to choose freely a value of \(E/B\) anymore, and so much the less can we set the intensity of both the electric and the magnetic field throughout the whole process. To follow a chosen polytropic process along the channel we have to relax the assumption of a constant \(E/B\) ratio.

Differentiating equation (3.10) with respect to \(z\) and properly combining it with the conservation equations, the Ohm’s law and the equation of state, we can derive the following expression which links together the Mach number, the non-dimensional parameter \(\mathcal{P}\) and the polytropic index, \(n\).
Eq. (3.11) is a relation between $M$ and $\Pi$ which is known for any choice of the polytropic exponent $n$. It thus enables us to draw on the $(M, \Pi)$ plane the curve representing the polytropic process of exponent $n$. A family of such curves is shown in Fig. 5.

$$n = k \frac{M^2(k-1)(1-\Pi) - \Pi}{(k-1)(1-\Pi) - \Pi}$$

(3.11)

Some of the values of $n$ used in the above diagram correspond to cases of special significance:

$n = 1$: in this case equation (3.10) becomes $p/\rho = \text{const}$; based on the state equation this implies: $RT = \text{cost} \Rightarrow T = \text{cost}$. The process described by this curve is therefore isothermal ($dT/dz = 0$).

$n = 0$: equation (3.10) becomes $p = \text{cost}$. The process described by this curve is therefore isobaric ($dp/dz = 0$).

For $n = -1$ we obtain the curve on which $dM/dz = 0$.

E. Polytropic Processes with Heat Losses

Let us write the energy equation in the following way:

$$\frac{j^2}{\sigma}(1 - \alpha) = \rho v c\, \frac{dT}{dz} - v \frac{dp}{dz}$$

(3.12)

Here the heat losses have been described as a fraction of the joule heating:

$$Q_{loss} = \alpha \frac{j^2}{\sigma}$$

(3.13)

If we follow the same procedure applied in the previous paragraph, we come to the following equation which links $M$, $\Pi$ and the polytropic index $n$ for a more general case where $\alpha \neq 0$.
\[ n = k \frac{M^2 (k-1)(1-\alpha)(1-\Pi) - \Pi}{(k-1)(1-\alpha)(1-\Pi) - \Pi} \]  

(3.14)

Plotting the diagrams relative to this new equation, we obtain:

**Fig. 6: Polytropic processes with heat losses for \( \alpha > 0 \)**

**Fig. 7: Polytropic processes with heat losses for \( \alpha < 0 \)**

For \( \alpha > 0 \) we can see that all the processes reach the condition \( M = 1 \) for lower values of \( P \) (consequently for lower value of flow velocity, \( v \)). This is obvious, because if \( \alpha > 0 \) that means the we are taking off heat from the flow and so the sound speed is decreased (since we work at lower temperatures). The contrary is true if we add heat to the flow (\( \alpha < 0 \)) and in that case (fig. 7) we reach the condition \( M = 1 \) for higher values of \( P \).

**IV. Coaxial High Current Thruster Analysis**

To understand something more about the acceleration process inside a real MPD thruster, Tikhonov extended his analysis to the case of a Coaxial High Current Thruster with a constant cross sectional area (fig. 8). He investigated
the system of governing equations, trying to attempt an analytical solution. The governing equations and the assumptions made for this case are very similar to the ones written before for the ideal MHD channel.

![Diagram of Coaxial High Current Thruster with cylindrical electrodes]

Fig. 8: Coaxial High Current Thruster with cylindrical electrodes

A. Assumptions
We deal with a constant cross section thruster. Both electrodes are cylinders and they have the same length. The flux is assumed 1D (so all the parameters are constant throughout each section). The current density and the electric field are both radially directed. There is no externally applied magnetic field (only the self-induced one is present, and it is everywhere in the azimuthal direction).

All the electric current flows in the inter-electrode region, consequently the self induced magnetic field is maximum at the base of the cathode and it is zero in the exit section. Electron and ions have the same temperature and number density ($T_e = T_i, n_e = n_i$). Hall parameter, $\beta$, is much lower than unity (which means highly collisional plasma). Specific heat ratio has a constant value ($\gamma = 5/3$, monoatomic gas). The fluid is considered a perfect gas (so the equation of state is simply: $p\nu = RT$). Once again, we assume to operate in steady conditions.

B. Equations
Under these hypotheses the governing equation are the following:

Continuity equation: $\rho \nu = \text{constant}$ (4.1)

Momentum conservation equation (obviously along z-axis):

$$\rho \nu \nu + dp = j B \nu$$ (4.2)

Energy conservation equation:

$$\rho \nu ^2 \nu + \rho \nu \nu \nu T = j E \nu$$ (4.3)

Equation of state:

$$p = \rho RT$$ (4.4)

Relation between plasma temperature and electrical conductivity:

$$\sigma = \sigma_0 \left( \frac{T}{T_0} \right)^{3/2}$$ (4.5)

Ohm’s law:
Maxwell equation to link the current density to the self induced magnetic field (that is the new equation! We do not need it in the MHD channel analysis, because there we work with an applied $B$ field):

$$
\nabla \times \vec{B} = \mu \vec{j} \quad \Rightarrow \quad j = -\frac{1}{\mu} \frac{dB}{dz}
$$

(4.7)

C. Solution

In order to solve our system of equations we introduce dimensionless quantities simply dividing each parameter by its initial value. Dimensionless variables are designated with an underscore.

Using this notation, we obtain:

\begin{align*}
\bar{\rho} &= 1 \\
\gamma \cdot M_0^2 \bar{\rho} + d\bar{p} + A_0 d(\bar{B}^2) &= 0 \\
\frac{\gamma - 1}{2} M_0^2 d(\bar{\rho}^2) + d\bar{T} + \frac{\gamma - 1}{\gamma} \frac{A_0}{\Pi_0} d\bar{B} &= 0 \\
\bar{p} &= \rho \bar{T} \\
\bar{\sigma} &= \bar{T}^{3/2} \\
\bar{J} &= \sigma (1 - \bar{\Pi} \Pi_0) \\
\bar{J} &= -\frac{d\bar{B}}{dz}
\end{align*}

(4.8)

$\Pi$ have been introduced in the previous section, while $A_0$ is simply the ratio between the magnetic pressure and the gas dynamic pressure, evaluated in the section where $M=1$:

$$
A_0 = \left( \frac{p_{\text{mag}}}{p_{\text{gas}}} \right)_{M=1}
$$

In the next paragraph we are going to show how and why $A_0$ and $\Pi$ are two key parameters for our analysis.

After some substitution we come to the following system of equations:

\begin{align*}
\gamma \cdot M_0^2 \cdot (\bar{T} - \bar{B}) + \frac{\bar{T}}{M^2} - \bar{B} + A_0 \bar{B}(\bar{B}^2 - 1) &= 0 \\
M_0^2 (\bar{T}^2 - \bar{B}^2) + \frac{2}{\gamma - 1} \left( \frac{\bar{T}^2}{M^2} - \bar{B}^2 \right) + \frac{4 A_0}{\gamma \Pi_0} (\bar{B} - 1) \bar{B}^2 &= 0 \\
\bar{z} &= -\int_{1}^{\bar{\Pi}} \frac{A_0}{M^2 \bar{B}^3} (1 - \bar{\Pi} \Pi_0) \frac{d\bar{B}}{\bar{B}^2}
\end{align*}

(4.9)

(4.10)

(4.11)
Since we know that $B$ varies between 1 (for $z = 0$) and 0 (for $z = L$) we can solve these equations, tracking the evolution of various dimensionless parameter along the thruster. Plotting $B$ versus $z$ for different values of initial conditions, we obtain the graph in Fig. 9.

A striking result is apparent: we cannot obtain an acceptable solution if $A_0$ exceeds a critical value ($z$ becomes imaginary, that is why the light blue curve is suddenly interrupted).

D. $A_0$, $\Pi$ and the Onset Phenomenon

$\Pi$ is defined as the ratio between $v$ times $B$ and the electric field $E$. If we have a closer look to the Ohm’s law we find that should $\Pi$ become greater than unity, the current density would change its sign. This means that the Lorenz force would then act in the opposite way, braking the flow instead of accelerating it. When the velocity is sufficiently reduced, $\Pi$ becomes less than unity again and the Lorentz force comes back to accelerate the fluid, until $\Pi$ exceeds unity again and so on. Such a phenomenon can be associated to an instability regime which causes very poor performance. For our simple case, this could be a reasonable explanation for the “onset problem” nature.

According to what was stated in the previous section, it is clearly possible to give a representation of the processes described by equations (4.1)-(4.7) on $(M, \Pi)$ plane. This new representation (fig. 10) show us that for certain $A_0$ values the plasma flow enters the instability region (where $\Pi > 1$).
$A_0$ is a non-dimensional parameter fully dependent on initial conditions. From this analysis it seems that a large value of $A_0$ can drive $\Pi$ beyond unity, thus generating an instability inside the thruster. We can conclude that $A_0$ is the parameter that controls the onset occurrence.

Developing $A_0$ expression we can derive a very interesting result:

$$A_0 = \frac{p_{magnetica}}{p_{gasdinamica}} = \frac{B^2}{2\mu} \frac{1}{\rho RT} = \frac{\gamma I^2}{2ma} 10^{-7} \tag{4.11}$$

For this simple configuration we have proved a result already well known from many experiments carried out on MPD thrusters. We cannot raise the ratio $I^2/m$ over a certain value.

It is worth underlying also that expression (4.11) includes the parameter $a$ (sound speed), equal to $a = \sqrt{\frac{\gamma RT}{\mu}}$, where $R$ is a function of propellant molecular mass; this suggest that different propellants could experience slightly different “onset limits”.

Moreover the previous analysis also gives a physical explanation of the onset phenomenon, even if it is quite difficult to apply the same considerations to a more complex thruster configuration.

**F. The Onset Limit: Tikhonov’s first criterion**

From CHCT analysis we have seen that to avoid the onset occurrence $A_0$ has to stay below a limit value. However we have not stated what this limit value is. Although it is not apparent from the previous analysis, experiments carried out by Prof. Tikhonov suggest that in a real self-field MPDT also the thruster geometry affects the onset occurrence. The following relation is known as the first Tikhonov’s criterion and it tells us the maximum value allowed for $A_0$ if we want the thruster to show a stable behaviour. The right-hand-side of this criterion takes into account the role played by thruster geometry and it has an empirical derivation:

$$Tikhonov’s \ criterion \ for \ self-induced \ MPDTs: \quad A_0 > \frac{3.6}{\frac{R_a}{R_c} - 0.5} \tag{4.12}$$

Where:  
$R_a$ = anode radius  
$R_c$ = cathode radius

**V. Butt-end High Current Thruster Analysis**

Let us now take into consideration a real thruster as the one represented in Fig. 11. Many of the previous assumptions are not valid anymore and the set of governing equations is far more complicated. It is impossible to attempt an analytical solution, so it is necessary to rely on experimental results if we want to delve into the onset nature.

![Fig. 11: MPDT with a short cathode and an externally applied magnetic field](image-url)
Prof. Tikhonov collected and analysed the results coming from a massive experimental campaign and extrapolated a general criterion for the stability of the discharge inside an MPD thruster (Tikhonov’s criterion for applied-field MPDTs). According to that criterion, any MPD thruster shows a stable behaviour unless:

\[
A_0' = \frac{I \left( B_{\text{self-induced}} + B_{\text{applied}} \right) (R_a - R_e)}{\mu a m} \times 10^{-7} > \frac{3.6}{R_a - R_e - 0.5} \quad (5.1)
\]

The criterion has been built maintaining for the right-hand-side the same expression already used in relation (4.12). Consequently, the left-hand-side has been changed introducing a new dimensionless parameter, \( A_0' \) instead of \( A_0 \). This criterion predicts the onset limit with rather good accuracy. Its only flaw is that the above relation has an empirical derivation and does not tell us anything about the possible causes that lie behind the onset occurrence. Anyway we cannot neglect the utility of this relation, as it shows us what are the parameters which influence the stability of an MPD thruster. Besides we can also get an idea of how to act on these parameters in order to hamper the onset occurrence:

a) Geometry: it is advisable to keep the ratio \( R_a/R_e \) on low values (say about 2-2.5) if we desire a higher onset limit.

b) Applied field: although the presence of an external magnetic field is a benefit for the total thrust, it is evident from relation (5.1) that an excessive intensity of that same field could finally lead to instability phenomena.

c) Propellant molecular mass: leaving all the other conditions unchanged, the sound speed is higher for a propellant with a lighter molecular mass (\( a \propto \sqrt{R_m/M_{\text{propellant}}} \)). According to equation (5.1) it seems advisable to use light propellants.

These precious suggestions hidden in Tikhonov’s criterion are useful in designing MPD thrusters where the onset is much delayed. Extending the region of stable behaviour is extremely important, because if higher values of the current intensity are allowed we can reach higher thrusts, specific impulses and efficiencies.

VI. Conclusion

This paper provides an overview of the work on MPD thrusters carried out by Professor Tikhonov. In order to present Tikhonov’s remarkable results in a systematic way we have gathered materials from several sources, most of which are scarcely known or accessible to non Russian-speaking readers.

Special emphasis has been given to the MHD channel analysis, where the systems of equations have been implemented and solved using MatLab. Each diagram has been drawn anew, showing the behaviour of a plasma flow accelerating inside a duct under different conditions.

In the second part of our work, we have revised Tikhonov’s theory on the MPD thruster and his analysis on the onset problem. We have presented the criteria he found to state if the electric discharge inside an MPDT has an unstable behaviour. This two stability criteria are probably the most important achievement of Tikhonov’s whole research in this field, because they predict the onset occurrence with a good accuracy and they “hide” some precious suggestions to design an MPD thruster with enhanced performance.
Appendix A: Tikhonov’s original diagrams

Fig. 12: MHD Channel: acceleration processes in a constant area channel with constant E/B and zero heat losses

Fig. 13: MHD Channel: polytropic processes
Fig. 14: CHCT: azimuthal magnetic field evolution along the thruster

Fig. 15: Plasma flows inside a CHCT represented on the (M,β) plane
Fig. 16: BHCT (Butt-end High Current Thruster)

Fig. 17: Graphic representation of Tikhonov’s stability criterion for MPD thrusters

References


