Effect of electrons non-mirror reflection from potential shield on plasma borders inside helicon and Hall effect thrusters

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1. Introduction

Electric propulsion thrusters [1] are widely used in modern astronautics for realization of different space missions such as interplanetary missions to Venus, Mars, Jupiter Troyan asteroids, main belt asteroids [2], near Earth inter-orbit operations with satellites delivery from basic orbit to geosynchronous earth orbit [3,4]. Traditional plasma-ion thrusters [5] and Hall effect thrusters [6] are used here as well as comparatively new conceptions of magneto-plasma dynamics thrusters [7].

Characteristic feature of electric propulsion devices is the fact that free path length of any process is higher than device size. It means that electrons reflection from near-the-surface potential barrier is stronger factor of all the gas dynamics parameters change than the collisions in the volume.

The levels of current inside metallic structure elements is not of such scale to result into considerable potential change inside the metal – any metallic surface can be considered as equal potential one.

The potential of different points on dielectric details surface is different but the spatial scale of potential change is great comparatively with near-the surface Langmuir layer thickness. Thus inside the layer the dielectric surface can be also considered as equal potential one.

2. Potential distribution inside near-the-surface layer

Just near the surface of solid body the equal potential surfaces inside the layer reproduce the solid body surface geometry (fig. 1).

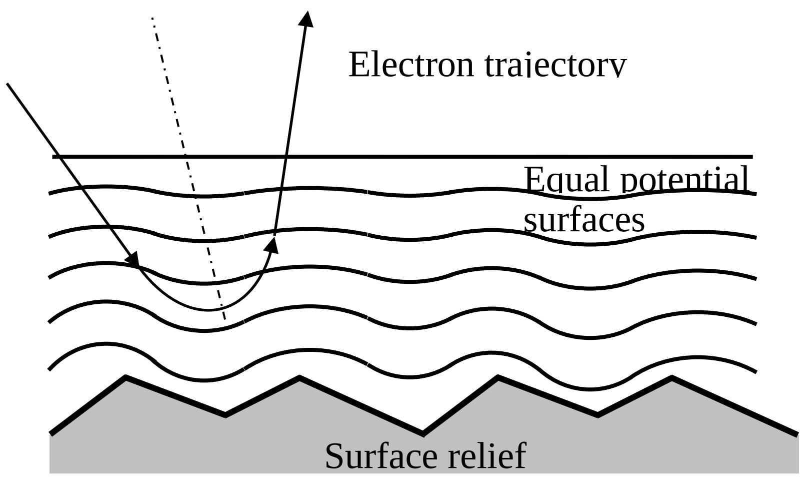

Fig. 1. Electrons reflection from near-the-surface potential barrier

As a result the potential distribution inside the layer is sufficiently inhomogeneous one. One of the consequences of this fact is elastic but non-mirror reflection of electrons from near-the-surface potential barrier – after collision electron returns into plasma with preservation of energy and motion absolute value but with the deviation from mirror reflection trajectory. So not only normal but also parallel to surface projection of motion changes in the reflection of electron from potential barrier.

The randomnicity of motion direction after reflection means, for example, the loss of parallel to surface motion projection – the part of direct movement energy is transferred into heat energy. It is necessary to take into account the fact that even in the case of ideally slim surface (that is impossible of course) the named effect takes place. As it had been shown in the work [8] even in steady-state and homogeneous boundary conditions own plasma oscillations take place, which do not need the external source. Plasma oscillations wave length is of the same order as screen length as well as Langmuir layer thickness. Plasma oscillations period and the time of electron's being inside the layer are also of the same order. So non-mirror reflection takes place also near ideally slim surface. Moreover, the same process takes place near the border of plasma and surrounding space.

The value of flow of the distribution function moment of n-th order onto the surface is equal to scalar product of (n+1)-th order moment and the ort of the normal to the surface:

$$\mathbf{M}_{\alpha}^{[n](S)} = \mathbf{M}_{\alpha}^{[n+1]} \cdot i_n = \int f_{\alpha}(\vec{v}) \mathbf{m}_{\alpha}^{[n+1]}(\vec{v}) \cdot i_n d\,\vec{v} = \int f_{\alpha}(\vec{v}) \mathbf{m}_{\alpha}^{[n]}(\vec{v}) v_n d\,\vec{v}\,. \tag{1}$$

Let's represent the integral in (1) as the sum of integrals in direct and reflected flow:

$$\mathbf{M}_{\alpha}^{[n+1]} \cdot i_n = \int\limits_{v_n \geq 0} \Gamma_{\alpha n}^{(r)}(\vec{v}) \mathbf{m}_{\alpha}^{[n]}(\vec{v}) d\,\vec{v} + \int\limits_{v_n < 0} \Gamma_{\alpha n}^{(r)}(\vec{v}) \mathbf{m}_{\alpha}^{[n]}(\vec{v}) d\,\vec{v}\,, \tag{2}$$

where

$$\mathbf{m}_{\alpha}^{[n]}(\vec{v}) = m_{\alpha} \vec{v}^{\lceil n \rceil}, \tag{3}$$

$$\Gamma_{\alpha n}^{(r)}(\vec{v}) = f_{\alpha}(\vec{v}) v_n\,. \tag{4}$$

In the case of mirror reflection the equivalence would take place in reflected flow with $v_n < 0$:

$$f_{\alpha 0}(\vec{v}) \Big|_{v_n < 0} = f_{\alpha}^{+}(\vec{v} - 2 i_n v_n), \tag{5}$$

where $f_{\alpha}^{+}(\vec{v})$ – distribution function in direct flow $v_n \geq 0$.

3. Electrons distribution function in reflected flow

Let's describe the electron movement after the reflection in spherically coordinate where the basic axis coincides with the direction i_v of mirror reflection (fig. 2). Let's also sign the unitary vector of trajectory direction after reflection as i'_v. So electron's velocity after collision can be expressed as:

$$\vec{v}' = v\left(i_v \cos\theta + (i_{\perp 1} \cos\psi + i_{\perp 2} \sin\psi) \sin\theta \right), \tag{6}$$

where v – velocity absolute value, which doesn't change in reflection;

$i_{\perp 1}$, $i_{\perp 2}$ – normal to i_v orts.

In the bulk of cases the deviation from the mirror reflection trajectory is small:

$$\theta << 1. \tag{7}$$

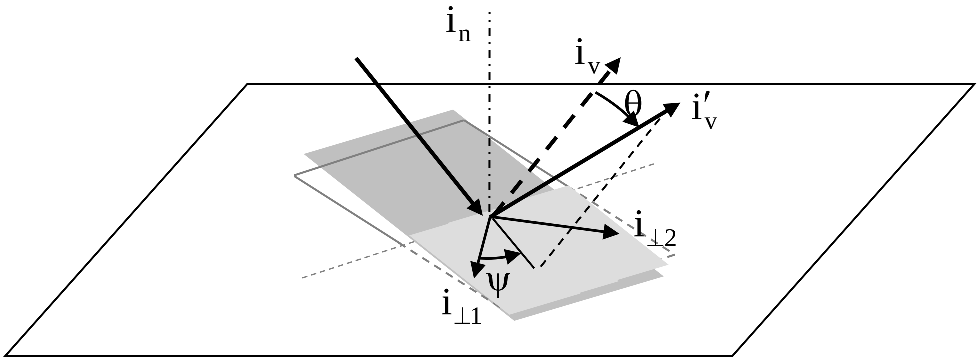

Fig. 2. Reflected movement coordinate system. i_n - normal to surface; i_v - basic axis coincides with the direction i_v of mirror reflection; i'_v - vector of trajectory direction after reflection; $i_{\perp 1}$, $i_{\perp 2}$ – normal to i_v orts; θ - deviation from the mirror reflection trajectory

It means that velocity $\vec{v}'$ differs small from mirror reflection velocity $\vec{v}$ as well as distribution function in reflected flow – from – от $f_{\alpha 0}(\vec{v})$. The analogue of collision integral in Landau form [9] can be used for determining of distribution function. The value of $\Gamma^{(r)}_{\alpha n}(\vec{v})$ can be represented in this case as:

$$\Gamma^{(r)}_{\alpha n}(\vec{v}) = \begin{cases} f^{+}_{\alpha}(\vec{v}) v_n, & v_n \geq 0 \\ f^{+}_{\alpha}(\vec{v} - 2 i_n v_n) v_n - \nabla_v \cdot \vec{J}^{(rv)}_{\alpha}(\vec{v}), & v_n < 0 \end{cases}, \tag{8}$$

where $\vec{J}^{(rv)}_{\alpha}(\vec{v})$ – electrons flow density in velocity space.

It is possible to write in Landau form for m-th projection of vector $\vec{J}^{(rv)}_{\alpha}(\vec{v})$:

$$J^{(rv)}_{\alpha m}(\vec{v}) = \frac{1}{2\pi} \int\limits_{\Delta v_m > 0} \int\limits_{v_m - \Delta v_m}^{v_m} \left(f_{\alpha 0}(\vec{v}) - f_{\alpha 0}(\vec{v} + \Delta\vec{v}) \right) v_n f(\theta) d\theta\, d\psi\, d v_m . \tag{9}$$

where $\Delta\vec{v}$ – electron's velocity deviation from mirror reflection velocity:

$$\Delta\vec{v} = v\left(i_{\perp 1} \cos\psi + i_{\perp 2} \sin\psi\right) \sin\theta , \tag{10}$$

The smallness of Δv value permits to transform the expression (9) to form:

$$J^{(rv)}_{\alpha m}(\vec{v}) = -\frac{1}{2\pi} \int\limits_{\Delta v_m > 0} \Delta v_m \Delta\vec{v} \cdot \nabla_v f_{\alpha 0}(\vec{v}) v_n f(\theta) d\theta\, d\psi , \tag{11}$$

which gives for vector $\vec{J}^{(rv)}_{\alpha}(\vec{v})$:

$$\vec{J}^{(rv)}_{\alpha}(\vec{v}) = -\frac{1}{2\pi} \int\limits_{\Delta v_m > 0} \Delta\vec{v}\, \Delta\vec{v} \cdot \nabla_v f_{\alpha 0}(\vec{v}) v_n f(\theta) d\theta\, d\psi . \tag{12}$$

The mean by angle ψ value of diadic product $\Delta\vec{v}\,\Delta\vec{v}$ is equal to:

$$\left| \Delta\vec{v}\, \Delta\vec{v} \right\rangle = v^2 \left(i_{\perp 1} i_{\perp 1} + i_{\perp 2} i_{\perp 2} \right) \frac{1}{2} \sin^2\theta = v^2 \left(\boldsymbol{\delta} - i_v i_v \right) \frac{1}{2} \sin^2\theta . \tag{13}$$

Thus:

$$\vec{J}_{\alpha}^{(rv)}(\vec{v}) = -\frac{\langle sin^2\,\theta\rangle}{4}\left(\boldsymbol{\delta}\,v^2 - \vec{v}\,\vec{v}\right)\cdot\nabla_v f_{\alpha}^{+}\left(\vec{v} - 2\,i_n v_n\right)v_n\,. \tag{14}$$

It follows from (2) in this case:

$$\mathbf{M}_{\alpha}^{[n+1]}\cdot i_n = \int\limits_{v_n\ge 0} f_{\alpha}^{+}(\vec{v})\mathbf{m}_{\alpha}^{[n]}(\vec{v})v_n d\,\vec{v} + \int\limits_{v_n<0} f_{\alpha}^{+}\left(\vec{v} - 2\,i_n v_n\right)\mathbf{m}_{\alpha}^{[n]}(\vec{v})v_n d\,\vec{v} - \\ - \int\limits_{v_n<0}\nabla_v\cdot\vec{J}_{\alpha}^{(rv)}(\vec{v})\,\mathbf{m}_{\alpha}^{[n]}(\vec{v})d\,\vec{v}\,. \tag{15}$$

The expression (15) can be transformed to the form:

$$\mathbf{M}_{\alpha}^{[n+1]}\cdot i_n = \int\limits_{v_n\ge 0} f_{\alpha}^{+}(\vec{v})\mathbf{m}_{\alpha}^{[n]}(\vec{v})v_n d\,\vec{v} + \int\limits_{v_n<0} f_{\alpha}^{+}\left(\vec{v} - 2\,i_n v_n\right)\mathbf{m}_{\alpha}^{[n]}(\vec{v})v_n d\,\vec{v} + \\ + \int\limits_{v_n<0}\vec{J}_{\alpha}^{(rv)}(\vec{v})\cdot\nabla_v\mathbf{m}_{\alpha}^{[n]}(\vec{v})d\,\vec{v}\,. \tag{16}$$

4. Boundary conditions for distribution function moments and kinetic equations

It is possible to write the boundary conditions for electrons dynamics equations using the expression (16). The boundary conditions for motion equation and motion flow equation:

$$\boldsymbol{\Pi}_{\alpha}\cdot i_n = i_n P_{\alpha}^{(nn)} + \eta_P\frac{v}{4}m_{\alpha}n_{\alpha}\vec{V}_{\alpha}\,, \tag{17}$$

$$\mathbf{Q}_{\alpha}\cdot i_n = \left(\vec{V}_{\alpha}i_n + i_n\vec{V}_{\alpha}\right)P_{\alpha}^{(nn)}, \tag{18}$$

where $\boldsymbol{\Pi}_{\alpha}$ – electron’s motion flow density tensor;

$\mathbf{Q}_{\alpha}$ – the third rank moment of distribution function [10];

v – electrons velocity mean absolute value;

η_P – electron’s motion relaxation factor:

$$\eta_P = \frac{3\langle sin^2\,\theta\rangle}{4}\,. \tag{19}$$

For velocity change equation and pressure tensor [10] equation boundary conditions it is possible to write:

$$\mathbf{P}_{\alpha}\cdot i_n = i_n P_{\alpha}^{(nn)} + \eta_P\frac{v}{4}m_{\alpha}n_{\alpha}\vec{V}_{\alpha}\,, \tag{20}$$

$$\mathbf{G}_{\alpha}\cdot i_n = -2\,\eta_P\frac{v}{4}m_{\alpha}n_{\alpha}\vec{V}_{\alpha}\vec{V}_{\alpha}\,. \tag{21}$$

where $\mathbf{G}_{\alpha}$ – the third rank moment of distribution function in motionless gas.

It can also be written for energy and temperature equations [8] boundary conditions:

$$q_{\alpha n} = \frac{Q_{\alpha}^{(11n)} + Q_{\alpha}^{(22n)} + Q_{\alpha}^{(nnn)}}{2} = 0\,, \tag{22}$$

$$g_{\alpha n} = \frac{G_{\alpha}^{(11n)} + G_{\alpha}^{(22n)} + G_{\alpha}^{(nnn)}}{2} = -\eta_P\frac{\langle v\rangle}{4}m_{\alpha}n_{\alpha}V_{\alpha}^{2}\,. \tag{23}$$

As it had to be expected energy density onto surface direction (22) in elastic reflection is equal to zero. Heat conduction (thermal energy flow density) is negative (23) – from the surface into plasma. The comparison of (20) and (23) shows that heat conduction flow is equal with the minus to "viscous forces work":

$$g_{\alpha n} = -\vec{V}_\alpha \cdot \mathbf{P}_\alpha \cdot i_n , \tag{24}$$

i.e. the part of electrons direct movement energy returns into plasma as thermal energy – the electrons temperature rises as the result of non-mirror reflection.

The complex $\boldsymbol{\delta} v^2 - \vec{v}\,\vec{v}$ in (14) has the following property:

$$\left(\boldsymbol{\delta} v^2 - \vec{v}\,\vec{v}\right)\cdot \vec{v} = \left(\boldsymbol{\delta} \cdot \vec{v}\right) v^2 - \vec{v}\left(\vec{v}\cdot\vec{v}\right) = \vec{v}\,v^2 - \vec{v}\,v^2 = 0 . \tag{25}$$

In any isotropic velocity distribution, when:

$$f_\alpha^+\left(\vec{v} - 2\,i_n v_n\right) = f_\alpha^+\left(\vec{v}\right) = f_\alpha^+\left(v\right), \tag{26}$$

we have:

$$\nabla_v f_\alpha^+\left(\vec{v} - 2\,i_n v_n\right) = \frac{\vec{v}}{v}\frac{d\ f_\alpha^+\left(v\right)}{d\,v} \tag{27}$$

or, according to (14), (25), (26):

$$\vec{J}_\alpha^{(r\,v)}\left(\vec{v}\right) = -\frac{\left\langle \sin^2\theta \right\rangle}{4}\frac{\left(\boldsymbol{\delta} v^2 - \vec{v}\,\vec{v}\right)\cdot\vec{v}}{v}\frac{d\ f_\alpha^+\left(v\right)}{d\,v} v_n = 0 . \tag{28}$$

So the disturbed part of distribution functions becomes to be equal to zero in isotropic distribution. The tendency exists of the transformation of electrons velocity distribution function into isotropic one because of electrons non-mirror reflection from the potential barrier near the surface.

5. Conclusions

The expressions obtained here (17), (18), (20), (21), (28) can and must be used in mathematic modeling of in electric propulsion devices of different types as the boundary conditions for differential equations in volume: motion equation, motion flow equation, Boltzman kinetic equation and their analogs depending on the level of detailing of processes description both in gas dynamics and kinetic approximation.

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