VUV Radiation in a Hall Effect Thruster: Preliminary Results

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A simplified 2D ray-tracing method has been applied to the simulation of the radiative transfer from the plasma to the internal walls of an Hall Effect Thruster. Departing from simulated axial profiles of neutral/electron densities and temperatures inside the thruster internal channel, local radiative emission and absorption coefficients have been obtained using the SPARTAN line-by-line code, coupled to a Xenon spectral database issued from NIST. Radiative transfer is then simulated in a tangent slab approximation. The predicted intensity and spectral distribution of the radiation impacting the thruster walls will help estimating the influence of radiation in wall energy deposition effects, as well as providing improved estimations of radiation-induced erosion of ceramics. Future extensions of this work will include a more general 3D ray-tracing description for radiative transfer.

Nomenclature

\begin{itemize}
\item $\alpha(\nu)$ = absorption coefficient (cm\textsuperscript{-1})
\item $\alpha'(\nu)$ = absorption coefficient corrected for self absorption (cm\textsuperscript{-1})
\item $\varepsilon(\nu)$ = emission coefficient (W/cm\textsuperscript{3}/cm\textsuperscript{-1}/sr)
\item $\theta$ = zenithal angle
\item $\nu$ = spectral frequency (cm\textsuperscript{-1})
\item $\phi$ = azimuthal angle
\item $N$ = species number density (cm\textsuperscript{-3})
\item $N_e$ = electronic level number density (cm\textsuperscript{-3})
\item $E_e$ = electronic level energy (J)
\item $I_\nu(\theta, \phi)$ = radiative energy (W/cm\textsuperscript{3}/cm\textsuperscript{-1}/sr)
\item $Q_{\text{tot}}$ = total partition function
\item $T_{el}$ = electrons temperature (K)
\item $T$ = heavy-species temperature (K)
\item $g_e$ = electronic level degeneracy
\item $l$ = ray length (cm)
\item $q_\nu$ = radiative wall fluxes (W/cm\textsuperscript{2})
\item $r_1$ = inner channel minimum radius (cm)
\item $r_2$ = inner channel maximum radius (cm)
\item $k_B$ = Boltzmann constant $1.38066 \times 10^{-23}$ (J/K)
\end{itemize}

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I. Introduction

The near-UV to near-IR optical range has been extensively used for the characterization of the ionization and acceleration zones in the annular insulated channel of the HET [1,2,3]. Recorded Time-resolved spectra has been used to study the “breathing mode” of the plasma. The properties of the erosion of the ceramics have been studied by the ratio of emitted lines as XeI, XeII, B with Corona and actinometry techniques [4,5,6].

The plasma discharge in a Hall Effect Thruster (HET) used for the station-keeping of geostationary satellites and interplanetary missions is optically emissive for a large wavelength range (VUV to IR). The near-UV to near-IR optical range has been extensively used for the characterization of the ionization and acceleration zones in the annular insulated channel of the HET [1,2,3]. Recorded Time-resolved spectra has been used to study the “breathing mode” of the plasma. The properties of the erosion of the ceramics have been studied by the ratio of emitted lines as XeI, XeII, B with Corona and actinometry techniques [4,5,6].

However, the VUV–UV optical range presents a large interest because a large part of the radiative energy could be transferred in this wavelength range (below 2000 A) and has to be considered for the energy deposition on the walls of the channel. Moreover, this radiation could play a role in the secondary electron emission (s.e.e.) rate of the ceramics. This rate is generally only correlated to the incident electron striking. It is also one of the main parameters controlling the plasma discharge inside the channel and consequently the performances of the thruster.

No data are today available on the VUV emission of the Hall effect plasma discharge mainly due to the difficulties to perform such as measurement with a spectrometer operating at reduced pressure and with an adapted detector.

A preliminary study has been performed to calculate the spectrum emitted in the VUV range for the plasma in the channel of the PPS100–ML thruster. The electrons properties (density, temperature) are taken from the results of a hybrid code calculation\(^1\). The Line-by-Line code SPARTAN\(^2\) is then utilized to obtain the local radiative properties of the plasma, and finally a Ray-Tracing routine is applied for simulating the spectral-dependent radiative fluxes impacting the wall. Several geometric simplifications have been assumed in this work, which is considered as a stepping stone towards a more definite model. These are described in detail in the next section.

II. Methods

We depart from the axial neutral and electron density profiles calculated by L. Garrigues,\(^1\) as well as the average electron energies. These are reported in Fig. 1.

![Figure 1](image)

Figure 1. Left: Axial profiles of the neutral densities (black); Right: Axial profiles of the electron densities (black) and average energy (black).

For the purpose of our calculation, we consider that the plasma flow is exclusively composed from the
species Xe, XeII, and electrons, and hence, the density of XeII is equivalent to the electronic density, and the density of Xe is equivalent to the neutrals density. For the purpose of this work, the distribution of the internal electronic levels of Xe and XeII is assumed to be in equilibrium with the average energy of the free electrons of the flow (owing to the high efficiency of e-E energy exchange):

\[
\frac{N_e}{N} = \frac{g_e \exp \left(\frac{-E}{k_B T_{el}}\right)}{Q_{tot}}
\]  

(1)

The assumption that the internal electronic levels of Xenon follow a Boltzmann distribution may be questioned. However, not considering such an hypothesis leads to the necessity of defining a complex collisional-radiative model for populating such internal levels. This is a very time-consuming task and remains for now outside the scope of this work.

Making use of these assumptions, spectra is calculated for the different points in the channel using the line-by-line code SPARTAN, using an internal levels and line-by-line transitions list issued from the NIST database.\(^4\)

A sample emission spectra, obtained at the exit of the thruster (\(x = 4\) cm), is presented in Fig. 2. We can verify that neutral Xe accounts for most of the volume radiation at the nozzle exit (35 mW/cm\(^3\)), mostly in the VUV (1000–2000 Å), with radiation of XeII accounting for only 0.6 mW/cm\(^3\) in the visible region (4000–7000 Å).

![Figure 2. Spectra from neutral Xenon (blue) and ionized Xenon (red) at the exit of a PPS100–ML, for a Xenon mass flow of 5 mg/s. An apparatus function of 10 Å is considered for a better visualization of the spectrum lines.](image)

A. Ray-Tracing Routine

Radiative transfer towards the thruster internal walls is treated according to the ray-tracing approach, which provides an exact solution to the problem, provided that a large enough number of rays is sampled (in other terms, the angular intervals \(d\theta\) and \(d\phi\) need to be small enough). Here we consider the zenithal angle \(\theta\) as the angle in the radial plane of the thruster, and the azimuthal angle \(\phi\) as the angle in the axial plane of the thruster. In addition, we can take into account the symmetry over the zenithal angle \(\theta\), and we may finally write:\(^5\)

\[
q_\nu = 2 \int_{-\pi/2}^{\pi/2} \int_0^{\pi/2} I_\nu(\theta, \phi) \cos \theta \sin \theta d\theta d\phi
\]  

(2)

and for each ray path, we have the traditional equation for radiative transfer (excluding scattering):

\[
\frac{dI}{dl} = \varepsilon_\nu - \alpha_\nu I_\nu
\]  

(3)
In this preliminary investigation, we are further considering a simpler slab approach for radiative transfer, with the tracing of rays restricted to the zenithal plane \((\phi = 0, d\phi = \pi)\). This simplification is justified by the fact that a full 3D ray-tracing routine requires additional modeling efforts which are still underway. A 2D approach will slightly underestimate the fluxes impacting the wall, but can be carried out in a much simpler semi-analytical fashion, and in a more expedite way. We further only consider radiative transfer to the internal walls of the outer ring. Fig. 3 shows the geometrical characteristics of the zenithal rays impacting the walls of a sample Hall thruster.

After some trigonometrical manipulation, the length of each ray may be analytically calculated, as a function of angle \(\theta\) (see Fig. 3):

\[
l(\theta) = r_2 \left( \frac{1 + \cos(2\theta)}{\cos(\theta)} \right) - r_2 \left( \cos(\theta) + \sqrt{\cos(\theta) + \left( \frac{r_1}{r_2} \right)^2 - 1} \right) \quad \theta < \cos^{-1}\left( \frac{r_2^2 - r_1^2}{r_2} \right), \quad (4)
\]

\[
l(\theta) = r_2 \left( \frac{1 + \cos(2\theta)}{\cos(\theta)} \right) \quad \theta > \cos^{-1}\left( \frac{r_2^2 - r_1^2}{r_2} \right). \quad (5)
\]

In the case of this approach, the plasma properties are homogeneous over the radial plane, and we may rewrite Eq. 3 such that:

\[
I_\nu = \frac{\varepsilon}{\alpha'} \left( 1 - \exp(-\alpha l(\theta)) \right), \quad (6)
\]

\[
\text{with} \quad \alpha' = \alpha \left( 1 - \exp\left( \frac{\nu}{k_B T} \right) \right), \quad (7)
\]

with the heavy-species translational temperature \(T\) being assumed to be an average of 5eV.\(^1\)

Taking into account that emission \(\varepsilon\) and absorption coefficients \(\alpha\) are constant in the zenithal plane, we may write the simplified form of Eq. 2 in the case of the slab approximation:
\[ q_{\nu} = 2\pi \int_0^{\pi/2} \frac{\varepsilon}{\alpha} [1 - \exp(-\alpha l(\theta))] \cos \theta \sin \theta d\theta. \] (8)

III. Results

Eq. 8 has been solved for different points in the HET internal channel, considering the emission and absorption coefficients calculated from the density and temperature profiles retrieved from Fig. 1. Eq. 8 has been solved for 90 equispaced zenithal rays, for the 13 axial positions of the HET channel. The calculated spectral-dependent wall fluxes are presented in Fig. 4.

The examination of the obtained results shows a close dependence between the spectral and axial distribution of the wall fluxes, and the neutral Xe and ionized XeII profiles of Fig. 1. VUV radiation is exclusively from neutral Xe (as well as a line at 7000\(\AA\)), and will decrease near the HET channel exit, line with the decrease in concentrations. There is some amount of absorption in the VUV, as the ratio from the VUV lines and the line at 7000\(\AA\) is smaller than in the case of the emission coefficients plot (see Fig. 2). However, the absorption in the VUV range remains limited, possible as a consequence of the small optical paths (8cm at most). In the case of visible radiation, it has peak near the nozzle exit, closely correlated to the peak in the concentration of XeII (see Fig. 2).

We then proceed to the integration of the spectral-dependent wall fluxes:

\[ q_{\nu} = \int_0^{\infty} q_{\nu} d\nu. \] (9)

The integrated wall fluxes are plotted in Fig. 5, for the overall length of the HET channel. As a comparison, the volumetric emission coefficients \((4\pi \varepsilon)\) are also represented.

We see that the fluxes in the external wall of the HET channel remain close to 0.12W/m\(^2\), then falling down to 0.04W/m\(^2\) in the channel exit, following a trend very close to the neutral Xe density profiles of Fig. 1. This is not surprising as the most prominent radiative transitions (from neutral Xe) are located in the VUV region, which is also the most energetic region of the spectra. This is also a trend that closely mirrors the volumetric emission coefficient inside the thruster channel.
Figure 5. Integrated emission coefficients and radiative wall fluxes in the HET channel.

IV. Prospectives

This study has laid down the groundwork for a detailed 3D ray-tracing simulation of radiative transfer inside the channel of an HET, while at the same time providing reasonably accurate values for the expected wall fluxes, using a simplified 2D slab approach. The results from this work should already provide sufficient quantitative insight on the radiative fluxes impacting the thruster channel outer walls, allowing to estimate the importance of radiation transfer, compared to other physical-chemical phenomena present in this class of thrusters.

We can outline some improvements that can be brought to the present approach, and their expected impact on the predicted results:

- Calculation of the radiative fluxes in the channel inner walls: This has not been carried out for a lack of time, and as the optical paths are considered as significantly shorter than for the outer walls, the wall fluxes are estimated to be 50–70% lower.

- Application of a full 3D ray-tracing routine instead of a 2D slab routine: It is expected that the overall fluxes (as represented in Fig. 5) will follow a more homogeneous trend (as radiation from the back of the channel will be allowed to carry to the walls in the front of the channel, and vice-versa). Differences from these two approaches should lead to around 30–50% variation in the obtained quantitative results, judging from other applications results.

- Improvement of the radiative model: Inclusion of VUV photoionization/radiative recombination cross-sections for Xe will be applied. This should not lead to significant increases in radiation unless radiative recombination effects are found to be significant.

- Application of a collisional-radiative model for the calculation of the Xe and XeII internal levels, instead of assuming a Boltzmann distribution of these: This is a significantly more complex and time-consuming task, but it is also the one which may potentially lead to larger variations in the predicted intensities.

- Considering a more detailed model in the flow inside the channel, namely accounting for the presence of boundary layers near the channel walls: This simply requires some minor updates of the routines for including a spatial grid. The presence of a colder boundary layer will lead to a stronger absorption and lower wall fluxes. However, in view of the short optical paths involved, decreases in the predicted wall fluxes should not be spectacular.

As this study is the first step before experimental measurement performed with a Hall effect thruster in the Pivoine test facility at ICARE laboratory (CNRS Orléans), it is expected that some validation of these predicted results might help determining the accuracy of the approach outlined in this work.
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References